Providing for others can make altruistic risk-averse individuals act as risk-lovers

Cécile Aubert*

Abstract

We consider an individual who uses her risky income to provide for a loved one (as a child or aged ascendant). We study how the Arrow-Pratt measure of absolute risk aversion varies according to the degree of altruism of the parent, and to her ability to adjust allowances to realized income.

We show that the ability to give out money to loved ones can create some ‘self-insurance’. A risk-averse individual may even exhibit a negative Arrow-Pratt measure of risk aversion, once the endogeneity of gifts is taken into account. Under CARA utility functions, altruism has no impact on risk tolerance, implying a loss of generality. We derive implications with respect to the impact of poverty and of children’s age on risk-taking.

JEL Codes: D10, D64, D82.

Keywords: Altruism, Risk aversion, Parental love, Care for dependents.

1 Introduction

This article studies whether an individual becomes more or less risk averse when she uses her risky income to provide for loved ones. Income pooling within the household has been studied for couples in the absence of altruism (Chiappori and Reny, 2016). We are interested in a different

*University of Bordeaux (GREThA) and Toulouse School of Economics (LERNA). Address: TSE - LERNA, Manufacture des Tabacs, 21 allée de Brienne, F - 31 042 Toulouse, France. E-mail address: Cecile.Aubert@tse-fr.eu
problem: An altruistic individual derives utility both from the income she uses and the income she gives out to the person(s) she cares for, but with a different marginal utility. This affects the impact risk has on her utility. Giving may mitigate risk in such a way that, as we show, even a risk averse individual may have the preferences of a risk-loving one, once altruism is taken into account.

The analysis also applies to situations in which individuals can make gifts out of altruism (rather than a warm glow effect à la Andreoni, 1989, 1990). We will frame it below in terms of the love of a parent for her child\(^1\). Using a more or less risky revenue to provide for one’s child is indeed the situation of most parents. While there are reasons why parents may have different preferences than childfree individuals, as discussed below, we are interested in a particular mechanism that applies to any given preferences: Deriving utility from altruistic gifts create a secondary source of utility. This source is risky, as the first one (income), but in a way that is partially controlled by the individual (to the extent that she can choose the level of gifts she makes). The gift then affects the total risk exposure of the individual. We highlight that the overall effect may be counterintuitive, with this secondary source of risk potentially leading to more risk taking, and even to risk-loving behavior.

**Risk preferences for parents**

There are a number of reasons why parents may have different preferences to risk than single, childfree individuals. Cameron, DeShazo and Johnson (2010) show that parents attach a higher value than non-parents to improvements in their own health, which is related to an assessment of health risks. Chaulk, Johnson and Bulcroft (2003) also show how having children reduces financial risk tolerance. These results may be due to different effects.

First, individuals deciding to become parents likely have different characteristics than individuals willfully choosing to remain childless.

Second, one’s preferences may also become affected by the existence of children. Berg and Wynne-Edwards (2001) find lower levels of testosterone and cortisol in new and soon-to-be

\(^1\)The loved one may also be a dependant, sick, disabled or aged relative one cares for, or any other person as long as he is not earning risky income himself, which would add an exogenous source of risk to the model.
fathers; Gray, Yang and Pope (2006) find lower testosterone levels in fathers than in unmarried men and married non-fathers. And Gettler, McDade, Feranil and Kuzawa (2011) show in a longitudinal study that testosterone levels decrease after the birth of a child (so that the lower testosterone level is indeed a consequence of fatherhood). These hormonal changes tend to favor more ‘maternal’ behavior and reduce aggression with respect to children\(^2\). More generally, they affect preferences and behavior.

The riskiness of an investment or of a work position is likely viewed in a different light when one’s mate or children may suffer from bad outcomes. Feelings for loved ones can thus affect both utility and behavior, an impact we study here.

