Land rent taxation and socially optimal allocation in economies with environmental externality

N. Thang Dao[1] and Ottmar Edenhofer[2]

Abstract

We consider an overlapping generations economy with "land" as a fixed factor of production and an environmental externality on production in which tax revenue from land rent and/or from other schemes such as labor income, capital income, or Pigouvian taxation can be used for environmental protection through mitigation investment. Notably, we show that, for any given target of stationary stock of pollution, the land rent taxation scheme leads to a higher steady state capital accumulation than the other schemes, and hence the steady state consumption of agents when young under a land rent taxation scheme is also higher than under the others. In addition, when the mitigation technology is relatively high compared to the dirtiness of production, under an ambitious target on mitigation, the land rent taxation also provides a higher steady state consumption when old, therefore higher social welfare, than the others. In the second part of the paper, we propose a *period-by-period* balanced budget policy, which includes land rent and capital income taxes with intergenerational transfers, to decentralize the socially optimal allocation during the transitional phase to the social planner's steady state as a competitive outcome.

Key words: overlapping generations economy, land rent taxation, capital income taxation, labor income taxation, Pigouvian taxation, socially optimal allocation.

JEL Classification: H23, I31, Q50.

1 Introduction

The crucial purpose of pollution mitigation is to reduce the long-term damage of climate change and hence to improve the social welfare towards sustainable development. The Paris Climate Agreement on limiting global warming to well below 2 degrees centigrade above pre-industrial level indeed requires strict long-term and persistent global emission mitigation efforts which are essentially financed through taxation schemes, hence probably lessening capital accumulation and economic growth. These efforts, however, may contradict the poverty reduction goal unless an appropriate climate policy is designed. This requires a tax policy to be designed efficiently to support the climate mitigation calls from both scientists and policy makers. We engage in this discussion by presenting in this paper an overlapping generations economy with a fixed factor of production and environmental externality on productivity in which pollution is a by-product of production. Within framework, we will evaluate the impacts of climate

[1]Mercator Research Institute on Global Commons and Climate Change, Torgauer Str. 12 - 15, 10829 Berlin, Germany. Email: daonguyen@mcc-berlin.net

[2]Mercator Research Institute on Global Commons and Climate Change, Torgauer Str. 12 - 15, 10829 Berlin, Germany. Potsdam Institute for Climate Impact Research (PIK), and Technische Universität Berlin (TU-Berlin). Email: edenhofer@pik-potsdam.de.

mitigation policy on capital accumulation and social welfare under alternative tax policies, as well as considering the possibility of obtaining the socially optimal allocation.

Our paper integrates two theoretical strands in the environmental economics and public finance literature. The first is a sizable literature concerning environmental quality and long-term growth, as well as the decentralization of socially optimal allocation, in which the papers by Howarth and Norgaard (1992) and John and Pecchennino (1994) are two of the pioneers. The second is the literature of incidence of rent taxation on fixed factor of production (Feldstein 1977, Calvo et al. 1979, Chamley and Wright 1987, and recently Edenhofer et al. 2015). While the first strand focuses on correcting environmental externalities, it ignores land as a factor of production as well as tax on its rent can be a source for public spending. The second strand does not take environmental issues into account. If we consider the fact that pollution is a by-product of production, then enhancing capital accumulation will degrade the environment which will have negative feedback on production in the future. Therefore, it is reasonable to think that the environmental protection policy needs to be combined with land rent taxation in order to guarantee an improvement in social welfare. This paper attempts to fill this gap in the literature. We will point out that, for a given target of stationary stock of pollution under the climate change mitigation agreement, land rent taxation can be an efficient instrument for mitigation in the sense that it will lead to higher capital accumulation and consumption, hence greater welfare, than other taxation schemes. In addition, the land rent tax can be combined with appropriate capital income tax and intergenerational transfers in order to decentralize the socially optimal allocation.

The role of tax on land was indeed addressed extensively by classical economists. In particular, the American political economist Henry George (1839 - 1897) is famous for the idea that land (or resource) rent should be taxed for public use or share. George (1879) proposed a single tax on land in lieu of distortion taxes on labor and productive investment. According to George's (1879) perspective of justice, the economic value derived from natural resources, including land, should belong equally to all citizens of a society.[3] In addition, contrast with taxation on other factors of production, tax on land does not change the size of land and may enhance capital accumulation through the portfolio effect of owning land and holding capital. Indeed, in an overlapping generations model without environmental externality or the effect of infrastructure, Feldstein (1977) points out that capital accumulation can be enhanced, and thus welfare improved, through the taxation on the rent of a fixed factor of production such as land. That is because taxing on land rent will shift the savings of a household's portfolio towards capital accumulation.[4] The theoretical results in this paper consolidate George's ideas in the context of investing in climate change mitigation.

The paper also relates to the literature of land tax and investment/spending in public goods or infrastructure (Arnott and Stiglitz 1979, Rangel 2005, Mattauch et al 2013, and recently Behrens et al. 2015). Arnott and Stiglitz (1979) and Behrens et al. (2015) focus on static models investigating the relationship between aggregate urban land rents and local public goods provision in which the authors identify general conditions under which the Henry George theorem holds. Rangel (2005) sets up a two-period two-generation political economy to study institutional implementation over alternative taxation and debt regimes, including a *land-tax-only scheme*, to obtain Pareto improvement through investing in intergenerational

[3]For more discussion about this single tax, see George (1890).

[4]This tax incidence in Feldstein (1977) is then analyzied in refined models such as Calvo et al. (1979) which introduces the bequest motive, and Chamley and Wright (1987) which analyzes both the static and dynamic effects of land rent tax. Petrucci (2006) and Koethenbuerger and Poutvaara (2009) also consider the effects of land rent taxation on capital accumulation and improving welfare but they study this in the framework of small and open economies.

public goods. In a continuous time Ramsey model, Mattauch et al. (2013) show that when land rent is sufficiently high, the social optimum can be obtained by using the land rent tax revenue to invest in public infrastructure. Such investment is less rigorous in the *laissez-faire* economy. Our paper differs from this body of literature by setting up an overlapping generations model à la Diamond (1965), incorporating environmental externality in order to study the incidence of land rent taxation on environmental protection and welfare. We will compare this to other taxation schemes such as capital income, labor income and Pigouvian taxation. In addition, we propose a combination between land rent tax, capital income tax, and intergenerational transfers in order to decentralize the socially optimal allocation during the entire transitional phase as a competitive outcome.

The rest of the paper is organized as follows. The basic model is presented in the next section and we define the competitive equilibrium and steady state of the economy in section 3. Section 4 examines the effects of taxation schemes on steady state stock of pollution as well as capital accumulation. We compare these effects on capital accumulation and welfare of taxation schemes in section 5. In section 6, we characterize the socially optimal allocation from the viewpoint of the social planner and we compare it with the purely competitive allocation. We propose a strategy to decentralize the social planner's optimal allocation as a competitive outcome in section 7. Finally, section 8 concludes the paper.

2 The model

We consider a two-period overlapping generations model in Diamond (1965) with a fixed factor of production and an environmental externality on the total factor productivity. The final output is produced out of capital, labor, and land (which is a fixed factor of production). Agents (or households) live for two periods (say young and old). They work when young to earn income for consumption during their youth, savings in terms of capital and land ownership. They buy land from the previous generation. Agents lend their capital and land to producing firms. Their capital savings (with its returns), land rent, and income from sale if land to the subsequent generation are fully consumed in the second period of life. In this set up, we do not consider the bequest motives of agents.

2.1 Dynamics of pollution stock

We assume the dynamics of pollution stock as follows

$$E_t = \bar{E} + (1-\delta)(E_{t-1} - \bar{E}) + P_t$$

where E_t is the pollution stock (i.e. the so-called carbon concentration) in period t; P_t is the net flow of emissions due to the economic activities of human (say production and mitigation) in period t; $\bar{E}$ is the natural state of carbon concentration in the atmosphere, i.e. the state of the ecological system without any human activity. The $\delta \in (0, 1]$ is the decay rate of pollution stock which measures the convergent speed of pollution stock to the natural state $\bar{E}$. For simplicity, we normalize $\bar{E} = 0$. Therefore, hereafter we rewrite the dynamics of pollution as

$$E_t = (1-\delta)E_{t-1} + P_t$$

We assume that the flow of pollution is determined by

$$P_t = \xi Y_t - \gamma M_t$$

where ξY_t is the pollution arising from the production Y_t reflecting that pollution is an inevitable by-product of production; γM_t is pollution abatement coming from the mitigation effort M_t. The $\xi > 0$ is the dirtiness coefficient of production and $\gamma > 0$ is the effectiveness coefficient of mitigation technology.

2.2 Production

The final good producing firm operates under perfect competition and follows the following production function

$$Y_t = z(E_{t-1})F(K_t, L_t, X) \tag{1}$$

where $z(E_{t-1})$ is total factor productivity in period t which depends on the stock of pollution in the previous period. Note that the past stock of pollution affects the current productivity, reflecting the long-term effect of pollution. We assume that $z(E) = \bar{z} > 0$ for all $E \leq 0$,[5] $z'(E) < 0$ for all $E > 0$, $z'(0^+) = 0$, and $\lim_{E\to+\infty} z(E) = 0$. The $F(K_t, L_t, X)$ exhibits constant returns to scale over capital K_t, labor L_t, and land X; and $F_i(K_t, L_t, X) > 0$, $F_{ii}(K_t, L_t, X) < 0$, $F_{ij}(K_t, L_t, X) > 0$ where $i, j \in \{K_t, L_t, X\}$, $i \neq j$. The returns to factors of production are determined by their marginal productivity, i.e.

$$r_t = z(E_{t-1})F_K(K_t, L_t, X) \tag{2}$$

$$w_t = z(E_{t-1})F_L(K_t, L_t, X) \tag{3}$$

$$x_t = z(E_{t-1})F_X(K_t, L_t, X) \tag{4}$$

where r_t, w_t, and x_t are returns to capital, labor, and land, respectively. For exposition purposes, we follow Feldstein (1977) by assuming, without loss of generality, that capital does not depreciate.

2.3 Agents

For simplification, we assume that each agent has one parent and one offspring (i.e. the population is constant) and the economy is populated by L_t such identical agents. Without lost of generality, we normalize $L_t = 1$ for all $t \in \mathbb{N}$. Each agent lives for two periods. We assign the time index t to an agent, say agent t, implying that the agent becomes adult (or young) in period t and old in period $t+1$. A similar idea is applied for a generation as a whole. In the first period of life (say young period), the agent t is endowed with one unit of labor which he supplies inelastically to the market to earn labor income. During this period, he allocates his income to consumption when young c_t^y, and savings in terms of physical capital k_{t+1}, which is lent to producing firms in the next period, and savinge in terms of holding land X, which will be lent to producing firms and sold to the succeeding generation in the next period. These savings will be fully consumed in the second period of life (i.e. when the agent

[5]For simplification, we assume $z(E)$ is a positive constant for all $E \leq 0$. One may argue that $z'(E) > 0$ for all $E < 0$. Indeed, capturing this assumption does not change crucially the analytical results of this paper. Moreover, the case $E < 0$ is not a focus of our paper.

is old). Agents born at date t have preferences over their consumptions when young and old $(c_t^y, c_{t+1}^o) \in \mathbb{R}_+^2$, represented by $U(c_t^y, c_{t+1}^o) = u(c_t^y) + v(c_{t+1}^o)$ with $u', v' > 0$ and $u'', v'' < 0$.[6]

The life-time utility maximization problem of the agent is as follows:

$$\max_{c_t^y, k_{t+1}, c_{t+1}^o, X} u(c_t^y) + v(c_{t+1}^o) \tag{5}$$

subject to

$$c_t^y + k_{t+1} + p_t X \leq (1 - \tau^L) w_t \tag{6}$$

$$c_{t+1}^o \leq \left[1 + (1 - \tau^K) r_{t+1}\right] k_{t+1} + \left[(1 - \tau^X) x_{t+1} + p_{t+1}\right] X \tag{7}$$

given w_t, perfect forseen r_{t+1}, and the prices of land p_t and p_{t+1} in period t and $t+1$, respectively; $\tau^X, \tau^K, \tau^L \in [0, 1]$ are the tax rate on land rent, capital income, labor income respectively.

Under standard assumptions guaranteeing the concavity of the objective function and the interiority of the optimal solution, the choice of the agent is characterized by the following first order conditions (FOCs)

$$\begin{pmatrix} u'(c_t^y) \\ 0 \\ v'(c_{t+1}^o) \\ 0 \end{pmatrix} = \lambda_t \begin{pmatrix} 1 \\ 1 \\ 0 \\ p_t \end{pmatrix} + \mu_t \begin{pmatrix} 0 \\ -1 - (1 - \tau^K) r_{t+1} \\ 1 \\ -(1 - \tau^X) x_{t+1} - p_{t+1} \end{pmatrix} \tag{8}$$

for some $\lambda_t, \mu_t > 0$, and the budget constraints (6) and (7) are binding; i.e.

$$\frac{u'(c_t^y)}{v'(c_{t+1}^o)} = 1 + (1 - \tau^K) r_{t+1} \tag{9}$$

$$c_t^y + k_{t+1} + p_t X = (1 - \tau^L) w_t \tag{10}$$

$$c_{t+1}^o = \left[1 + (1 - \tau^K) r_{t+1}\right] k_{t+1} + (1 - \tau^X) x_{t+1} X + p_{t+1} X \tag{11}$$

$$\left[1 + (1 - \tau^K) r_{t+1}\right] p_t = (1 - \tau^X) x_{t+1} + p_{t+1} \tag{12}$$

where the last equation is the no-arbitrage condition between savings in terms of land and capital.

[6] We can extend the model by introducing the disutility of pollution in the preference, i.e. the stock of pollution not only has an externality on production but also an externality on utility. The utility function can be presented as $\tilde{U}(c_t^y, c_{t+1}^o, E_{t+1}) = u(c_t^y) + v(c_{t+1}^o) + \phi(E_{t+1})$ where for all $E > 0$, $\phi'(E) < 0$ and $\phi''(E) > 0$. We assume that each agent born in t is negligible to affect the stock of pollution, hence he/she has no incentive to internalize the environmental externality and always treats E_{t+1} as given in solving his/her optimization problem. However, the qualitative results do not change when we model preference in this way. So, in order to illustrate the ideas, we keep the model simple and abstract disutility of pollution from the utility function.

3 Equilibrium and steady state

Without of loss of any generality, let normalize $X = 1$ for all t. In order to lighten the notation, we denote the function $F(K_t, L; X)$ and its derivatives as $F^t = F(K_t, L, X)$, $F_i^t = F_i(K_t, L, X)$, and $F_{ij}^t = F_{ij}(K_t, L, X)$ where $i, j \in \{K_t, L, X\}$. Since we normalize the size of population by 1, then the aggregate capital is also the average capital, i.e. $K_t = k_t$ for all t. We assume that all tax revenue will be used for emission mitigation. Hence, the mitigation in period t is

$$M_t = \tau^K z(E_{t-1}) F_K^t k_t + \tau^L z(E_{t-1}) F_L^t + \tau^X z(E_{t-1}) F_X^t = z(E_{t-1}) F^t \sum_i \sigma_t^i \tau^i$$

where σ_t^i and τ^i are respectively the share in final output and the tax rate on the income of production factor $i \in \{K, L, X\}$.

We define the equilibrium and steady state as follows:

Equilibrium: Under the given tax rates $\tau^X, \tau^K, \tau^L \in [0, 1]$ and sequence of land prices $\{p_t\}_{t=0}^{+\infty}$, the competitive equilibrium of the economy is characterized by: (i) the agent's utility maximization (5) under the budget constraints (6) and (7); (ii) the final good producing firm maximization determining the returns of the production factors as in (2), (3), and (4); (iii) the dynamics of the environment. Therefore, the competitive equilibrium allocation $\{c_t^y, k_{t+1}, c_{t+1}^o, E_t\}_t$, which fully characterizes the competitive equilibrium of the economy, is the solution to the following system of equations:

$$\frac{u'(c_t^y)}{v'(c_{t+1}^o)} = 1 + (1 - \tau^K) z(E_t) F_K^{t+1} \tag{13}$$

$$c_t^y + k_{t+1} + p_t = (1 - \tau^L) z(E_{t-1}) F_L^t \tag{14}$$

$$c_{t+1}^o = \left[1 + (1 - \tau^K) z(E_t) F_K^{t+1}\right] (k_{t+1} + p_t) \tag{15}$$

$$E_t = (1 - \delta) E_{t-1} + \left(\xi - \gamma \sum_i \sigma_t^i \tau^i\right) z(E_{t-1}) F^t \tag{16}$$

for given E_{t-1} and k_t.

By doing some simple substitutions and transformations, the competitive equilibrium of the economy can be fully characterized by the following reduced system:

$$\frac{u'\left((1 - \tau^L) z(E_{t-1}) F_L^t - k_{t+1} - p_t\right)}{v'\left(\left[1 + (1 - \tau^K) z(E_t) F_K^{t+1}\right] (k_{t+1} + p_t)\right)} = 1 + (1 - \tau^K) z(E_t) F_K^{t+1} \tag{17}$$

$$E_t = (1 - \delta) E_{t-1} + \left(\xi - \gamma \sum_i \sigma_t^i \tau^i\right) z(E_{t-1}) F^t \tag{18}$$

$$p_{t+1} = \left[1 + (1 - \tau^K) F_K^{t+1}\right] p_t - (1 - \tau^X) F_X^{t+1} \tag{19}$$

for given E_{-1}, k_0, and p_0.

The equation (17) is the Euler equation which implies the optimal allocation between consumptions and savings from the perspective of the agents. The equation (18) represents the dynamics of pollution stock.

Competitive steady state: Under the tax rates $\tau^X, \tau^K, \tau^L \in [0,1]$, the competitive steady state of the economy is characterized by

$$\frac{u'\left((1-\tau^L)z(E)F_L - k - \frac{(1-\tau^X)F_X}{(1-\tau^K)F_K}\right)}{v'\left([1+(1-\tau^K)z(E)F_K]\left[k+\frac{(1-\tau^X)F_X}{(1-\tau^K)F_K}\right]\right)} = 1+(1-\tau^K)z(E)F_K$$

$$\frac{\delta E}{z(E)} = \left(\xi - \gamma \sum_i \sigma^i \tau^i\right) F$$

The existence and uniqueness of the steady state will be discussed in the next section in which we visualize some functional forms of utility and production. We also assume that under each taxation scheme, the economy will converge to the corresponding steady state.

4 Effects of tax schemes

In this section we study the effect of each taxation scheme on capital accumulation, given a target on stationary stock of pollution. We then compare these effects on consumptions and welfare between taxation schemes. In order to analyze the impact of each taxation scheme on the steady state of the economy, for the purpose of exposition, we specify the functional forms of utility and production in this section. We consider the logarithmical preference, i.e. $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$ where $\beta \in (0,1)$ is the time preference parameter of the household, and the Cobb-Douglas production function $z(E)F(K,L,X) = z(E)K^{\alpha_K}L^{\alpha_L}X^{\alpha_X}$, $0 < \alpha_K, \alpha_L, \alpha_X < 1$, $\alpha_K + \alpha_L + \alpha_X = 1$. For these functional forms, the system of equations characterizing the steady state now boils down to

$$\left[1 + \frac{(1-\tau^X)\alpha_X}{(1-\tau^K)\alpha_K}\right] k - \frac{\beta\alpha_L(1-\tau^L)}{1+\beta} z(E)k^{\alpha_K} = 0 \tag{20}$$

$$\frac{\delta E}{z(E)} - \left(\xi - \gamma \sum_i \alpha_i \tau^i\right) k^{\alpha_K} = 0, \quad i \in \{K, L, X\} \tag{21}$$

Note that for the Cobb-Douglas production function form, α_i is the share in final output of production factor $i \in \{K, L, X\}$.

4.1 *Land rent taxation* $\tau^X \in (0,1)$, $\tau^K = 0$, $\tau^L = 0$

Under the land rent taxation scheme only, the system (20) - (21) becomes

$$\frac{\alpha_K + (1-\tau^X)\alpha_X}{\alpha_K} k - \frac{\beta\alpha_L}{1+\beta} z(E)k^{\alpha_K} = 0 \tag{22}$$

$$\frac{\delta E}{z(E)} - (\xi - \gamma\tau^X\alpha_X)k^{\alpha_K} = 0 \tag{23}$$

The proposition 1 below states the existence and uniqueness of a steady state under the land rent taxation scheme $\tau^x \in (0,1)$ and its effects on steady state stock of pollution, as well as on capital accumulation.

Proposition 1: *In the overlapping generations economies with* $F(K, L, X) = K^{\alpha_K} L^{\alpha_L} X^{\alpha_X}$ *and* $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$ *set up above:*

(i) *For all given* $\tau^X \in [0, 1)$*, there always exists a unique competitive steady state.*

(ii) *If* $\alpha_K < 1/2$ *and* $\xi/\gamma < 1 - \alpha_K + \alpha_X$ *then* $\partial E_X / \partial \tau^X < 0$ *and* $\partial k_X / \partial \tau^X > 0$*; where* E_X *and* k_X *are respectively steady state stock of pollution and capital per capita under the land rent taxation scheme.*

Proof: See Appendix A1.

The statement (i) in proposition 1 states the existence and uniqueness of the steady state. This property will hold for other taxation schemes, as will be clarified in the later subsections. The statement (ii) of proposition 1 implies that if the share of capital to final output is not too high, particularly $\alpha_K < 1/2$, and the mitigation is sufficiently effective, particularly $\xi/\gamma < 1 - \alpha_K + \alpha_X$, then the higher the land rent tax $\tau^X \in [0, 1]$ the lower the steady state stock of pollution and the higher the steady state capital per capita. This is because, when the mitigation technology is relatively effective compared to the dirtiness of production, the effect of mitigation will be strong when decreasing the stock of pollution. The better environmental quality increases the output and hence enhances capital accumulation, which in turn, degrades the environment. On the other hand, when the share of physical capital to final output is not too high then the effect of capital accumulation on carbon concentration will be dominated by the mitigation which is supported by land rent tax revenue.

4.2 *Capital income taxation* $\tau^X = 0$, $\tau^K \in (0, 1)$, $\tau^L = 0$

Under the capital income taxation scheme only, the system (20) - (21) becomes

$$\frac{\alpha_X + (1 - \tau^K)\alpha_K}{(1 - \tau^K)\alpha_K} k - \frac{\beta \alpha_L}{1 + \beta} z(E) k^{\alpha_K} = 0 \tag{24}$$

$$\delta E - (\xi - \gamma \tau^K \alpha_K) z(E) k^{\alpha_K} = 0 \tag{25}$$

Proposition 2: *In the overlapping generations economies with* $F(K, L, X) = K^{\alpha_K} L^{\alpha_L} X^{\alpha_X}$ *and* $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$ *set up above:*

(i) For all given $\tau^K \in [0, 1)$*, there always exists a unique competitive steady state.*

(ii) For all $\tau^K \in [0, 1)$*, then* $\partial E_K / \partial \tau^K < 0$*, where* E_K *is the steady state stock of pollution under the capital income taxation scheme.*

Proof:

(i) The proof for this statement is similar to the proof for statement (i) of proposition 1, hence it is fairly straightforward.

(ii) From (24) and (25) we have

$$E_K = \tilde{g}(\tau^K) z(E_K)^{\frac{1}{1-\alpha_K}} \tag{26}$$

where

$$\tilde{g}(\tau^K) = \frac{\xi - \gamma\tau^K\alpha_K}{\delta}\left(\frac{\beta\alpha_L\alpha_K(1-\tau^K)}{(1+\beta)[\alpha_X + \alpha_K(1-\tau^K)]}\right)^{\frac{\alpha_K}{1-\alpha_K}} \tag{27}$$

Applying the implicit function theorem for (26) with respect to E and τ^k, we have (note that from (27) it is straightforward to derive $-\infty < \tilde{g}'(\tau^K) < 0$),

$$\frac{\partial E_K}{\partial \tau^K} = \frac{(1-\alpha_K)z(E_K)^{\frac{2-\alpha_K}{1-\alpha_K}}\tilde{g}'(\tau^K)}{(1-\alpha_K)z(E_K) - z'(E_K)E_K} < 0 \tag{28}$$

Q.E.D.

Statement (ii) of proposition 2 mentions the effect of the capital income tax on the steady state stock of pollution. The higher tax rate on capital income leads to a lower steady state stock of pollution. This result is rather intuitive. Capital income taxation provides two reinforcing effects on environmental protection. First, the tax revenue from capital income is used for mitigation, hence it directly decreases the stock of pollution. Second, when capital income is taxed, households will shift the savings of their portfolio towards land value ownership, thereby reduceing capital accumulation, which in turn leads to lower pollution. It is interesting to note that although under the capital income taxation scheme households tend to shift the savings of their portfolio towards land value ownership, the steady state capital per capita under this taxation scheme is not necessarily lower than that under a *laisserz-faire* economy without any intervention. That is because the lower stock of pollution increases the total factor productivity, which increases labor income. In this way, capital accumulation is accelerated. Indeed, lets examine the impact of capital income taxation on capital accumulation. From (24) we have

$$k_K = \left[\frac{\beta\alpha_L\alpha_K(1-\tau^K)}{(1+\beta)[\alpha_X + (1-\tau^K)\alpha_K]}z(E_K)\right]^{\frac{1}{1-\alpha_K}} \tag{29}$$

Hence,

$$\frac{\partial k_K}{\partial \tau^K} = \left[\frac{\beta\alpha_L\alpha_K(1-\tau^K)z(E)}{(1+\beta)[\alpha_X + (1-\tau^K)\alpha_K]}\right]^{\frac{1}{1-\alpha_K}} \frac{\left[\frac{z'(E_K)}{z(E_K)}\frac{\partial E_K}{\partial \tau^K} - \frac{1}{1-\tau^K}\right]\left[\alpha_X + (1-\tau^K)\alpha_K\right] + \alpha_K}{(1-\alpha_K)\left[\alpha_X + (1-\tau^K)\alpha_K\right]} \tag{30}$$

From (30) we find that contrast to the effect of land rent taxation on capital accumulation (which is stated in proposition 1), the effect of capital income taxation is ambiguous and the direction of this effect seems to depend on the level of the tax rate on capital income τ^k. