Endogenous Interest rate Wedge and Productivity Growth in an Integrated World

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Abstract

In one open OLG economy with endogenous interest rate wedge, higher productivity growth rate raises both marginal saving rate by households and investment by firms. If saving increases more than investment, one fast-growing experiences net total capital outflows. Therefore, the model can solve simultaneously two puzzles in international macroeconomics literature: Feldstein-Horioka puzzle and Allocation puzzle. The empirical evidence supports the theoretical implications.

JEL Classification: F21, F32, F41.

Key words: International Capital Flows, Endogenous financial friction, Financial development, Productivity growth rate.

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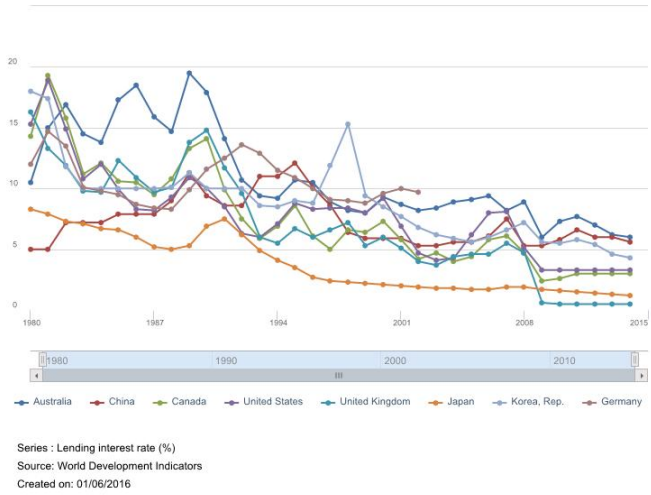
1 Introduction

The integration of developing economies into the world economy for the last decades is featured by two stylized facts.

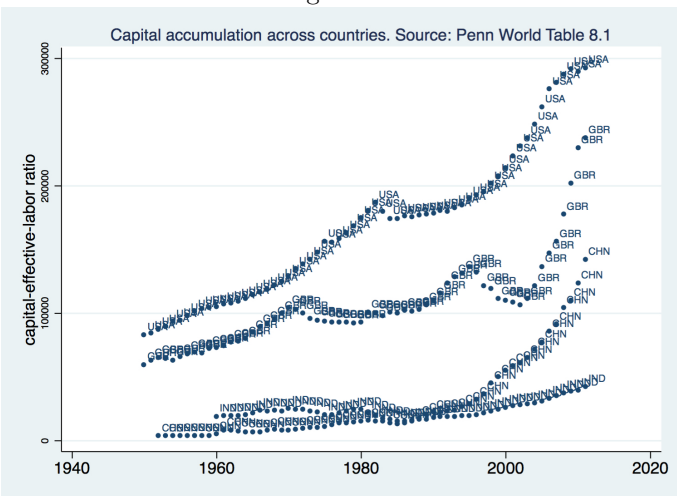
(1) The convergence of lending interest rate across countries (Figure 1). This fact along with the increasing financial integration process implies that the capital should flow across countries to establish the same capital-effective-labor ratio (let say, κ_t), since interest rate is equal to marginal product of capital in Neo-Classical growth model: $R_t = \alpha(\kappa_t)^{(\alpha-1)}$. Developing countries (China,

Figure 1:



India...), however, have lower capital-effective-labor ratio than advanced economy (United States, United Kingdoms...) even when the financial markets integrate together for last decades (Figure 2). This second fact contradicts to free mobility of capital and convergence of lending interest rate across countries.

Figure 2:



(2) The pattern of capital flows in some economies is reversal to prediction by theory. The advanced economies continuously run current account deficit which is increasingly financed by the emerging economies who grow faster and invest more. This fact was documented for the first time as Lucas paradox (Lucas 1990) which concerns about the capital flows does not go from capital-abundant to capital-scare economies. Recently, the allocation puzzle (Gourinchas and Jeanne 2013) provides the empirical evidence about the pattern of capital flows from fast-growing to slow growing economy.

Figure 3:

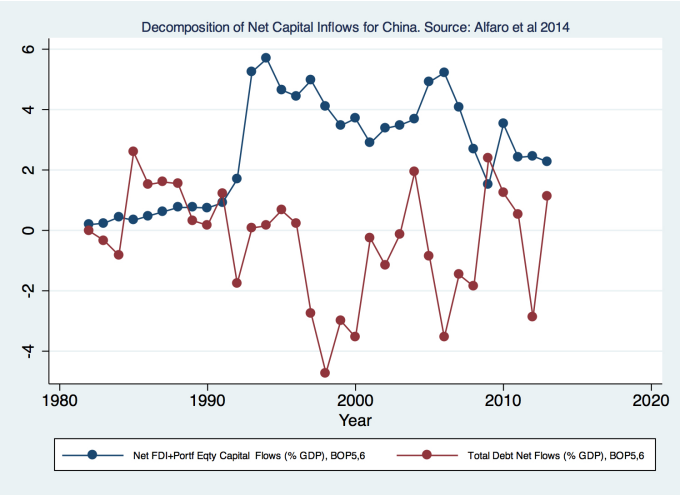
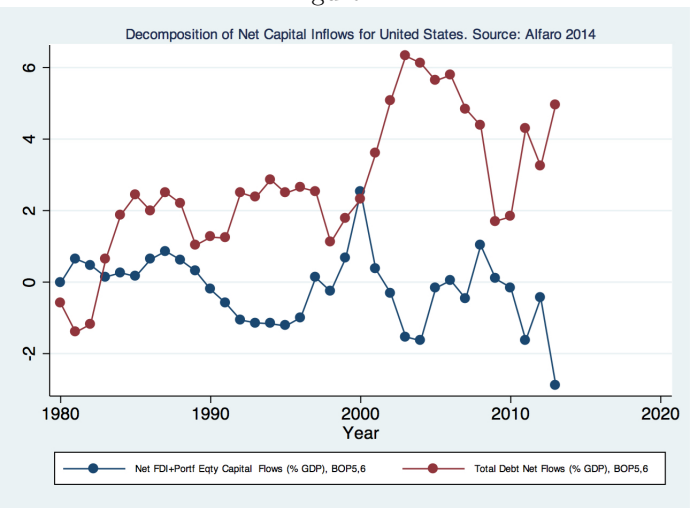


Figure 4:



In fact, the decomposition of net total capital inflows into sub inflows shows that fast-growing economies (like China) still experiences net portfolio and FDI investment inflows while slow-growing economies (like United States) experiences net investment outflows. Therefore, the puzzling fact remains on why households' savings increases in fast-growing economies (Feldstein-Horioka puzzle).

Both the Lucas paradox and Allocation puzzle raise new challenge for the Neo-Classical growth model, where capital should flows from high to low capital-effective-labor economies, taking the interest rate as the driving signal. For the literature on this up-hill capital flows, the financial friction usually is used

to depress the autarky interest rate in fast-growing economies, then, at integration, capital would flows out for a higher return on abroad (see Gourinchas and Rey 2014 as a recent survey). However, in these model, the excess saving in autarky economy also results in a high level of capital accumulation, because the saving is converted directly into investment, such that the interest rate equalizes to marginal product of capital. Then, if one type of financial frictions generates the excess saving, it would also generate the high capital accumulation to have low interest rate at equilibrium. On other word, the fact that low persistent capital-effective-labor ratio in capital-exporting economies is still a challenge for existing theoretical models. My objective is to fill in this gap.

The research question is why some developing and fast-growing economies have lower capital-effective-labor ratio than advanced slow-growing economies, even for a long time of integration into world economy. And, on that case, I seek the explanation why capital flows from the former to latter group. To answer these question, I build up a general equilibrium OLG model of one small open economy[2].

Our paper offers a theory of endogenous interest rate wedge. The capital taxation on bank's revenue generates the wedge between lending interest rate (exogenous given at world level) and domestic deposit interest rate. When the public expenditure is complementary for the private output, higher productivity growth rate requires higher public expenditure, which, in turn, requires higher capital tax rate and then, higher wedge. Therefore, the interest rate wedge is endogenous determined by productivity growth rate. The increase of wedge depresses the domestic deposit interest rate, then, raises the saving by households if the substitution effect dominates the income effect effect on the utility function. Moreover, higher productivity growth rate also raises the demand for investment. Therefore, if the productivity growth rate induces saving to increase more than investment, then, there would be net capital outflows. Based on this mechanism, I introduce the borrowing constraint for households and analyze the age-saving profile. Indeed, productivity growth rate only raises the saving by middle-age agents but depresses the saving by young and old agents. Then, the net total capital inflows relies on the reduction of saving by young and old agents.

On the low capital accumulation for developing economies, in the model with credit constraint for domestic firms, the capital-effective-labor is increasing on tightness of domestic credit constraint. This is because firm's rate of return on capital is higher than world lending interest rate in case of binding credit constraint. Then, relaxing the constraint would motivate domestic firms to borrow more from banks to build up capital, in seeking higher return on capital investment. If the Rest of World includes the advanced economies with unbinded credit constraint, then, one small open economy with too tight constraint would converge to a lower capital-effective-labor ratio than advanced economies' ratio in the long run. On other word, we can observe a persistent low capital accumulation in long run in one developing economy, even when its interest rate converges to world interest rate in a world integrated lending

[2]The benchmark model with credit constraint for firms is an extension of Song, Storesletten and Zilibotti's (2011), Gourinchas and Jeanne (2013). I employ the model of heterogenous agents (worker-saver and entrepreneur-borrower) with separation between lending and deposit interest rates in Song et al. Then, we introduce a saving wedge like Gourinchas et al, but endogenize that wedge by modeling it as one type of capital taxation to finance the public expenditure. For the model with borrowing constraint, we modifies the model by Coeurdacier, Guibaud, and Jin's 2015 to analyze the case of small open economy with endogenous interest rate wedge.

markets with free mobile capital.

The paper belongs to the literature on pattern of capital flows across developing and developed economies, where financial friction is used to generate the uphill capital flows from the former to latter group. Frictions that impact saving includes the quality of institution (Lucas 1990, Alfaro, Kalemli-Ozcan, and Volosovych 2008, Ventura and Martin 2015), the uninsurable idiosyncratic risks (Angeletos and Panousi 2008, Mendoza, Quadrini, and Rios-Rull 2009), the borrowing constraint (Coeurdacier, Guibaud and Jin 2015), the comparative advantage on providing the safe asset (Caballero, Farhi, and Gourinchas 2008). The financial frictions on investment includes the credit constraint for entrepreneurs (Matsuyama 2004, Martin and Taddei 2013), agency cost in economy with asymmetric information (Boyd and Smith 1998, Gertler and Rogoff 1990), efficiency of financial intermediation (Song, Storesletten and Zilibotti's 2011). There are two key features that distinguishes our theory to the rest of literature. The first is the endogenous financial friction which generates the channel for productivity growth rate affect the saving - instead of exogenous financial friction which is usually more severe in developing economies. Second, we focus on the heterogeneity both within and across countries. Within one country, the existence of workers and entrepreneurs generate two different source of savings, while, the difference of tightness of credit (or borrowing) constraint across countries stresses the interaction between financial development level and productivity growth rate on international capital flows.

Our paper also sheds light on the nexus between productivity growth, saving and long-run capital accumulation. The capital accumulation is affected by the the households' saving to insure against the idiosyncratic risk (Aiyagari 1994), bequest motive with uncertain lifetimes (Abel 1986), transition income shocks (Kraay and Ventura 2000). The first key distinguishable feature on our paper is that the marginal saving rate of households depends on the output growth rate. Indeed, the future output growth rate raises the wedge, which, depresses the future deposit interest rate as the expected return on saving for households. Then, for a higher future output growth rate, the saving supply by households could change both in quantity since future income increase and in marginal saving rate since future rate of return on saving decreases. Therefore, higher productivity or labor force growth rates does not necessary result in lower capital accumulation in long-run since they can raise the total saving as the supply of fund for capital accumulation. The raising of both saving and investment induced by higher productivity growth rate provides one explanation for the high correlation between savings and investment (Feldstein-Horioka puzzle). The second key feature is that we stress the role of financial development on long-run capital accumulation. Compared with the the rest of literature, we indicates the international capital flows as one important channel for level of financial development to affect the long-run capital accumulation. Indeed, one country with better financial development can attract capital inflows in term of debt or portfolio, FDI investment to build up the domestic capital stock.

Finally, our paper also makes contribution on the literature on interest rate spread: Elton, Gruber, Agrawal and Mann (2001), Dotsey (1998), Campbell and Shiller (1991), McMillan (2002), Friedman and Kuttner (1992), Bernanke and Blinder (1992), Ho and Saunders (1981)...Our different position is to emphasize the capital taxation rate on banking revenue as the key to create the gap between lending and deposit interest rate. Moreover, since the government in our model collects tax to finance a public expen-

diture which is complementary to private output, then, the future output growth rate is an important predictor for the interest rate spread, not vice verso like the rest of literature.

The paper is organized as follows. Section 2 provides the definition of interest rate wedge and the empirical analysis on the relationship between productivity growth and international capital flows. Section 3 and 4 lay out the theoretical models to explain the empirical evidence. Section 5 concludes and followed by Appendix.

2 Productivity growth rate and International capital flows: an empirical analysis

2.1 Interest rate wedge

The interest rate wedge is introduced by Gourinchas and Jeanne (2013) as the difference between the domestic interest rate (R^j) and world interest rate (R^w) which is exogenous given for one small open economy j.

$$R^j = (1 - wedge^j)R^w$$

They calibrate the wedge such that the net total capital flows predicted by model fits the data. The result is that the wedge is higher in African economies and lower for emerging Asian economies (for instance, China and Korea): countries with low productivity growth rate tax saving while countries with high productivity growth rate subsidy saving. Moreover, the wedge is decreasing on level of financial development and on the income per capita.

The saving wedge is one type of financial friction that creates the gap between the rental rate of capital for domestic firms (which equals to world interest rate for one small open economy without credit constraint) and the rate of return on savings for households. Since there exist no one world lending interest rate across countries, we can not compute the exact measure of saving wedge by data. In fact, we can only observe the deposit interest rate ($R_t^{d,j}$) which is the rate of return on saving and the lending interest rate ($R_t^{l,j}$) as the cost of loans supplied by banks in one country j at time t. Given that the lending interest rate tends to converge across countries (figure 1), however, we can use the domestic lending interest rate as the proxy for the "theoretical" world interest rate in the model.

$$wedge_t^j \equiv \frac{R_t^{l,j} - R_t^{d,j}}{R_t^{l,j}}$$

The wedge should be decreasing on output growth rate and on the income levels. Indeed, one country having high economic growth or income level can be affordable to finance better but costly financial system (Lucas 1988, Greenwood and Jovanovic 1990) or better institutional quality (Levine, Loayza and Beck 2000, Rajan and Zingales 1998, La Porta, Lopez-de-Silanes, Shleifer and Vishny 1997). Higher level of financial development can, in turn, reduce the financial friction level, such as the cost induced by the asymmetric information such as state-verifying cost, agency cost (Hubbard 1998, Stiglitz 2000, Bernanke and Gertler 1989, Boyd and Smith 1997, Gertler and Rogoff 1990). Moreover, higher level

financial development can push up the economic growth, because better financial system can allocate efficiently the capital to the most productive project (Beck and Levine 2004, King and Levine 1993). In a nutshell, on controlling for the level of financial development, economic growth and income level should have negative impacts on the interest rate wedge.

2.2 Empirical evidence

We carry out empirical test for a sample of 169 countries from 1980 to 2013. The objective is to use the interest rate wedge and financial development level to shed a new light on the relationship between productivity growth rate and net total capital inflows. We perform fixed-effect panel data analysis to capture the pattern of capital inflows over time. For instance, the capital inflows in China was reversal in 1990s from deficit to surplus or reduced substantially after world financial crisis 2007. Furthermore, the fixed-effect regression can control for the unobserved heterogeneity which are constant over time in each country, such as the quality of institution.

2.2.1 Productivity growth rate, International capital flows and Interest rate wedge

The first three columns in Table 1 investigate the relationship between interest rate wedge and productivity growth. Column 1 shows that the wedge is increasing on productivity growth. We repeat the same regression of the wedge on output growth rate and output per capita in column 2 and 3. The wedge is still increasing on output growth rate and output per capita. In all these columns, the wedge is decreasing on level of financial development, a confirmation that wedge can be one type of financial friction. The first three columns prove that the direct impact of productivity growth rate on interest rate wedge is positive, on controlling for the level of financial development.

The last two columns report the regression of saving and capital flows on interest rate wedge. Column 4 shows that the saving-output ratio is increasing on output growth rate and on interest rate wedge, while decreasing on the lending interest rate. Therefore, on average, in one country, higher interest rate wedge raises the total saving rate while higher interest rate reduce it. Indeed, 1 percent of increase in wedge leads to an increase of about 1.8 percent on the saving rate, which is even higher than the direct effect of income per capita on saving rate (0.173 percent). Column 5 reveals one new evidence that the wedge plays one important role on determining the net total capital inflows: 1 percent of increase on wedge reduces the net total capital inflows (or raises net total capital outflows) by 5.15 percent. These two columns prove that the wedge can play an important role on determining the pattern of saving and capital flows.

TABLE 1

Fixed-effect estimation results: regression of interest rate wedge ($wedge$), annual net total capital inflows ($negCA2y\_ifs$) and annual gross saving per gross national income ($S2GNI$) on time trend (t), real productivity growth rate ($TFPgrowth$), financial development mesured by domestic credit to private sector ($FinDev$), Chinn-Ito index of openness ($kaopen$), real output growth rate ($GDPpcgrowth$), output per capita ($GDPpc$), labor force growth ($Lgrowth$), interest rate wedge ($wedge$), real lending interest rate ($Rgross$)

VARIABLES	(1) wedge	(2) wedge	(3) wedge	(4) S2GNI	(5) negCA2y_ifs
Time trend (t)	0.0141***	0.0128***	0.0135***	0.0691***	-0.0625***
	(0.000525)	(0.000379)	(0.000592)	(0.0199)	(0.0199)
Productivity growth rate ($TFPgrowth$)	0.242***				-7.078**
	(0.0845)				(3.349)
Financial Development ($FinDev$)	-0.00164***	-0.00177***	-0.00219***		
	(0.000192)	(0.000169)	(0.000269)		
Openness ($kaopen$)	0.0133***	0.0195***			
	(0.00387)	(0.00307)			
Output growth rate ($GDPgrowth$)		0.0376**		0.173***	
		(0.0182)		(0.0321)	
Output per capita ($GDPpc$)			1.52e-06*		
			(8.10e-07)		
Labor force growth ($Lgrowth$)					-31.81**
					(15.14)
Interest rate wedge ($wedge$)				1.818**	-5.148***
				(0.914)	(0.856)
Real lending interest rate ($Rgross$)				-5.801***	6.027***
				(0.772)	(1.166)
Constant	-27.57***	-24.95***	-26.48***	-112.3***	122.6***
	(1.044)	(0.753)	(1.176)	(39.56)	(39.58)
Observations	2,254	3,721	4,026	3,477	2,243
R-squared	0.333	0.315	0.157	0.036	0.058
Number of countries	101	161	169	164	104

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

In a nutshell, the puzzling positive relationship between the interest rate wedge and productivity growth suggests that the wedge should illustrate much more than only the level of financial friction. Moreover, the wedge can be an important channel through which the productivity growth rate can affect the savings and capital flows.

2.2.2 Productivity growth rate, International capital flows and Financial development

Table 2 reports the regression result of net capital inflows and its decomposition on growth rate of total effective units of labor. In column 1, the growth rate of total effective units of labor reduce net total capital inflow (well documented as Lucas paradox 1990, Allocation puzzle 2013) but its impact depends on the level of financial development. In particular, with the same magnitude of increase in productivity growth rate, an increase of 1 percent on the financial development level would raise 31.34 percent of increase on the net total capital inflows. Column 2 shows that an increase of productivity growth rate leads to an increase of net total debt outflows and reserves accumulation. And the impact of productivity

growth on net total debt flows depends on the level of financial development. Indeed, with the same magnitude of increase in productivity growth rate, an increase of 1 percent on the financial development would raise 23.52 percent of increase on the net total debt inflows. In column 4, the regression of net FDI and portfolio inflows on productivity growth rate is insignificant. That result is just consistent with Alfaro et al 2008 where they find that the productivity growth can not explain the FDI and portfolio inflows. In column 5, I regress the net FDI and portfolio inflows on the output per capita growth rate. The result shows that the net FDI and portfolio inflows is increasing on the output per capita growth rate. In both columns, however, the interaction of financial development with productivity growth rate and with output per capita growth rate still have positive impacts on the net FDI and portfolio inflows. In a nutshell, higher productivity growth rate reduces net total capital inflows and this reduction depends on the level of financial development.

TABLE 2

Fixed-effect estimation results: regression of annual net total capital inflows ($negCA2y\_ifs$) and its decomposition of annual net total debt inflows ($TDebtNetF2y\_ifs$), annual reserves and related items accumulation ($ResRelF2y\_ifs$), annual net Portfolio and FDI inflows ($EqtNetF2y\_ifs$) on time trend (t), financial development measured by domestic credit to private sector ($FinDev$), growth rate of total effective units of labor ($TFPLgrowth$), GDP per capita growth rate, and Chinn-Ito index of openness ($kaopen$).

	(1)	(2)	(3)	(4)	(5)
VARIABLES	negCA2y_ifs	TDebtNetF2y_ifs	ResRelF2y_ifs	EqtNetF2y_ifs	EqtNetF2y_ifs
Time trend (t)	-0.167***	-0.194***	0.0823***	0.0546***	0.100***
	(0.0210)	(0.0288)	(0.0161)	(0.0131)	(0.0117)
Financial development	0.0347***	0.0242**	-0.0218***	-0.00889**	-0.00968**
($FinDev$)	(0.00695)	(0.00949)	(0.00533)	(0.00432)	(0.00463)
Growth rate of total effective	-40.04***	-42.66***	10.91***	2.185	
units of labor ($TFPLgrowth$)	(3.799)	(5.184)	(2.910)	(2.648)	
$FinDev$ * $TFPLgrowth$	31.34***	31.69***	0.428	5.585**	
	(3.328)	(4.540)	(2.549)	(2.333)	
Openness ($kaopen$)	0.379**	0.978***	0.567***	0.262**	0.0559
	(0.169)	(0.232)	(0.130)	(0.106)	(0.107)
GDPpc growth rate					0.0905***
($annual\_ygr05wb$)					(0.0190)
$FinDev$ * $annual\_ygr05wb$					0.0446***
					(0.0130)
Constant	334.7***	386.8***	-163.6***	-107.1***	-198.0***
	(41.74)	(57.15)	(31.97)	(26.01)	(23.19)
Observations	2,765	2,762	2,765	2,709	4,283
R-squared	0.078	0.050	0.035	0.019	0.034
Number of countries	105	105	105	105	170

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

2.2.3 Capital accumulation and Financial development

Table 3 examines the role of financial development on determination of capital-effective-labor and output-effective-labor ratios. In column 1, higher level of financial development increases the long-run capital

accumulation, as measured by the capital-effective-labor ratio. In column 2, financial development level still has positive impact on capital accumulation, even when we control for the productivity growth rate, government_size and human capital [3]. Column 3 and 4 shows that the financial development level has positive impact on output-effective-labor ratio, after controlling for the productivity growth rate, public expenditure and human capital. On final words, on average across countries, better financial market increases the long-run output by raising the capital accumulation.

TABLE 3

Fixed-effect estimation results: regression of output-effective-labor ratio and capital-effective-labor ratio on time trend (t), financial development measured by domestic credit to private sector ($FinDev$), productivity growth rate ($TFPgrowth$), index of human capital per person based on years of schooling (Barro/Lee, 2012) and returns to education (Psacharopoulos 1994) ($human\_capital$), public expenditure per output ratio ($government\_size$)

	(1)	(2)	(3)	(4)
VARIABLES	K2AL	K2AL	Y2AL	Y2AL
Time trend (t)	957.4***	460.9***	331.3***	196.2***
	(208.3)	(97.15)	(12.24)	(21.51)
Financial development ($FinDev$)	193.8***	215.6***	45.84***	57.37***
	(68.88)	(17.81)	(4.465)	(3.943)
Productivity growth rate ($TFPgrowth$)	-60,364***	-47,758***	-8,395***	-5,621***
	(19,285)	(8,157)	(2,058)	(1,806)
$Human\_capital$		26,841***		6,090***
		(4,190)		(927.5)
$Government\_size$		652.6***		-146.7***
		(123.5)		(27.34)
Constant	-1.824e+06***	-910,739***	-629,798***	-373,431***
	(414,271)	(184,902)	(24,312)	(40,933)
Observations	3,049	2,988	3,049	2,988
R-squared	0.235	0.287	0.339	0.418
Number of countries	109	109	109	109

Note: robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Capital-effective-labor ratio equals to capital stock at constant 2005 national prices (in million USD) divided by total effective units of labor, which is the productivity level at constant 2005 national prices multiplied by number of people engaged. Output-effective-labor ratio equals to GDP at constant 2005 national prices (in million USD) divided by total effective units of labor.

The empirical analysis reveals one puzzling fact that the Interest rate wedge is increasing on productivity growth rate. This evidence suggest that the wedge illustrates much more than just level of financial friction. And the wedge is an important channel for productivity growth rate affects the cross-border capital flows. Moreover, the financial development level not only affects positively the long-run capital accumulation but also determines the impact of productivity growth rate on net capital inflows.

In the next sections, we would develop a theory of relationship between productivity growth rate and

[3] Levine, Loayza and Beck (2000a, b) find that the human capital and government size have positive impact on capital accumulation and output

international capital flows taking into account the interest rate wedge and financial development level to be consistent with these empirical stylized facts.

3 Small open economy with credit constraint

The model is one small open economy with exogenous world lending interest rate ($R^{l,w}$). At each time period t, a constant number of N new born workers and a measure μN of new born entrepreneurs live for only two periods: young and old. There exists heterogeneous skill across agents: the worker only supply labor force while the entrepreneur inherits the entrepreneurial skill from their parent (cf. Caselli and Nicola Gennaioli 2006). There is one homogeneous good, which is used for consumption and investment, and is traded freely and costlessly. Capital is free mobile between the country and the rest of world (ROW) but labor is immobile, and firms are subject to changes in productivity.

3.1 Production

The firm j employs the constant return to scale technology to produces one common good, used as numeraire for price.

$$y_t = k_t^{\alpha}(A_t n_t \hat{g}_t)^{1-\alpha} = \kappa_t^{\alpha}(A_t \hat{g}_t)^{1-\alpha} n_t = \bar{\kappa}_t^{\alpha} A_t n_t \hat{g}_t^{(1-\alpha)} \tag{1}$$

Where $0 < \alpha < 1$ and κ_t is the capital per unit of labor ratio, $\bar{\kappa}_t$ is the capital per effective unit of labor. Firms are symmetric, then capital-effective-labor ratio is common across firms. I employ the production function from Barro and Sala-i-Martin (2002) where public expenditure is a factor of production.

With full depreciation rate, the capital stock at period t is augmented by the self-saving of the old entrepreneur (who saves (s_{t-1}^E) a part of the management rate at youth) and her demand for loans (l_{t-1}) from banks at the end of period $(t-1)$: $k_t = s_{t-1}^E + l_{t-1}$.

The old entrepreneur is the owner of firm and is residual claimants on the profit. She would choose the demand for labor and loans to maximize the value of firm, taking the self-saving as given:

$$\Pi_t(s_{t-1}^E) = max_{\{n_t, l_{t-1}\}}\{y_t - m_t - w_t n_t - R^{l,w} l_{t-1}\} \tag{2}$$

where m_t is the wage rate for hiring her children as manager. I assume that[4] $m_t = \psi y_t$.

The labor market is assumed to be competitive so that worker earns the marginal product of labor, after the management rate has been paid.

$$w_t = (1-\alpha)(1-\psi)(\kappa_t)^{\alpha}(A_t \hat{g}_t)^{1-\alpha} \tag{3}$$

The net output is the fraction of output left after allocated for management and wage rates.

$$\bar{y}_t \equiv y_t - m_t - w_t n_t = \alpha(1-\psi) y_t = \alpha(1-\psi)(\bar{\kappa}_t)^{(\alpha-1)}(\hat{g}_t)^{(1-\alpha)} k_t \equiv \rho_t k_t \tag{4}$$

[4] In Song, Storesletten, and Zilibotti's 2011, by delegation, the ψ represents the fraction of output could be subtracted by the manager.

where I define the net return on capital ρ_t such that: $\rho_t k_t = \bar{y}_t$.

The entrepreneurs face the credit constraint when borrowing from banks: the value of loans could not excess a fraction θ of expected net output in the next period. The θ measures the level of financial development.

$$R^{l,w} l_{t-1} \leq \theta \bar{y}_t \tag{5}$$

With too tight credit constraint, the constraint is binding and the firms can not borrow such that the marginal product of capital equals to lending interest rate ($\rho_t \geq R^{l,w}$). Then, the demand for loans equals is as following[5] .

$$l_{t-1} = \frac{\theta \rho_t}{R^{l,w} - \theta \rho_t} s^E_{t-1} \tag{6}$$

With unbinded credit constraint, marginal product of capital equals to lending interest rate ($\rho_t = R^{l,w}$).

$$l_{t-1} = \left[\frac{\alpha(1-\psi)}{R^{l,w}} \right]^{1/(1-\alpha)} A_t n_t \hat{g}_t - s^E_{t-1} \tag{7}$$

With unbinded credit constraint, the self-saving of entrepreneur offsets the indirect financing source from banks. With binding credit constraint, however, entrepreneurs need to use their self-saving as collateral to borrow from banks. To borrow more, they need to save more. Moreover, the amount of borrowing is increasing on level of financial development.

Finally, the productivity grows at exogenous rates g^A_t so that $A_t = (1 + g^A_t) A_{t-1}$.

3.2 Households

Agents have the utility function $u(c) = \dfrac{c^{1-1/\sigma} - 1}{1 - 1/\sigma}$. I will focus on the case that the coefficient[6] of substitution satisfies: $0 < \sigma \leq 1$.

Let $(c^w_{y,t}, c^w_{o,t+1})$ denote the consumption of one worker when young and old, respectively. The lifetime utility of the worker at period t is:

$$u(c^w_{y,t}) + \beta u(c^w_{o,t+1}) \tag{8}$$

with the discount factor β satisfies $0 < \beta < 1$.

Let s^w_t denote the saving at the end of period t of a young worker. A worker born in period t faces the following sequence of budget constraints:

$$c^w_{y,t} + s^w_t = w_t \tag{9}$$

$$c^w_{o,t+1} = R^d_{t+1} s^w_t \tag{10}$$

The worker i born in period t earns the competitive wage rate w_t when young. He allocates the income between consumption and saving as deposit into bank. When old, he receives the deposit rate R^d_{t+1} as the rate of return on saving, then he consumes all available resource. Therefore, the saving by worker depends on the future deposit interest rate.

$$s^w_t = \frac{w_t}{1 + \beta^{-\sigma} (R^d_{t+1})^{(1-\sigma)}} \tag{11}$$

[5] Rewrite (5) as $R^{l,w} l_{t-1} = \theta \bar{y}_t = \theta \rho_t k_t = \theta \rho_t (s^E_{t-1} + l_{t-1})$. Then, l_{t-1} follows.

[6] This range of value is consistent with literature on asset pricing (Guvenen 2006 for one discussion) and many empirical papers (Hall 1988, Ogaki and Reinhart 1998, Yogo 2004) which estimate the coefficient of substitution below 1.

Let $(c^E_{y,t}, c^E_{o,t+1})$ denote the consumption of the entrepreneur j when young and old, respectively. The lifetime utility of the entrepreneur in period t is as following:

$$u(c^E_{y,t}) + \beta u(c^E_{o,t+1}) \tag{12}$$

Let s^E_t denote the saving at the end of period t of a young entrepreneur. The sequence of budget constraints for each entrepreneur born in period t:

$$c^E_{y,t} + s^E_t = m_t \tag{13}$$

$$c^E_{o,t+1} = \Pi_{t+1}(s^E_t) = \rho_{t+1} s^E_t + (\rho_{t+1} - R^{l,w}) l_{t-1} \tag{14}$$

The entrepreneur works as the manager to get the management rate when young. She allocates income between consumption and self-saving. At the end of youth period, she borrows an amount of loan l_t from the bank, augmented by his own saving to build up the capital stock k_{t+1} for the next period. At old, she runs the firm to get the net profit $\Pi_{t+1}(s^E_t)$. Then, she consume all the available resource.
With binding credit constraint, we plug the solution for l_t from (6) to get the net profit. And the self-saving by entrepreneur is solution for the utility maximization problem[7].

$$\Pi_{t+1}(s^E_t) = \frac{(1-\theta)\rho_{t+1} R^{l,w}}{(R^{l,w} - \theta\rho_{t+1})} s^E_t \tag{15}$$

$$s^E_t = \frac{m_t}{1 + \beta^{-\sigma}\left[\frac{(1-\theta)\rho_{t+1} R^{l,w}}{(R^{l,w} - \theta\rho_{t+1})}\right]^{(1-\sigma)}} \tag{16}$$

Similarly, with unbinded credit constraint,

$$\Pi_{t+1}(s^E_t) = R^{l,w} s^E_t \tag{17}$$

$$s^{E,ub}_t = \frac{m_t}{1 + \beta^{-\sigma}(R^{l,w})^{(1-\sigma)}} \tag{18}$$

3.3 Bank

One representative bank earns profit Π^B_t by collecting the saving from workers (D^B_{t-1}) and provides the loans (L^B_{t-1}) to the firms. The domestic government imposes the tax τ_t on the total revenue on loans to firms. I assume the governments follows the residence principle on taxation: it only levies tax on the domestic bank, no matter the source of income.

$$\Pi^B_t = R^{l,w} L^B_{t-1} - R^d_t D^B_{t-1} - \tau_t R^{l,w} L^B_{t-1}$$

With constant return to scale, the net profit is zero for bank at equilibrium. Then, the capital tax rate is a wedge between domestic deposit interest rate and world lending interest rate.

$$R^d_t = (1 - \tau_t) R^{l,w} \tag{19}$$

Therefore, the domestic deposit interest rate is decreasing function on capital tax rate: $\partial R^d_t / \partial \tau_t < 0$.
Two market clearing conditions need to be held. First, saving market equalizes the deposit demand by

[7] On Song et al 2011, the return on saving of workers is also different to the return on saving by entrepreneurs because the latter own the firms.

domestic bank (D_t^B) and supply of saving by workers ($\Sigma_i s_t^{w,i}$). Second, the balance sheet of bank clears such that total supply of loans equalizes to total demand of deposit.

$$\begin{aligned} \Sigma_i s_t^{w,i} &= D_t^B \\ L_t^B &= D_t^B \end{aligned}$$

The lending market is assumed to integrate into the world economy, then, the domestic bank is free to choose to lend to domestic firms or foreign firms. Therefore, the total demand for loans by domestic firms $\Sigma_j l_t^j$ is not necessarily equal to total supply of loans L_t^B. And the inequality gives the raise of cross-border capital flows.

3.4 Government

The government collects tax to finance the public expenditure[8]: $G_t = T_t = \tau_t R^{l,w} L_{t-1}^B$. As Barro and Sala-i-Martin (2002), the government demands the level of public expenditure such that its marginal product ($\partial Y_t / \partial G_t$) equalizes to the price of public good (which equals to 1). Therefore, level of public expenditure equals to a constant fraction of domestic output ($Y_t = \Sigma_j y_t^j$).

$$G_t = (1-\alpha) Y_t \tag{20}$$

3.5 Equilibrium

Since the capital tax rate solves the budget balance of government, it is increasing on the output growth rate[9]: $\partial \tau_t / \partial (Y_t / Y_{t-1}) > 0$.

$$\frac{\tau_t R^{l,w}}{1 + \beta^{-\sigma_w} (R^{l,w})^{(1-\sigma)} (1-\tau_t)^{(1-\sigma)}} = \frac{1}{(1-\psi)} \frac{Y_t}{Y_{t-1}} \tag{21}$$

Because of the complementary between public expenditure and private output, higher output growth rate raises the demand for higher public expenditure (for instance, to provide more public infrastructure). Higher expenditure, in turn, requires government to raise capital tax rate.

With the existence of endogenous wedge, the marginal saving rate adjusts to output growth rate. Indeed, since domestic deposit interest rate is decreasing on capital tax rate, higher output growth rate depresses domestic deposit interest rate. This reduction of interest rate raises the marginal saving rate by households as the relative price of future consumption in term of present consumption becomes higher.

$$\frac{S_{t-1}^w}{Y_{t-1}} \equiv \frac{\Sigma_i s_{t-1}^{w,i}}{Y_{t-1}} = \frac{(1-\alpha)(1-\psi)}{[1 + \beta^{-\sigma} (R^{l,w})^{(1-\sigma)} (1-\tau_t)^{(1-\sigma)}]} \tag{22}$$

The law of capital accumulation depends on the tightness of credit constraint. For unbinded credit constraint, the domestic firms could borrow as much as to depress the domestic return on capital to be

[8]The balanced government's budget constraint differs our theory to the literature on international taxation where government involves in debt and has to issue the bond and increase tax to finance public debt

[9]In general, $\frac{\partial \tau_t}{\partial (Y_t / Y_{t-1})} > 0 \Leftrightarrow \sigma < \bar{\sigma} \equiv 1 + \frac{(1-\tau_t)}{\tau_t} (1 + \beta^{-\sigma} (R^{l,w})^{(1-\sigma)} (1-\tau_t)^{(1-\sigma)})$. Then, for $0 < \sigma < 1$, this condition always holds.

equal to world lending interest rate. For binding credit constraint, there exists a gap between domestic return on capital and world lending interest rate.

$$\text{Binding credit constraint: } K_{t+1} = \Sigma_j k_{t+1}^j = \Sigma_i (s_t^{E,j} + l_t^j) = \frac{R^{l,w}}{(R^{l,w} - \theta\rho_{t+1})} \Sigma_j s_t^{E,j}$$

$$\text{Unbinded credit constraint: } K_{t+1} = \left[\frac{\alpha}{R^{l,w}}\right]^{1/(1-\alpha)} A_{t+1} N \hat{g}_{t+1}$$

3.5.1 Long-run capital accumulation

I seek an explanation for the low persistent capital accumulation in one small open economy based on the tightness of credit constraint, as a measure of financial development level. For simplicity, we focus on case with $\sigma = 1$. Then, I can characterize the existence and uniqueness of long-run equilibrium with binding credit constraint for one small open economy.

Theorem. *There exists an unique, stable steady state for one small open economy. All else equal, at long-run equilibrium with binding credit constraint, one country would have lower the capital-effective-labor ratio if it has lower financial development level :* $\frac{\partial \bar{\kappa}}{\partial \theta} > 0$. *Furthermore,* $\lim_{\theta \to 1} (\frac{\bar{\kappa}}{\hat{g}}) = (\frac{\bar{\kappa}^w}{\hat{g}^w})$

In deed, the capital-effective-labor ratio in the autarky long-run equilibrium with unbinded credit constraint and binding credit constraint follows:

$$\bar{\kappa}^{ub} = \left[\frac{\alpha(1-\psi)}{R^{l,w}}\right]^{1/(1-\alpha)} \hat{g} \tag{23}$$

$$\bar{\kappa}^{b} = \left[\frac{\theta\alpha(1-\psi)}{R^{l,w}} + \frac{\beta}{(1+\beta)} \frac{\psi}{(1+g^A)}\right]^{1/(1-\alpha)} \hat{g} \tag{24}$$

The lending interest rate affects negatively the capital accumulation because it is the cost of capital for firms. The productivity growth rate, however, affects the capital-effective-labor ratio in long run in case of binding credit constraint. With binding constraint, domestic firms can not borrow as much as to span capital to higher productivity growth rate. Therefore, higher productivity growth rate reduces the capital accumulation.

I assume that rest of world (ROW) includes advanced economies with high level of financial development such that the credit constraint never binds. Then, ROW's capital-effective-labor ratio in long run is determined such that the rate of return on capital equalizes the world lending interest rate.

$$\bar{\kappa}^w = \left[\frac{\alpha(1-\psi)}{R^{l,w}}\right]^{1/(1-\alpha)} \hat{g}^w \tag{25}$$

Therefore, we can find one condition for that one small open economy would have lower capital-effective-labor ratio in long run than ROW's, even when the capital is free mobile.

Proposition 1. *Suppose that long-run productivity growth rate is the same between small open economy and ROW. If* $\theta < \bar{\theta} \equiv \left[\frac{\hat{g}^w}{\hat{g}} - \frac{\beta}{(1+\beta)} \frac{\psi}{(1+g^A)} \frac{R^{l,w}}{\alpha(1+\psi)}\right]$, *then* $\bar{\kappa}^b < \bar{\kappa}^w$.

Too low level of financial development generates the persistently low capital accumulation in the long run for one small open economy. Since firms can only borrow a quite low amount of loans from the banks, the small open economy could not accumulate enough capital to depress its domestic rate of return on capital to be equal to world lending interest rate.

Corollary. *The country-specific threshold on level of financial development is increasing on productivity growth rate, decreasing on world lending interest rate:* $\frac{\partial\bar{\theta}}{\partial g^A} > 0$; $\frac{\partial\bar{\theta}}{\partial R^{l,w}} < 0$.

An increase of productivity growth rate or decrease of world lending interest rate both raise the threshold on financial development for the small open economy. With higher threshold, one country tends to fall into the area of low long run capital-effective-labor ratio. When the world lending interest rate declines over last decades (figure 1), the country-specific threshold goes up, then, there would be more countries fall into the club of low persistent capital accumulation in long run.

Note that on the alternative set-up with representative firm, the capital accumulation on case of binding credit constraint turns to be independent to the productivity growth rate and the threshold on level of financial development does not depend on the world lending interest rate: $\bar{\kappa} = \frac{\theta(1-\alpha)}{R^{l,w}}\hat{g}$ and $\bar{\theta} = \frac{\alpha}{(1-\alpha)}$.

3.5.2 International capital flows

At time s, the current account is the difference between total saving supplied by workers ($S_s^w \equiv \Sigma_i s_s^{w,i}$) and total loans demanded by firms ($L_t \equiv \Sigma_j l_s^j$).

$$\frac{CA_s}{Y_s} = \frac{CA_{s-1}}{Y_{s-1}} + \left[\Delta\left(\frac{S_s^w}{Y_s}\right) - \Delta\left(\frac{L_s}{Y_s}\right)\right] \tag{26}$$

Therefore, the change of current account per output ratio between two time point is the results of changes in stock of savings by domestic workers subtracting the changes in stock of loans' demanded by domestic firms. In case of binding credit constraint[10], the current account at steady state is as following.

$$\left(\frac{CA_s}{Y_s}\right)|_{ss} = \frac{(1-\alpha)(1-\psi)}{[1+\beta^{-\sigma}(R^{l,w})^{(1-\sigma)}(1-\tau)^{(1-\sigma)}]} - \frac{\theta\rho}{(R^{l,w}-\theta\rho)}\frac{\psi}{1+\beta^{-\sigma}\left[\frac{(1-\theta)\rho R^{l,w}}{(R^{l,w}-\theta\rho)}\right]^{(1-\sigma)}} \tag{27}$$

Proposition 2. *The impact of productivity growth rate on current account*:
$\frac{\partial(S_t^w/Y_t)\,|_{ss}}{\partial g^A} > 0$; $\frac{\partial(L_t/Y_t)\,|_{ss}}{\partial g^A} > 0$ iff $\varepsilon_{g^A}^{\varphi^{s,E}} < \bar{\varepsilon}_{g^A}^{\varphi^{s,E}}$; $\frac{\partial(CA_t/Y_t)\,|_{ss}}{\partial g^A}$ is *indeterminacy*;
$\frac{\partial^2(CA_t/Y_t)\,|_{ss}}{\partial\theta\partial g^A} < 0$. Whereby, $\bar{\varepsilon}_{g^A}^{\varphi^{s,E}} \equiv \frac{R^{l,w}(\partial\rho/\partial g^A)}{(R^{l,w}-\theta g^A)}\frac{g^A}{\rho}$).

The endogenous interest rate wedge is the key channel through which the productivity growth rate affects the saving rate. High productivity growth rate requires the domestic government to raise capital tax rate to finance the increase of public expenditure which complements to the private output. Then, given world lending interest rate, higher tax generates higher wedge, which depresses the domestic deposit interest rate. Therefore, the worker would raises their saving rate in expecting that the future return on saving is

[10]The analysis for case of unbinded credit constraint is similar.

lower. Without the endogenous wedge, the productivity growth rate can not affect the marginal saving rate.

The loans-output ratio raises for an increase of productivity growth rate, only if the elasticity of marginal saving rate by entrepreneur w.r.t the productivity growth rate is small enough. Indeed, with binding credit constraint, the productivity growth rate affects loans-output ratio by two channels. First, it raises rate of return on capital which raises loans demand. Second, it reduces the self-saving, since saving is decreasing on rate of return for the small value of substitution coefficient $0 < \sigma < 1$.

The change in current account induced by productivity growth rate is ***indeterminacy***. Indeed, one country experiences an increasing marginal saving rate dued to the increase of the interest rate wedge and also an increasing loans-output ratio dued to an increase of rate of return on capital. Therefore, if the saving-output ratio increases more than the loans-output ratio, higher productivity growth rate leads to an increase of net total capital outflows. On the case that the marginal saving rate is unaffected by the productivity growth rate (such as with log utility), an increase of productivity growth rate turns the current account turns to be deficit since only loans-output ratio increases. Therefore, without the endogenous wedge, the productivity growth rate only raises the investment demand, then net capital inflows as implication of Neo-Classical growth model (Gourinchas and Jeanne 2013).

Another important result is that the impact of productivity growth rate on current account depends on the level of financial development. Given the same magnitude increase of productivity growth rate, one small open economy with higher level of financial development would experience more net capital inflow. Relaxing the credit constraint, the domestic firm can borrow more to finance more productive investment. Since saving is unaffected, there would be an increase of net capital inflow to serve the increase of domestic investment demand.

4 Small open economy with borrowing constraint

The impact of the interaction between productivity growth rate and financial development level on savings is silent in the previous model, since there is only the credit constraint for firm. On this section, I study that issue by introducing the borrowing constraint for workers on a three-period OLG model. The tightness of borrowing constraint can be interpreted as the level of financial development for one country. I also introduce the population growth rate to capture the demographic changes[11].

Agent lives for three periods. Let $z \in \{y, m, o\}$ denote the young, middle and old age respectively. At each time t, there are L_t^y units of labor provided by young workers and L_t^m units of labor provided by middle-age workers. Then, total units of labor supplied at time t: $N_t = e_t L_t^y + L_t^m$, whereby, $e_t < 1$ measures the relative efficiency of young to middle-age workers. The labor force and productivity (A_t) grow with exogenous rates $L_t^y = (1 + g_t^L)L_{t-1}^y$ and $A_t = (1 + g_t^A)A_{t-1}$.

[11] The model is an extended version of Coeurdacier, Guibaud and Jin's 2015 where agents live three-periods and there exists the borrowing constraint for young agent. We introduce the endogenous interest rate wedge in their model and focus on the small open economy case.

4.1 Production

One representative firm hires labors (N_t) and capital (K_t) to maximize the profit, using one constant return to scale production function with public expenditure as one factor of production.

$$\Pi_t = K_t^{\alpha}(A_t N_t)^{(1-\alpha)}\hat{g}_t^{(1-\alpha)} - R^{l,w}K_t - w_t N_t + (1-\delta)K_t \tag{28}$$

With depreciation rate $0 < \delta < 1$, capital stock at time t is augmented by the investment from last period I_{t-1}.

$$K_t = I_{t-1} + (1-\delta)K_{t-1} \tag{29}$$

Labor market is competitive so that the wage rates equals to marginal product of labor. Free mobile of capital implies that domestic firm invests until the marginal product of capital equals to world lending interest rate. Therefore, the capital-effective-labor ratio (κ_t) is constant: $\kappa = \left[\frac{\alpha}{R^{l,w} - 1 + \delta}\right]^{1/(1-\alpha)}$.

$$K_t = \left[\frac{\alpha}{R^{l,w} - 1 + \delta}\right]^{1/(1-\alpha)} A_t[e_t L_t^y + L_t^m]\hat{g}_t \tag{30}$$

$$w_t^y = e_t(1-\alpha)A_t\kappa^{\alpha}\hat{g}_t^{(1-\alpha)} \tag{31}$$

$$w_t^m = (1-\alpha)A_t\kappa^{\alpha}\hat{g}_t^{(1-\alpha)} \tag{32}$$

4.2 Households

Let c_t^z denote the consumption of an agent belonging to generation z. The life time utility of a consumer born at time t is as following.

$$u(c_t^y) + \beta u(c_{t+1}^m) + \beta^2 u(c_{t+2}^o) \tag{33}$$

with the utility function $u(c) = \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma}$. The intertemporal elasticity of substitution coefficient satisfies $0 < \sigma < 1$.

Agent born at time t earns the competitive wage rate w_t^y when young and w_{t+1}^m when middle-age. Let a_{t+1}^y and a_{t+2}^m are net asset holding by an agent at young and middle-age respectively. The sequence of budget constraints for one agent born at t is:

$$c_t^y + a_{t+1}^y = w_t^y \tag{34}$$

$$c_{t+1}^m + a_{t+2}^m = w_{t+1}^m + R_{t+1}^d a_{t+1}^y \tag{35}$$

$$c_{t+2}^o = R_{t+2}^d a_{t+2}^m \tag{36}$$

$$a_{t+1}^y \leq -\eta\frac{w_{t+1}^m}{R_{t+1}^d} \tag{37}$$

At youth, agents can borrow to consume against the future income at middle-age, but there exists the borrowing constraint ($0 < \eta < 1$), which is tighter in the country with low level of financial development. At middle-age, they earn the competitive wage, repay their loans, consume and save. At old-age, they consume all available resources.

When the borrowing constraint is tight enough or the age-saving profile is steep enough, the constraint

is binding, then we find out the net asset position by young agents. The Euler equation that relates marginal utility at $(t+1)$ and $(t+2)$ gives the net asset position by middle-aged agents[12].

$$a^y_{t+1} = -\eta \frac{w^m_{t+1}}{R^d_{t+1}} \tag{38}$$

$$a^m_{t+2} = \frac{(1-\eta)w^m_{t+1}}{[1+\beta^{-\sigma}(R^d_{t+2})^{(1-\sigma)}]} \tag{39}$$

4.3 Banking system and Government budget constraint

One representative bank collects the saving and lends to firms. Given the world lending interest rate, the capital taxation on bank's revenue generates the wedge between lending and deposit interest rate: $R^d_t = (1-\tau_t)R^{l,w}$. Moreover, the government keeps the balanced budget.

$$\tau_t R^{l,w}\left[L^y_{t-1}a^y_t + L^m_{t-1}a^m_t\right] = G_t \tag{40}$$

4.4 Equilibrium

The interest rate wedge is the solution for the government's budget constraint and on the case of binding borrowing constraint, the wedge is increasing on domestic output growth rate[13] : $\dfrac{\partial \tau_t}{\partial(Y_t/Y_{t-1})} > 0$. Since the public expenditure is complementary for private output, higher output growth rate requires higher public expenditure. With balanced budget, government needs to raise the capital tax rate, which results in higher interest rate wedge.

Given the lending interest rate as world exogenous given, the deposit interest rate is negatively related to the interest rate wedge. With three-period living worker, we can decompose the total saving into age-profile saving:

$$\frac{S^y_t}{Y_t} = -\frac{(1+g^A_{t+1})(1+g^L_{t+1})}{1+e_t(1+g^L_t)}\frac{\eta(1-\alpha)}{R^{l,w}(1-\tau_{t+1})}\left[\frac{\kappa_{t+1}/\hat{g}_{t+1}}{\kappa_t/\hat{g}_t}\right]^\alpha \tag{41}$$

$$\frac{S^m_t}{Y_t} = \left[\frac{(1-\eta)}{1+\beta^{-\sigma}(R^{l,w})^{(1-\sigma)}(1-\tau_t)^{(1-\sigma)}} + \frac{\eta}{R^{l,w}(1-\tau_{t+1})}\right]\frac{(1-\alpha)}{[1+e_t(1+g^L_t)]} \tag{42}$$

$$\frac{S^o_t}{Y_t} = -\frac{(1-\eta)}{[1+\beta^{-\sigma}(R^{l,w})^{1-\sigma}(1-\tau_t)^{(1-\sigma)}]}\frac{(1-\alpha)}{(1+g^A_t)(1+g^L_t)}\frac{1}{1+e_t(1+g^L_t)}\left[\frac{\kappa_{t-1}/\hat{g}_{t-1}}{\kappa_t/\hat{g}_t}\right]^\alpha \tag{43}$$

[12]The F.O.C condition on the $(c^m_{t+1}, c^o_{t+2}, a^m_{t+1})$ $\dfrac{\partial u(c^m_{t+1})}{\partial c^m_{t+1}} = \beta\dfrac{\partial u(c^o_{t+2})}{\partial c^o_{t+2}}$. Since $c^m_{t+1} = (1-\eta)w^m_{t+1} - a^m_{t+2}$ and $c^o_{t+2} = R^d_{t+2}a^m_{t+2}$, then, we find a^m_{t+2}.

[13]Tax rate is solution for the equation:

$$\frac{(1-\eta)R^{l,w}}{1+e_{t-1}(1+g^L_{t-1})}\frac{Y_{t-1}/Y_t}{[1+\beta^{-\sigma}(R^{l,w})^{(1-\sigma)}(1-\tau_t)^{(1-\sigma)}]}{\tau_t}^2 + \left[\frac{\eta}{1+e_t(1+g^L_t)} - \frac{(1-\eta)R^{l,w}}{1+e_{t-1}(1+g^L_{t-1})}\frac{Y_{t-1}/Y_t}{[1+\beta^{-\sigma}(R^{l,w})^{(1-\sigma)}(1-\tau_t)^{(1-\sigma)}]}\right]\tau_t + 1 = 0$$

. Then the implicit function theorem implies that $\dfrac{\partial \tau_t}{\partial(Y_t/Y_{t-1})} > 0$.

The domestic firm invests until the marginal product of aggregate capital equals to the world lending interest rate net of depreciation rate[14].

$$\frac{I_t}{Y_t} = \left[\frac{\alpha}{R^{l,w} - 1 + \delta}\right]^{1/(1-\alpha)} \left[(1+g^A_{t+1})(1+g^L_t)\frac{1+e_{t+1}(1+g^L_{t+1})}{1+e_t(1+g^L_t)}\frac{\hat{g}_{t+1}}{\hat{g}_t} - 1 + \delta\right] \tag{44}$$

4.4.1 International Capital Flows

With endogenous interest rate wedge, productivity growth rate affects the net total capital inflows through both saving and investment. Moreover, I can decompose the total saving into age profile.

Proposition 2. *The impact of productivity growth rate on current account.*

$\frac{\partial(S^y_t/Y_t)\mid_{ss}}{\partial g^A} < 0$	$\frac{\partial(S^m_t/Y_t)\mid_{ss}}{\partial g^A} > 0$	$\frac{\partial(S^o_t/Y_t)\mid_{ss}}{\partial g^A} > 0$ iff $\varepsilon^{\varphi^S}_{g^A} < \bar{\varepsilon}^{\varphi^S}_{g^A} \equiv \frac{g^A}{(1+g^A)}$	$\frac{\partial(I_t/Y_t)\mid_{ss}}{\partial g^A} > 0$
$\frac{\partial^2(S^y_t/Y_t)\mid_{ss}}{\partial\eta\partial g^A} < 0$	$\frac{\partial^2(S^m_t/Y_t)\mid_{ss}}{\partial\eta\partial g^A} < 0$ iff $\varphi^S > \frac{(\beta R^d)^\sigma}{(1-\sigma)}$	$\frac{\partial^2(S^o_t/Y_t)\mid_{ss}}{\partial\eta\partial g^A} < 0$ iff $\varepsilon^{\varphi^S}_{g^A} < \frac{g^A}{(1+g^A)}$	$\frac{\partial^2(I_t/Y_t)\mid_{ss}}{\partial\eta\partial g^A} = 0$

The impact of productivity growth rate on marginal saving rate depends on the age profile. With higher growth rate, young agent borrows more while middle-aged agent save more. The old agent, however, raises the saving rate if the productivity growth rate is high enough ($\frac{\partial\bar{\varepsilon}^{\varphi^S}_{g^A}}{\partial g^A} > 0$). In a nutshell, if productivity growth rate increases, one country can run current account surplus only the saving rate by middle-age worker raises more than the increase of investment demand and more than decreases of saving rate by young and old workers.

Furthermore, the impact of productivity growth rate on current account depends on the level of financial development. For a given increase of productivity growth rate, in one country with better financial system, the young worker would save less and borrow more while the middle-age worker would save less if the world lending interest rate is small enough. Moreover, the old worker would save more only if the productivity growth rate is high enough. In sum, the interaction term between productivity growth rate and borrowing constraint can make one country to experience net capital inflow.

5 Conclusion

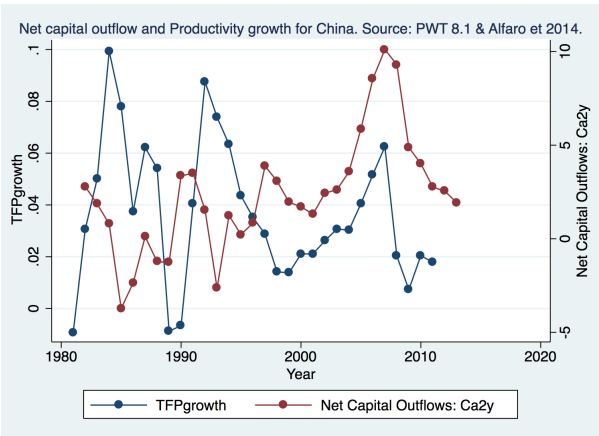
Why do developing economies in integrated world has lower capital-effective-labor than advanced economies, even with the free mobile of capital? and why the capital flows from former to latter group of countries? If the level of financial development (measured by tightness of credit constraint) of one developing country

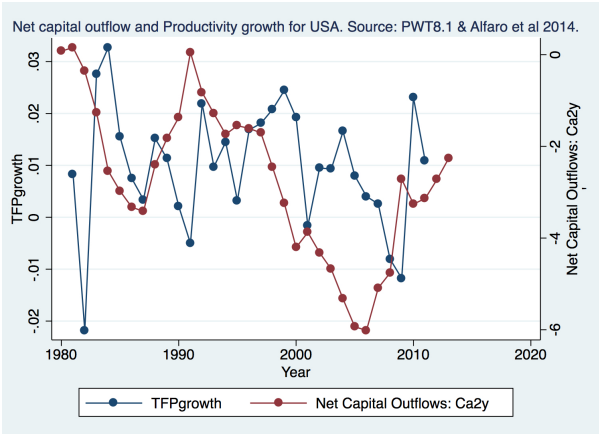
[14]In detail, $I_t = \left[\frac{\alpha}{R^{l,w} - 1 + \delta}\right]^{1/(1-\alpha)} \left[A_{t+1}(e_{t+1}L^y_{t+1} + L^m_{t+1})\hat{g}_{t+1} - A_t(e_tL^y_t + L^m_t)\hat{g}_t\right]$

is tow low (below a threshold), it would accumulate low capital accumulation in long run then advanced economies. Moreover, since the threshold of credit constraint is country specific and decreasing on the world lending interest rate, the decline of lending interest rate over last decades (figure 1) would push more developing economies into the club of low capital-effective-labor ratio.

Given that the lending interest rate is exogenous in integration, the capital flows is the result of interaction between saving-output and loans-output ratio. Both of them are exposure by increase of output growth rate. On one hand, the saving-output ratio goes up because of the decrease of deposit interest rate which is dued to the higher interest rate wedge. On other hand, the loans-output ratio increases because of an increase of rate of return on capital. Therefore, the capital flows out from low to high capital-effective-labor ratio economy only if the saving-output ratio increases more than loans-output ratio. The model implies that the current account could be turned from deficit to surplus or vice versa, depending on the relative magnitude of saving and investment adjustment conditional on the output growth rate. Therefore, the model could account for the reversal pattern of capital flows over time, like the case of China in 1990s or right after 2008 world financial crisis.

The graphs plots the productivity growth rate and current account per output ratio as an illustration for theirs positive relationship over time in both China as an emerging economy and United States as an advanced economy. It is quite clear that the change of productivity growth rate is an important predictor of net capital outflows, measured by current account per output ratio.





Furthermore, we show that the impact of productivity growth rate on saving and investment depends on the level of financial development. Given an increase in productivity growth rate, one country with better financial system would experience net capital inflows because the investment demand increases more (by relaxing the credit constraint for domestic firms) or because the saving reduces (by relaxing the borrowing constraint for domestic households). Indeed, for set-up with only credit constraint for domestic firms, the saving of worker is unchanged since it does not depends on the credit constraint. For the investment side, the increase of credit constraint allows firms to borrow more from banks, then the loans-output ratio goes up. Therefore, the economy needs to borrow from abroad to finance the increase in the demand of investment. In the extended set-up with borrowing constraint for households, we can trace down the source of saving adjustment by decomposing the saving into age-profile. For a given

increase of productivity growth rate, higher level of financial development can reduce the saving by all ages. This reduction of saving would, unambiguously, raises net total capital inflows since one economy needs to borrow abroad to finance the domestic demand of investment.

For future research avenue, an important extension could be to provide micro foundation for banking system. Indeed, the bank could lend more than the value of deposit that it receives. Moreover, the capital flows across banking system in different countries is driven by both risky and riskfree assets. Therefore, introducing the risky investment could generate the motivation for cross-border portfolio flows. Another line of research can focus on separation between public and private flows. To do that, we can allow the government to issue debt to finance the budget deficit.

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7 Appendix

7.1 Data Appendix

Data on international capital flows is extracted from the updated and extended version of dataset of net private and public capital flows constructed by Alfaro, Kalemli-Ozcan and Volosovych (JEEA 2014). The net capital inflows is measured by the minus of current account per output ratio, denoted by ($annual\_negCA2y\_ifs$). Then, the net capital inflows is decomposed into subflows of the debt ($annual\_TDebtNetF2y\_ifs$), reserves ($annual\_ResRelF2y\_ifs$), portfolio and FDI inflows ($annual\_EqtNetF2y\_ifs$).

$$annual\_negCA2y\_ifs = annual\_TDebtNetF2y\_ifs + annual\_EqtNetF2y\_ifs - annual\_ResRelF2y\_ifs$$

The productivity level at constant 2005 USD is explored by the Penn World Table 8.1 (updated version 2015). On this updated version, the productivity level is computed with three novelty. First, the share of factor income is different across countries and across time. Second, the physical capital stocks is decomposed by type of assets, each of which has one specific depreciation rate. These are combined with human capital which takes into account average year of schooling to result in total factor productivity. One key feature is that the production employed in PWT 8.1 is Harrod-neutral production function, which is also exactly the one we use in the model. Therefore, we prefer this measure on our analysis. In comparison with the approach by Gourinchas and Jeanne 2013, Caselli and Fayrer 2005, the computation of productivity level does not take into account the natural land and capital. However, on case that these natural capital does not change much over time, the productivity level from PWT 8.1 still validates the theoretical implication in our model because we use the growth rate of productivity, not its level. Moreover, Alfaro et al 2014 use the growth rate of output per capita, in stead of productivity growth rate, on explaining the pattern of capital flows across countries. That measure does not differ the source of growth rate induced by productivity to which induced by the capital accumulation. In our paper, the focal point would be on the role of productivity growth rate on pattern of capital flows across countries. But we also use an alternative measure as the output per capita growth rate from Alfaro et al database if necessary.

The output and capital stock at constant 2005 national prices (in millions USD) and number of people engaged are from Penn World Table 8.1(2015). Then, the capital-effective-labor ratio, a measure of capital accumulation, equals to the real capital stock divided by the total effective units of labor, which is the productivity level multiplied by the number of people engaged. And the output-effective-labor ratio equals to the real output divided by total effective units of labor.

The level of financial development is measured by there alternatives ratios from World Development Indicators: (1) Domestic credit to private sector as percentage of GDP; (2) Private credit by deposit money banks and other financial institutions to GDP and (3) Private credit by deposit money banks to GDP. We would focus on the first measure on the analysis, but, the regressions with two other alternative measures of financial development give the similar results.

The interest rate wedge is computed from the data on lending and deposit interest rate, which are from

World Development Indicators 2016. The terms and conditions attached to these rates differ by country[15]. Therefore, even we can construct one common world interest rate based on the weight that equalizes to the share of output of each country over the world economy, the result would be no accurate. Then, our strategy is to use directly the domestic lending interest rate to compute the interest rate wedge. This measure can still validate our set-up where the wedge illustrates the financial friction between the cost of capital that domestic firms have to pay and the rate of return on saving that domestic households receives.

Other independent variables includes (1) the government size, measured by the ratio of public expenditure to output ratio from WDI; (2) index of human capital per person based on years of schooling (Barro/Lee, 2012) and returns to education (Psacharopoulos 1994) from PWT 8.1. These two variables are proved to have positive impact on economic growth (Levine, Loayza and Beck (2000a, b)). And the openness of one country is measured by the Chinn-Ito index of capital account openness, which measures the extensity of capital controls based on the information from the IMF's Annual Report on Exchange Arrangement and Exchange Restrictions. It combines variables which indicates the presence of multiple exchange rate, the restrictions on current account transactions, restrictions on capital account transactions and requirement of the surrender of export proceeds.

TABLE 4

Descriptive Statistics for Variables

Variables	Obs	Mean	Std. Dev.	Min	Max
Net total capital inflows ($annual\_negCA2y\_ifs$)	4973	-3.751759	13.55661	-240.4958	304.0221
Net total debt inflows ($annual\_TDebtNetF2y\_ifs$)	4934	-.489817	18.0906	-388.1733	350.4075
Net portfolio and FDI inflows ($annual\_EqtNetF2y\_ifs$)	4720	2.867072	13.70745	-128.2574	381.6855
Net reserves and related Items Flow ($annual\_ResRelF2y\_ifs$)	4956	-.4757101	8.786179	-123.8326	217.7711
Gross saving per Gross national income ($annual\_S2GNI$)	4697	19.96542	13.14928	-181.269	107.8741
Interest rate wedge ($wedge$)	4219	.4750364	.2934043	-11.49206	.998
Real lending interest rate ($Rgross$)	4227	1.069825	.2187266	.0218794	8.897989
Finnancial Development ($FinDev$)	5378	43.17747	40.54411	.1982856	311.063
Output-effective-labor ratio ($Y2AL$)	3369	33582.95	30395.53	984.8127	276551.6
Capital-effective-labor ratio ($K2AL$)	3369	95312.51	86905.77	971.2172	685227.4
Productivity growth rate ($TFPgrowth$)	3259	.0010538	.0534612	-.6607364	.5259184
Labor force growth rate ($Lgrowth$)	3259	.0149039	.0156076	.2377644	2.211171
Output growth rate ($GDPgrowth$)	6034	.0748376	.1504582	-.072565	.2042816
Output per capita ($GDPpc$)	6244	8752.573	15722.04	64.35624	186242.9
Public expenditure-output ratio ($Government\_size$)	5501	16.76715	8.28374	0	156.5315
Chinn-Ito index of Capital account Openness ($kaopen$)	5462	.0736314	1.559328	-1.888895	2.389668
Human capital	4064	2.315615	.5850802	1.086181	3.618748

Table 1 provides summary statistics on the panel sample of 160 countries from 1980 to 2013. There is considerable variation for each variable. For examples, the net total capital inflows has a mean of -3.751759 with a standard deviation of about 13.5. The interest rate wedge has a mean of .4750364, with about .2934043 of standard deviation. Similarly, the productivity growth rate and output growth rate also exhibit quite large deviation, with means of .0010538 and .0748376 respectively. Thus, the data

[15] In details, the data appendix in WDI states that: "Countries use a variety of reporting formats, sample designs, interest compounding formulas, averaging methods, and data presentations for indices and other data series on interest rates".

set offers rich variation for exploring the relationship between productivity growth and net total capital inflows.

7.2 Definition of equilibrium

7.2.1 The temporary equilibrium

Given the variables from the previous period $(s_{t-1}^{E,i}, s_{t-1}^{w,i}, l_{t-1}^{i})$ and the exogenous world lending interest rate $(R^{l,w})$, the temporary equilirium of time t is defined by:

- The wage rate w_t, the management rate m_t, the saving wedge τ_t.
- The aggregate variables: $K_t, N_t, Y_t, \bar{\kappa}_t$.
- The individual variables: $c_{y,t}^{w,i}, c_{o,t}^{w,i}, c_{y,t}^{E,j}, c_{o,t}^{E,j}, s_t^{w,i}, s_t^{E,j}, l_t^j$

such that:

- Utility maximization subjected to the budget constraint for workers and entrepreneurs.
- Profit maximization for each firm j subjected to the credit constraint.
- The market clearing condition:
  Labor market clearing (immobile workers): $\Sigma_i n_t^i = N$
  Saving market clearing: $\Sigma_i s_t^{w,i} = D_t^B$
  Interbank market clearing: $L_t^B = D_t^B$
  Market clearing for consumption and investment good: $\Sigma_{i=1}(c_{y,t}^{w,i} + s_t^{w,i}) + \Sigma_{i=1} c_{o,t}^{w,i} + \Sigma_{i=1}(c_{y,t}^{E,i} + s_t^{E,i}) + \Sigma_{i=1} c_{o,t}^{E,i} + G_t = Y_t$

7.2.2 The inter-temporal equilibrium with perfect foresight

Given an initial capital stock, $\bar{\kappa}_0 = \frac{K_0}{(A_{-1}N_{-1})}$ and a sequence of public spending $(G_t)_{t\geq 0}$, an inter-temporal equilibrium with perfect foresight is a sequence of temporary equilibria that satisfy for all $t \geq 0$ the law of capital accumulation.

Small open economy with unbinded credit constraint.

$$K_{t+1} = \Sigma_j k_{t+1}^j = \Sigma_i (s_t^{E,j} + l_t^j) = \left[\frac{\alpha(1-\psi)}{R^{l,w}}\right]^{1/(1-\alpha)} A_{t+1}\hat{g}_{t+1}N \tag{45}$$

Small open economy with binding credit constraint.

$$K_{t+1} = \Sigma_j k_{t+1}^j = \Sigma_j (s_t^{E,j} + l_t^j) = \frac{R^{l,w}}{(R^{l,w} - \theta\rho_{t+1}^e)} \Sigma_i s_t^{E,j} \tag{46}$$

Where $\rho_{t+1}^e = \rho_{t+1}$ under the perfect foresight.

Lemma 1.

Proof. The Lagrange formula for the profit maximization by the old entrepreneur j at time t follows:

$$L(n_t^{j,D}, l_{t-1}^{j,D}, \lambda_t) = y_t^j - m_t - w_t n_t^{j,D} - R^{l,w} l_{t-1}^{j,D} + \lambda_t(\theta(y_t^j - m_t - w_t n_t^{j,D}) - R^{l,w} l_t^{j,D}) \quad (47)$$

$$= (1-\psi)y_t^j - w_t n_t^{j,D} - R^{l,w} l_{t-1}^{j,D} + \lambda_t(\theta((1-\psi)y_t^j - w_t n_t^{j,D}) - R^{l,w} l_t^{j,D}) \quad (48)$$

The FOCs:

$$(n_t^{j,D}) : (\frac{\partial(1-\psi)y_t^j}{\partial n_t^{j,D}} - w_t)(1+\lambda_t\theta) = 0 \quad (49)$$

$$(l_{t-1}^{j,D}) : (\frac{\partial(1-\psi)y_t^j}{\partial l_{t-1}^{j,D}})(1+\lambda_t\theta) - (1+\lambda_t)R^{l,w} = 0 \quad (50)$$

$$\lambda_t(\theta(y_t^j - m_t - w_t n_t^{j,D}) - R^{l,w} l_t^{j,D}) = 0 \quad (51)$$

Thus, 3 equations on 3 variables : $(n_t^{j,D}, l_t^{j,D}, \lambda_t)$.
Case (1): Unbinding credit constraint. $\lambda_t = 0$; $\dfrac{\partial(1-\psi)y_t^j}{\partial n_t^{j,D}} = w_t$; $\dfrac{\partial(1-\psi)y_t^j}{\partial l_{t-1}^{j,D}} = R^{l,w}$.

$$\Rightarrow l_{t-1}^{j,D} = \left[\frac{\alpha(1-\psi)}{R^{l,w}}\right]^{1/(1-\alpha)} A_t \hat{g}_t N - s_{t-1}^{E,j} \quad (52)$$

Case (2): Binding credit constraint. $\lambda_t \geq 0$; $\dfrac{\partial(1-\psi)y_t^j}{\partial n_t^{j,D}} = w_t$.

$$\theta\alpha(1-\psi)y_t^j = R^{l,w} l_{t-1}^{i,D} \Rightarrow l_{t-1}^{i,D} = \frac{\theta\rho_t}{(R^{l,w} - \theta\rho_t)} s_{t-1}^{E,i} \quad (53)$$

wherey, $\rho_t \equiv \alpha(1-\psi)(\bar{\kappa}_t)^{(\alpha-1)}(\hat{g}_t)^{(1-\alpha)}$ such that $\rho_t k_t^j = \alpha(1-\psi)y_t^j$. □

Theorem.

Proof. The law of capital accumulation in case of binding credit constraint for autarky economy follows:

$$K_{t+1} = \frac{R^{l,w}}{(R^{l,w} - \theta\rho_{t+1})} \Sigma_i s_t^{E,i} \quad (54)$$

For log utility for entrepreneurs, the marginal saving rate entrepreneur is independent to the interest rate, then the law of capital accumulation follows.

$$\Delta(\bar{\kappa}_{t+1}, \bar{\kappa}_t) = \bar{\kappa}_{t+1} - \frac{R^{l,w}}{[R^{l,w} - \theta\alpha(1-\psi)(\bar{\kappa}_{t+1})^{(\alpha-1)}(\hat{g}_{t+1})^{(1-\alpha)}]} \frac{\beta}{(1+\beta)} \frac{\psi}{(1+g_{t+1}^A)} (\bar{\kappa}_t)^\alpha (\hat{g}_t)^{(1-\alpha)} = 0 \quad (55)$$

1 Existence of steady state: $\lim_{\bar{\kappa}_{t+1}\to 0} \Delta(\bar{\kappa}_{t+1}, \bar{\kappa}_t) < 0$ and $\lim_{\bar{\kappa}_{t+1}\to\infty} \Delta(\bar{\kappa}_{t+1}, \bar{\kappa}_t) > 0$.

2 Uniqueness and Global stability
By implicit function theorem, $\dfrac{\partial\Delta(\bar{\kappa}_{t+1}, \bar{\kappa}_t)}{\partial\bar{\kappa}_{t+1}} \neq 0$, then we can express $\bar{\kappa}_{t+1}$ as a function of $\bar{\kappa}_t$. Let denote by: $\bar{\kappa}_{t+1} = h(\bar{\kappa}_t)$. Then, by implicit function theorem, $\dfrac{\partial\bar{\kappa}_{t+1}}{\partial\bar{\kappa}_t} > 0$. Therefore, $h(\bar{\kappa}_t)$ is a monotonic increasing function, then, it would converge to one positive value if any.

Moreover, $K_{t+1} < Y_t = (\bar{\kappa}_t)^{\alpha} A_t N_t (\hat{g}_t)^{(1-\alpha)} \Rightarrow \bar{\kappa}_{t+1} < (\bar{\kappa}_t)^{\alpha} (\hat{g}_t)^{(1-\alpha)}$. Then, $\frac{\bar{\kappa}_{t+1}}{\bar{\kappa}_t} = \frac{h(\bar{\kappa}_t)}{\bar{\kappa}_t} < (\bar{\kappa}_t)^{(\alpha-1)} (\hat{g}_t)^{(1-\alpha)}$. And,

$$\lim_{\bar{\kappa}_t \to \infty} \frac{\bar{\kappa}_{t+1}}{\bar{\kappa}_t} = 0 < 1 \tag{56}$$

Then, the function $h(\bar{\kappa}_t)$ intersect the 45 degree line. by two points 0 and $\bar{\kappa} > 0$. And it is concave since its graph would be below the 45 degree line for a large enough value of $\bar{\kappa}_t$, then we know for sure that $0 < \bar{\kappa} < \infty$.

For the comparative static, at steady state, $\bar{\kappa} = \left[\frac{\theta \alpha (1-\psi)}{R^{l,w}} + \frac{\beta}{(1+\beta)} \frac{\psi}{(1+g^A)} \right]^{1/(1-\alpha)} \hat{g}$.

Therefore, $\frac{\partial \bar{\kappa}}{\partial \theta} > 0$.

For the last part. ROW is assumed to have unbinded credit constraint, then:

$$\rho^w = R^{l,w} \Rightarrow \bar{\kappa}^w = \left[\frac{\alpha(1-\psi)}{R^{l,w}} \right]^{1/(1-\alpha)} \hat{g}^w \tag{57}$$

Then, at steady state, $\bar{\kappa} = \left[\theta (\frac{\bar{\kappa}^w}{\hat{g}^w})^{(1-\alpha)} + \frac{\beta}{(1+\beta)} \frac{\psi}{(1+g^A)} \right]^{1/(1-\alpha)} \hat{g}$. Therefore, $\theta (\frac{\bar{\kappa}^w}{\hat{g}^w})^{(1-\alpha)} < (\frac{\bar{\kappa}}{\hat{g}})^{(1-\alpha)}$.

But, when the credit constraint is binding, $\rho \geq R^{l,w}$, then, $(\frac{\bar{\kappa}}{\hat{g}})^{(1-\alpha)} \leq (\frac{\bar{\kappa}^w}{\hat{g}^w})^{(1-\alpha)}$. Therefore,

$$\theta (\frac{\bar{\kappa}^w}{\hat{g}^w})^{(1-\alpha)} < (\frac{\bar{\kappa}}{\hat{g}})^{(1-\alpha)} \leq (\frac{\bar{\kappa}^w}{\hat{g}^w})^{(1-\alpha)} \Rightarrow \lim_{\theta \to 1} (\frac{\bar{\kappa}}{\hat{g}}) = (\frac{\bar{\kappa}^w}{\hat{g}^w}) \tag{58}$$

□

Proposition 1.

Proof. $\bar{\theta}$ is defined such that $\bar{\kappa}^b = \bar{\kappa}^w$. Then, if $\theta < \bar{\theta}$, then, $\bar{\kappa}^b < \bar{\kappa}^w$. □

Corollary.

Proof. The proof follows directly by taking the derivative w.r.t $g^A, \lambda, R^{l,w}$.

$$\frac{\partial \bar{\theta}}{\partial g^A} = \frac{\beta}{(1+\beta)} \frac{\psi}{(1+g^A)^2} \frac{R^{l,w}}{\alpha(1+\psi)} > 0 \tag{59}$$

$$\frac{\partial \hat{\theta}}{\partial R^{l,w}} = -\frac{\beta}{(1+\beta)} \frac{\psi}{(1+g^A)} \frac{1}{\alpha(1+\psi)} < 0 \tag{60}$$

□

Lemma 2.

Proof. By implicit function theorem, let define G_t as equation defining the τ_t. Then, for $0 < \tau_t < \frac{1}{2}$ and $0 < \sigma < 1$,

$$\frac{\partial G_t}{\partial \tau_t} = \frac{(1-\eta) R^{l,w}}{[1 + e_{t-1}(1+g^L_{t-1})]} \frac{Y_{t-1}}{Y_t} \left[\frac{\partial \varphi^{s,w}_t}{\partial \tau_t} \tau_t (\tau_t - 1) + \varphi^{s,w}_t (2\tau_t - 1) + (\frac{\eta}{1 + e_t(1+g^L_{t-1})} - 1) \right] \tag{61}$$

$$\frac{\partial G_t}{\partial (Y_{t-1}/Y_t)} = \frac{(1-\eta) R^{l,w}}{[1 + e_{t-1}(1+g^L_{t-1})]} \left(\frac{Y_{t-1}}{Y_t} \right)^{-2} \varphi^{s,w}_t \tau_t (1 - \tau_t) > 0 \tag{62}$$

$$\frac{\partial \tau_t}{\partial (Y_t/Y_{t-1})} = -\frac{\partial G_t / \partial \tau_t}{\partial G_t / \partial (Y_{t-1}/Y_t)} > 0 \tag{63}$$

□

Proposition 2.

Proof. On the impact of productivity growth rate:

$$\Delta(\frac{S_t^w}{Y_t})\mid_{ss} \approx \frac{\partial(S_t^w/Y_t)\mid_{ss}}{\partial\tau}\frac{\partial\tau}{\partial g^A} > 0 \tag{64}$$

$$\Delta(\frac{L_t}{Y_t})\mid_{ss} \approx \frac{\partial(L_t/Y_t)\mid_{ss}}{\partial g^A} = \frac{\psi\theta\varphi^{s,E}}{(R^{l,w}-\theta\rho)}\left[\frac{(\partial\rho/\partial g)R^{l,w}}{(R^{l,w}-\theta\rho)} - \frac{\rho}{g^A}\varepsilon_{g^A}^{\varphi^{s,E}}\right] \tag{65}$$

$$\frac{\partial^2(CA_t/Y_t)\mid_{ss}}{\partial\theta\partial g^A} = -\frac{\beta}{(1+\beta)}\frac{\partial\rho}{\partial g^A}\frac{\psi R^{l,w}}{(R^{l,w}-\theta\rho)^3}(R^{l,w}-\theta\rho+2\rho) < 0 \tag{66}$$

whereby, $\varepsilon_{g^A}^{\varphi^{s,E}} \equiv -\frac{\partial\varphi^{s,E}}{\partial g^A}\frac{g^A}{\varphi^{s,E}} > 0$ as the elasticity of marginal saving rate by entrepreneur w.r.t the productivity growth rate.

On the impact of financial development level:

$$\frac{\tau R^{l,w}}{1+\beta^{-\sigma}(R^{l,w})^{(1-\sigma)}(1-\tau)^{(1-\sigma)}} - \frac{(1+g^A)}{(1-\psi)} = 0 \Rightarrow \frac{\partial\tau}{\partial\theta} = 0 \Rightarrow \Delta(\frac{S_t^w}{Y_t})\mid_{ss} = 0 \tag{67}$$

For $\sigma = 1$,

$$\Delta(\frac{L_t}{Y_t})\mid_{ss} \approx \frac{\partial(L_t/Y_t)\mid_{ss}}{\partial\theta} = \frac{(\partial(\theta\rho)/\partial\theta)R^{l,w}}{(R^{l,w}-\theta\rho)^2}\frac{\beta}{(1+\beta)}\psi > 0 \tag{68}$$

since: $\theta\rho = \left[(R^{l,w})^{-1} + \frac{\beta}{(1+\beta)}\frac{\psi}{\alpha(1-\psi)}\frac{1}{(1+g^A)}\frac{1}{\theta}\right]^{-1}$, then, $\frac{\partial(\theta\rho)}{\partial\theta} > 0$.

For $0 < \sigma \leq 1$,

$$\Delta(\frac{L_t}{Y_t})\mid_{ss} \approx \frac{\varphi^{s,E}}{(R^{l,w}-\theta\rho)}\left[\frac{\partial(\theta\rho)/\partial\theta}{(R^{l,w}-\theta\rho)}R^{l,w} - \rho\varepsilon_\theta^{\varphi^{s,E}}\right] \tag{69}$$

Whereby, $\varepsilon_\theta^{\varphi^{s,E}} \equiv -\frac{\partial\varphi^{s,E}}{\partial\theta}\frac{\theta}{\varphi^{s,E}} > 0$ as the elasticity coefficient of marginal saving rate w.r.t the credit constraint. Then, if the elasticity is relative small ($i.e, \varepsilon_\theta^{\varphi^{s,E}} < \bar{\varepsilon}_\theta^{\varphi^{s,E}} \equiv \frac{\partial(\theta\rho)/\partial\theta)R^{l,w}}{(R^{l,w}-\theta\rho)\rho}$), the loans-output ratio increases for the relaxed credit constraint.

Finally,

$$\frac{\partial^2(CA_t/Y_t)\mid_{ss}}{\partial\theta\partial g^A} = -\frac{\beta}{(1+\beta)}\frac{\partial\rho}{\partial g^A}\frac{\psi R^{l,w}}{(R^{l,w}-\theta\rho)^3}(R^{l,w}-\theta\rho+2\rho) < 0 \tag{70}$$

□

Proposition 3.

The age-profile saving in case of binding borrowing constraint follows:

$$S_t^y = L_t^y a_t^y = -\eta\frac{w_{t+1}^m L_t^y}{R_{t+1}^d} \tag{71}$$

$$S_t^m = L_t^m\left[w_t^m + (R^d-1)a_t^y - c_t^m\right] = (a_{t+1}^m - a_t^y)L_t^m = \left[\frac{(1-\eta)}{1+\beta^{-\sigma}(R_{t+1}^d)^{1-\sigma}} + \frac{\eta}{R_t^d}\right]w_t^m L_t^m \tag{72}$$

$$S_t^o = (R_t^d-1)L_{t-1}^m a_t^m - L_t^o c_t^o = (R_t^d-1)L_{t-1}^m a_t^m - R_t^d L_{t-1}^m a_t^m = -L_{t-1}^m a_t^m \tag{73}$$

The output and wage rates at time t follows:

$$Y_t = A_t[e_t L_t^y + L_t^m](\kappa_t)^\alpha \tag{74}$$

$$w_{t+1}^y L_{t+1}^y = (1-\alpha)A_{t+1}L_{t+1}^y(\kappa_{t+1})^\alpha \tag{75}$$

$$w_t^m L_t^m = (1-\alpha)A_t L_t^m(\kappa_t)^\alpha \tag{76}$$

Therefore, the age-profile saving-output ratio follows:

$$\frac{S_t^y}{Y_t} = -\frac{\theta(1-\alpha)}{R^{l,w}(1-\tau_{t+1})}\frac{A_{t+1}L_t^y(\kappa_{t+1})^\alpha}{A_t[e_t L_t^y + L_t^m](\kappa_t)^\alpha} = -\frac{(1+g_{t+1}^A)}{e_t + \frac{1}{(1+g_t^L)}}\frac{\eta(1-\alpha)}{R^{l,w}(1-\tau_{t+1})}\left(\frac{\kappa_{t+1}}{\kappa_t}\right)^\alpha \tag{77}$$

$$= -\frac{(1+g_{t+1}^A)(1+g_t^L)}{1+e_t(1+g_t^L)}\frac{\eta(1-\alpha)}{R^{l,w}(1-\tau_{t+1})}\left(\frac{\kappa_{t+1}}{\kappa_t}\right)^\alpha \tag{78}$$

$$\frac{S_t^m}{Y_t} = \left[\frac{(1-\eta)}{1+\beta^{-\sigma}(R_{t+1}^d)^{1-\sigma}} + \frac{\eta}{R_t^d}\right]\frac{(1-\alpha)A_t L_t^m(\kappa_t)^\alpha}{A_t[e_t L_t^y + L_t^m](\kappa_t)^\alpha} \tag{79}$$

$$= \left[\frac{(1-\eta)}{1+\beta^{-\sigma}(R_{t+1}^d)^{1-\sigma}} + \frac{\eta}{R_t^d}\right]\frac{(1-\alpha)}{1+e_t(1+g_t^L)} \tag{80}$$

$$\frac{S_t^o}{Y_t} = -\frac{(1-\eta)}{1+\beta^{-\sigma}(R_t^d)^{(1-\sigma)}}\frac{(1-\alpha)A_{t-1}L_{t-1}^m(\kappa_{t-1})^\alpha}{A_t[e_t L_t^y + L_t^m](\kappa_t)^\alpha} \tag{81}$$

$$= -\frac{(1-\eta)}{1+\beta^{-\sigma}(R_t^d)^{(1-\sigma)}}\frac{(1-\alpha)}{(1+g_t^A)(1+g_{t-1}^L)}\frac{1}{1+e_t(1+g_t^L)}\left(\frac{\kappa_{t-1}}{\kappa_t}\right)^\alpha \tag{82}$$

The derivatives just follow directly.