Last, and less directly, households may also exhibit less risk aversion than their members, due to sharing\(^3\). We are interested in a different situation, in which one individual only is making decisions. There is evidence indicating that risk aversion is heightened by the fact that the risky decision affects others. Bolton and Ockenfels (2010) present experimental evidence that individuals are less likely to choose a risky option over a safe one when choosing the former extends risk to a third party. In their experiment, the third party is randomly and anonymously chosen among participants (undergraduate students); one might expect a stronger – and possibly different – effect for friends and family members. In this experiment however, the participants could not choose the amounts obtained by the affected third party. In contrast parents are often able to adjust gifts or allowances made to family members to their own circumstances. This is an important element, as our theoretical study will highlight.

**Roadmap**

We consider the impact of altruism on risk aversion in a simple model in which an individual, a parent, (‘she’) can allocate her revenues between spendings on herself and gifts or transfers to another party (‘he’) she cares for. Her utility depends on her own spendings and on the utility

\(^2\)together with progesterone receptors mediation, Schneider et al. (2003). For the record, decreases in testosterone levels appear to be triggered by the smell of the infant, for marmosets at least, see Prudom et al. (2008).  

\(^3\)Sharing between members of a group is indeed expected to have such an effect; Chambers and Echenique (2012) provide a social welfare approach to aggregating preferences within a household, or another group, in a way that is consistent with reduced aggregate risk aversion.
of the individual she is providing for. She is thus altruistic à la Becker (1974). ‘Altruism’ can be parental love, or an empathy sufficiently strong to induce sizable gifts and concerns. The existence of altruism between parents and young children is supported by much experimental evidence (see e.g., Dickie and Messman, 2004, and Dickie and Gerking, 2007). It is well-known that taking into account altruism can lead to departures from standard results in economics (Becker, 1974, 1976, 1991, Buchanan, 1975, Barro, 1974). But as the literature uses additive separable preferences, it can say little as to the impact of altruism on preferences towards risk, as we will see. We allow for a more general specification, and consider additive separable and CARA preferences as special cases. We show that the family situation of an individual can make her more or less prone to engage in risky activities, depending on the shape of her preferences and on her status quo situation.

To isolate the impact of altruism from the monetary insurance that the loved individual could provide thanks to his own resources, we assume that he is passive: he may be a child, but also an aged, dependent, parent, or a non-working spouse with no personal revenues. Altruism changes the value of monetary revenues, since they can be used directly on oneself, but also to provide utility to the other individual, and thereby indirectly to oneself. As we will see, this can generate some sort of non-monetary insurance, even when the ‘loved one’ has no resources to provide.

Parents are taking risks not only for themselves but also for loved ones, which might compound their risk aversion in some sense. However, the gains obtained in case of a good outcome will also have more value if shared with loved ones, while the effect of losses may be mitigated by an appropriate sharing of resources: being able to adapt the amounts given out as gifts to loved

---

4 In the case of spouses, each individual may be able to provide monetary insurance to the household – see Attanasio et al. (2008) on female labor participation as an insurance device against idiosyncratic risks. Other effects may also come into play, as two adults will bargain and reach an agreement. It has been shown that attitudes to risk depend on whether or not decisions are taken by individuals or by couples (Bateman and Munro, 2005, 2009). For instance, Bateman and Munro (2009) find that couples may show a higher degree of risk aversion than individuals taken separately, with respect to dietary health risks. We abstract from these effects by considering a passive child, who has little voice in the economic decisions taken by the parent.

5 Dunn, Aknin and Norton (2008) argue, based on experimental evidence, that spending on others improves happiness better than spending on oneself.
ones allows more flexibility in dealing with risk. These features affect risk-taking in a complex way.

The remainder of this article is organized as follows. Section 2 describes a simple model, and Section 3 shows how the Arrow-Pratt measure of absolute risk aversion depends on altruism. Section 4 concludes.

2 The model

2.1 The household

Consider two individuals, a ‘parent’, $P$, and a ‘child’ $C$: $P$ is altruistic and cares for $C$. The parent, $P$, also has the ability to give out part of her income to others, and in particular to $C$ (the resources given to $C$ are ‘intra-household transfers’ or ‘gifts’). Individual $C$ cannot directly insure $P$. The parent owns some initial endowment (that needs not be explicited) and may participate in a lottery, e.g., a risky investment or a job yielding a risky wage, $y$.

The utility parent $P$ obtains from her risky income, $y$, depends on the utility level of the child, $c$. It is represented by a ‘von Neumann-Morgenstern’-type bi-variate function $u(y, \alpha c)$, where parameter $\alpha \geq 0$ is a measure of the degree to which individual $P$ cares for $C$. Writing the utility function in that way allows isolating the effect of variations in the weight that the parent gives to the welfare of individual $C$, from effects due to the shape of the function $c(.)$. Moreover, we assume that the utility functions are concave and increasing: $u_1(.,.) > 0$, $u_{11}(.,.) \leq 0$, $u_2(.,.) > 0$, $u_{22}(.,.) \leq 0$. The sign of $u_{12}(.,.)$ cannot be posited: Altruism may increase or decrease the marginal utility from income.

The utility of child $C$ will be represented by the von Neumann-Morgenstern concave utility function $c(.)$ ($c'(.) > 0, c''(.) \leq 0$). We will not consider altruism on the child’s part, in order to avoid cycles and because he is passive. Note that $c(.)$ may represent the perception by the parent of her child’s utility rather than her true utility.

We do not make assumptions regarding the relative degree of risk aversion of parents and children, to keep our analysis general. While there is growing research on preference transmission across parents and children, the existing evidence is not sufficient to rule out a variety of
situations. Kimball, Sahm and Shapiro (2009) find positive correlation between parents’ and children’s risk aversion using PSID data for the US. Munro and Tanaka (2014) find that teenagers tend to be more risk averse than their parents.

Suppose that the ‘parent’ \( P \) is the only member in her household – or is purely selfish and does not care for anyone else: \( \alpha = 0 \). We will refer to this situation as to that of a childless individual. Her utility function is \( u(y, 0) \), strictly concave and increasing in \( y \). The utility when not participating in a risky activity, \( u(0, 0) \), can be normalized to zero.\(^6\)

Let us now consider an ‘altruistic’ parent \( P \) who cares for the utility level obtained by \( C \) (\( \alpha > 0 \)) and gives him some amount \( x \). We normalize to zero the optimal choice \( x(0) \) made by \( P \) when her total resources are exactly equal to her initial endowment (the lottery yields no additional revenues, so that \( y = 0 \)). A negative gift \( x \) means that \( P \) gives fewer resources to \( C \) than in this benchmark situation.\(^7\)

2.2 The parent’s utility when sharing resources

The parent \( P \) will choose the amount \( x \) that she gives to child or dependent \( C \), once uncertainty has been realized, so as to maximize her own utility, satisfying:

\[
u_1(y - x, \alpha c(x)) = \alpha c'(x) u_2(y - x, \alpha c(x)).\]

The parent’s program is concave under a technical condition given in the appendix.\(^8\). We denote by \( x_y \) this solution (recall that \( x_0 = 0 \)), and will omit the argument when there is no risk of mistake.

The marginal utility of \( C \) is equalized to the marginal cost for the parent of the gift, weighted by the marginal utility that \( P \) derives from an increase in \( C \)’s welfare. The larger the degree of altruism, \( \alpha \), the larger the intra-household gift \( x \).

Let us denote by \( \tilde{u}(y) \) the utility for \( P \) of a transfer \( y \) under optimal sharing:

\[
\tilde{u}(y) \equiv \max_x u(y - x, \alpha c(x)) = u(y - x_y, \alpha c(x_y)).
\]

\(^6\)Note that \( c(0) \) may not equal zero, and \( u(y, \alpha c(0)) \neq u(y, 0 \times c) \) in general, so that the normalization does not involve a loss of generality.

\(^7\)This normalization allows to better compare the impact of risk, without adding level effects.

\(^8\)Note that when \( u_1(0, z) \) and \( c'(0) \) both tend to \(+\infty\), \( \forall z \), boundary solutions are not optimal.
Whatever the value of income $y$, $\tilde{u}(y)$ is weakly larger than $u(y, \alpha c(0))$. The possibility of transferring resources to $C$ increases the utility for $P$ of a given gain. It is as if the parent had access to two different technologies to ‘produce well-being’, i) spending directly on herself, and ii) using the child’s welfare as an intermediate good.

If $c(0) \geq 0$, then $\tilde{u}(y) \geq u(y, \alpha c(0)) \geq u(y, 0)$: A loving parent is necessarily happier than a childless individual when receiving the same income, if the child obtains a strictly positive utility in the absence of intra-household gifts ($c(0) \geq 0$). This can correspond to the situation of rather well-off families. However in poor, vulnerable or single-parent households, the utility of individual $C$ in the absence of intra-household transfers may be quite low, with $c(0) < 0$.

Last, one should note that as the optimal value of $x$ increases with $y$ (but by less than a factor one), the two arguments in the parent’s utility function are positively correlated at the optimal sharing. The study of behavior in the presence of risk is technically somewhat complicated for bi-variate functions, and correlation between the risks bearing on the two arguments matters (see Richard, 1975, for a seminal analysis of this issue, and Eeckhoudt, Rey and Schlesinger, 2007, Denuit and Eeckhoudt, 2010, Abbas, 2011).

3 Results: Altruism and risk aversion

Let us denote by $A^u(y) \equiv -\frac{u''(y)}{u'(y)}$ the Arrow-Pratt coefficient of absolute risk aversion with regard to monetary payments for an individual with utility function $u(y)$. For an altruistic parent, the relevant utility function to use is the value function $\tilde{u}(\cdot)$.

3.1 Altruism and risk aversion with fixed allowances

Assume first that the parent is not able to adjust her gift to her child to her circumstances. This may arise for poor individuals, as we discuss in the next section. This assumption is also of interest as it allows to see how concern for others modify risk aversion in the absence of adjustment in behavior.

Consider $x$ fixed. Then the parent is multi-variate risk averse whenever $u_{12}(y) \leq 0$ on the
relevant range. The Arrow-Pratt coefficient of absolute risk aversion is

\[ A^\tilde{\alpha}(y) = \frac{-u_{11}(y - x, \alpha c(x))}{u_1(y - x, \alpha c(x))}. \]

Given our normalization, it is coherent to consider a fixed level \( x = 0 \) as when \( y = 0 \). Then the coefficient becomes

\[ A^\delta(y) = \frac{-u_{11}(y, \alpha c(0))}{u_1(y, \alpha c(0))}, \]

to be compared with

\[ A^i(y) = \frac{-u_{11}(y, 0)}{u_1(y, 0)} \]

for a childless individual\(^9\). The comparison highlights the importance of the sign of \( c(0) \) and of cross-derivatives \( u_{12}(.,.) \) and \( u_{112}(.,.) \). In words, for the allowance corresponding to the initial endowment, is the child suffering from a low level of utility that deteriorates the utility of the loving parent (compared with being childless or selfish)? If so, \( c(0) \) is negative, a situation we refer to as a ‘vulnerable household’. This reduces the marginal utility from income spent on oneself whenever \( u_{12}(.,.) \) is positive.

It is easy to compute the marginal impact of a higher degree of altruism (or parental love) on the degree of risk aversion:

\[
\frac{\partial A^\delta(y)}{\partial \alpha} = \frac{-c(0)}{(u_1(y, \alpha c(0)))^2} \left[ u_{112}(y, \alpha c(0))u_1(y, \alpha c(0)) - u_{11}(y, \alpha c(0))u_{12}(y, \alpha c(0)) \right].
\]

- If the parent’s utility is additive separable \( (u_{12}(y, z) = 0 \text{ for all } y, z) \), then this derivative is null: Under additive separability, more altruism has no impact on the degree of risk aversion of the parent, when the latter is not able to adjust gifts to the child to her own circumstances (as we will see, this does not hold for optimally chosen gifts).

- If \( u_{12}(.,.) \) and \( u_{112}(.,.) \) are both positive, then more altruism reduces risk aversion when \( c(0) \geq 0 \), and increases it otherwise.

- If \( u_{12}(.,.) \) and \( u_{112}(.,.) \) are both negative, then more altruism reduces risk aversion when \( c(0) \leq 0 \), and increases it otherwise.

\(^9\)With an abuse of notations, we replace the superscript corresponding to the utility function by an \( i \) in the case of a childless individual, who therefore takes ‘individualistic’ decision.
It is important to stress that the direct impact of a concern for another on the concavity of the parent’s utility with respect to money does not allow to conclude as to the overall impact of altruism on the degree of risk aversion.

Result 1  Altruism has an impact on risk aversion even when it has no direct impact on the concavity of the parent’s utility function with respect to money, i.e., even when $u_{112}(.,.) \equiv 0$.
In that case, risk aversion is increased (resp. reduced) by altruism when $c(0)$ and $u_{12}(.,.)$ are of opposite signs (resp. of identical sign), so that the marginal utility of income spent on oneself is reduced (resp. increased) by parental love.

When the cross derivatives $u_{12}(.,.)$ and $u_{112}(.,.)$ are of opposite signs, the impact of altruism is ambiguous. More information can however be gained by comparing the degree of risk aversion of a caring parent with that of a childless or selfish individual.

Result 2  Consider situations in which the outcome of the lottery cannot be used to adjust the endowment of the child. Suppose that $u_{112}(.,.) \geq 0$.
Then, if $c(0)$ and $u_{12}(.,.)$ are both positive, altruism reduces the parent’s coefficient of risk aversion.
If $c(0) < 0$ (vulnerable household) and $u_{12}(.,.) \leq 0$, then altruism increases the parent’s coefficient of risk aversion.

In words, we consider a situation in which altruism has either no impact on the concavity of the parent’s utility with respect to money spent on herself, or it softens this concavity. Our results then mean that for relatively well-off households, if altruism enhances the enjoyment of money on oneself (the marginal utility of money increases with the concern for the child’s wellbeing, $u_{12}(.,.) > 0$), then more altruism tends to reduce risk aversion. To the contrary, in vulnerable households, a similar impact of altruism will tend to make the parent more risk averse. Indeed in the case considered, the parent knows that the well-being of the child will not be modified by the outcome of the lottery. The latter will directly affect the parent’s own consumption only, but the marginal value of this consumption depends (unless utility is separable) on the child’s well-being. Hence the opposite results obtained depending on whether the child is well or not.
As the parent is not able to adjust the gift she is making to her child, our results measure the direct impact of feelings on preferences, in the absence of any action by the parent to optimally adjust behavior to these preferences. If the parent was able to adjust allowances, as we assume below, a new effect would come into play.

### 3.2 Altruism and risk aversion with optimal allowances

Assume now that the parent can modify her child’s allowance according to realization of her income. Computations of derivatives for the optimal resource sharing yield that the degree of risk aversion is

$$A\tilde{u}(y) = \frac{-u_{11}(y-x, \alpha c(x))}{u_1(y-x, \alpha c(x))} + \frac{dx}{dy} \frac{u_{11}(y-x, \alpha c(x)) + \alpha c'(x) u_{12}(y-x, \alpha c(x))}{u_1(y-x, \alpha c(x))}$$

at the optimal sharing $x = x_y$.

The first term represents the coefficient of absolute risk aversion one would have if a variation in income $y$ did not imply a modification in $x$, as in the previous subsection. The ability to adjust $x$ enables individual $P$ to reach a higher utility level than in this fictitious situation; and the second term, that represents the impact of adjusting the gift $x$, is negative\(^{10}\), reducing risk aversion. In other words, the ability to choose the gift is an ability to buy an increase in one argument of the parent’s utility function by reducing the other argument, and this allows to better cope with risk.

**Result 3** Being able to optimally adjust the amount $x$ given to the child to the realized outcome reduces the Arrow-Pratt coefficient of absolute risk aversion of the parent.

Using the expression of $\frac{dx}{dy}$, one can further simplify the coefficient of risk aversion:

$$A\tilde{u}(y) = \frac{-u_{11}(y-x, \alpha c(x))}{u_1(y-x, \alpha c(x))} \left[ \alpha(c'(x))^2 \left( u_{11}(y-x, \alpha c(x)) u_{22}(y-x, \alpha c(x)) + u_{12}(y-x, \alpha c(x))^2 \right) -c''(x) u_{2}(y-x, \alpha c(x) u_{11}(y-x, \alpha c(x)) \right].$$

\(^{10}\)This is always true under the assumption that $\frac{dx}{dy} \geq 0$, i.e., $u_{11}(y-x, \alpha c(x)) - \alpha c'(x) u_{12}(y-x, \alpha c(x)) \geq 0$. 

10
This expression increases when the marginal utility of the child is large. A large value for \( c'(x) \) can happen because the child has little of his own in the absence of a gift, or because the initial endowment of the parent are too low to allow for large gifts.

To the contrary, the second and last term in the expression contributes to improving the risk tolerance of the parent. This is more so when the parent values much improvements in the child’s welfare (\( u_2(t - x, \alpha c(x)) \) is large), and if the child is very risk-averse (\( c''(x) \) is large). Indeed, gifts then provide the parent much value by avoiding bad outcomes for the child – this is more valuable if the parent is also very risk averse to begin with. Paradoxically, gifts then allow the parent to self-insure to such an extent that he may no longer exhibit risk aversion, even when he is initially very risk averse.

**Proposition 1** A risk averse individual can exhibit risk loving when she can transfer resources to loved ones and when the following condition on her utility function holds:

\[
\alpha(c'(x))^2[u_{11}(y - x, \alpha c(x))u_{22}(y - x, \alpha c(x)) + (u_{12}(y - x, \alpha c(x)))^2] < c''(x)u_2(y - x, \alpha c(x))u_{11}(y - x, \alpha c(x)).
\]

In this extreme case, gifts allow the individual to self-insure so well that she becomes risk-lover. Then, an altruistic individual has an unambiguously lower coefficient of absolute risk aversion than an individualistic one.

Comparing risk tolerance for individualistic persons and altruistic ones is made difficult by the fact that functions are bi-variate; yet one can show that the coefficient of absolute risk aversion is lower for an altruistic parent than for an individualistic person when the following conditions are simultaneously satisfied:

- \( u_{12}(\ldots) \geq 0 \) and \( c(\cdot) \geq 0 \) on the relevant range of income \( y \) – so that altruism increases the marginal utility of the income,
- and \( u_{111}(\ldots) \geq 0 \) and \( u_{112}(\ldots) \leq 0 \).

A particular example for the last two conditions is a utility function that is concave quadratic in \( y \) and multiplicative in \( c(x) \). Note that with \( u_{111}(\ldots) \geq 0 \), the parent’s preferences do not entail prudence with respect to money. These sufficient conditions for an altruistic parent to exhibit less absolute risk aversion that an individualistic one are thus clearly restrictive.
3.3 Common preferences

3.3.1 Additive separability

Assume linear separability of altruism, e.g. \( u(y, c) = h(y) + ac \) with \( h(.) \) a strictly concave function. Despite the restrictions it imposes, this specification is used in the very large majority of papers on altruism (see e.g., Barro, 1974, Bernheim and Stark, 1988).

Then an altruistic individual exhibits a lower coefficient of absolute risk aversion than an individualistic one when \( h'''(.) < 0 \) – this condition being the opposite case of the absolute prudence defined in Kimball (1990). This is a sufficient but not a necessary condition\(^{11}\).

If this condition is not satisfied – as for instance for absolute prudence, \( h'''(.) < 0 \) – an altruistic parent may still exhibit less absolute risk aversion than an individualistic one, if \( \frac{\alpha c''(x)}{h''(y-x)+ac''(x)} \) is sufficiently low. Note that under additive separability, the fact that \( P \) cares for the welfare of \( C \) and chooses gifts to maximize her utility implies that \( P \) always derives a (weakly) higher utility level from a revenue than a less altruistic parent.

3.3.2 CARA preferences

When considering risky contexts, it is common to assume that risk aversion does not depend on income, by choosing a CARA utility function. We consider below the consequences of making such an assumption with regards to the parent’s vnm function.

Assume that \( u(y, c) = -e^{-r(y-x+ac(x))} \), where \( r \) is the constant degree of risk aversion of the parent. Note that we assume here that \( r \) applies linearly to both components of the parent’s utility function. We would obtain similar qualitative results with a specification in which \( r \) applied only to monetary spendings as discussed in a footnote below. A quasi-linear formulation of the two arguments in the exponential is necessary for \( r \) to indeed measure risk aversion in the absence of altruism.

\(^{11}\)One has \( A^h(y) = \frac{-h''(y-x)}{h'(y-x)} + \frac{(h''(y-x))^2}{h'(y-x)[h''(y-x)+ac''(x)]} \), that is \( A^h(y) = A^h(y-x) \frac{\alpha c''(x)}{h''(y-x)+ac''(x)} \) at the optimal sharing \( x = x_y \). The multiplicative term \( \frac{\alpha c''(x)}{h''(y-x)+ac''(x)} \) is lower than one. Since the marginal utility from money, \( h'(.) \) is decreasing, we have \( A^h(y-x) < A^h(y) \) when the marginal utility from money is a (decreasing) concave function \( (h'''(.) < 0) \).
For interior solutions the sharing rule depends only on $\alpha$ ($x_y$ is independent of $y$, $\frac{dx}{dy} = 0$ and $\frac{dx}{d\alpha} > 1$). One can check that $u_{12}(y - x, \alpha c(x)) = r^2 \alpha u(y - x, \alpha c(x))$.

Parameter $r$ measures the Arrow-Pratt coefficient of risk aversion, whatever the degree of altruism. It is identical to that of an individualistic parent or childless individual (and independent from $\alpha$).\footnote{Let us assume instead that parameter $r$ applies only to monetary spendings: $u(y, c) = -e^{-r(y-x) - \alpha c(x)}$. Then the solution $x_y$ is characterized by $c'(x) = \frac{r}{\alpha}$, provided $\alpha \neq 0$, and $x = 0$ for $\alpha = 0$. The optimal gift thus remains independent of $y$, and the Arrow-Pratt coefficient of risk aversion is still measured by $r = \frac{-u''(y)}{u'(y)} = \frac{-u_1(t-x_y, \alpha c(x))}{u_1(t-x_y, \alpha c(x))}$. Note that we could also have applied a weakly concave, increasing function to the ‘altruistic part’, $\alpha c(x)$ without changing our qualitative results.}

**Result 4** For CARA preferences, altruism does not affect the attitude towards risk of the altruistic parent.

This standard specification of preferences under risk thus leads to a loss of generality. Although CARA preferences are widely used, the fact that they suppress any impact of feelings for others in attitudes to risk make them ill-suited to study a number of issues, such as health or professional risk-taking in adults.

### 3.4 Discussion

Interestingly, Livingstone and Lunt (1992) find, from questionnaires distributed in the UK, that indebted individuals have fewer children on average – even though having more children is generally thought to imply greater monetary demands. Independent workers, who tend to be less risk averse, are also less likely to have children than salaried workers; Colombier and Masclet (2007) find that having children has a significant negative impact on the probability of being self-employed in agriculture. And according to the European Community Household Panel, 60.8 percent of salaried workers have children, compared with 52.5 percent of independent, self-employed, workers. This data would seem to indicate that parents behave more cautiously than childless individuals, as if the risk-aversion of the former was stronger than that of the latter.

However, it would be erroneous to use these data as indicating the impact of altruism on risk aversion. As mentioned in the introduction, individuals deciding to become parents are likely to
have different preferences from individuals who decide to remain childless. As having children tends to be the widespread social norm in most societies, not abiding by this rule may signal a more independent personality. Working as an independent worker may also require too much time and attention to allow for much family life, so that is is likely to attract mostly individuals not interested in this family life. In order to assess empirically in a satisfactory way whether caring for one’s children heightens risk aversion, one would need to observe the choices made by the same individual before and after the birth of his children, assuming one can control for age- and context-related effects. It might be easier to test the effects we highlight using specific, more narrowly-defined contexts, and especially in specifically designed experiments associated to personality questionnaires.

4 Conclusion

We have seen how Arrow-Pratt coefficients of absolute risk aversion are affected by altruism. Gifts to loved ones provide some self-insurance, and the more so when the loved ones are very risk averse. Common specifications such as CARA preferences entail a loss of generality in this context. This loss of generality is of importance, as it suppresses a major impact of loving and providing for others in risky situations.

Our analysis can be relevant for many decisions in which parental status and risk both matter, such as employment, health insurance, credit or retirement savings.

References


**Appendix**

**A.1. The sharing rule**

The first order condition of the maximization program that determines the amount given by $P$ to $C$ is:

$$-u_1(y - x, \alpha c(x)) + \alpha c'(x) u_2(y - x, \alpha c(x)) = 0.$$ 

Since this has to be satisfied for any interior solution $x_{y}$, we can differentiate it with respect to the first argument of the utility function of individual $P$, which gives (omitting the argument of
For the second-order condition to hold and our program to be concave, we assume
\[
\begin{align*}
&\quad u_{11}(y - x, \alpha c(x)) < 2\alpha \alpha'(x) u_{12}(y - x, \alpha c(x)) - \alpha^2 \alpha''(x) u_2(y - x, \alpha c(x)) \\
&\quad - (\alpha' c(x))^2 u_{22}(y - x, \alpha c(x)).
\end{align*}
\]
This is always satisfied when \(u_{12}(y - x, \alpha c(x)) \geq 0\).

Omitting all arguments, \(\frac{dx}{dy} = \frac{u_{11} - \alpha c'(x) u_{12}}{u_{11} - 2\alpha \alpha'(x) u_{12} + (\alpha c(x))^2 u_{22}}\).

And \(\frac{dx}{dy} \geq 0\) if and only if \(-u_{11}(y - x, \alpha c(x)) + \alpha \alpha'(x) u_{21}(y - x, \alpha c(x)) \geq 0\) for all \(y\) in the relevant range.

**A.2. The coefficient of risk aversion**

To simplify notations, let us omit the arguments in \(u(., .)\), that is taken at its maximum value, for \(x\) solution to the sharing rule.

Using the expression of \(\frac{dx}{dy}\) characterized above, one gets

\[
\mathcal{A}^\ddot{u}(y) = \frac{-\ddot{u}(y)}{\dot{u}(y)}
= -\frac{1}{u_1}[u_{11} + \frac{dx}{dy}(-u_{11} + \alpha c'(x) u_{12})]
= -\frac{1}{u_1}[u_{11} + \frac{-(u_{11} - \alpha c'(x) u_{12})^2}{u_{11} - 2\alpha \alpha'(x) u_{12} - (\alpha c(x))^2 u_{22} + \alpha c''(x) u_2}]
= -\frac{\alpha}{u_1}[-\alpha c'(x))^2(u_{11} u_{22} + (u_{12})^2) + c''(x) u_2 u_{11}].
\]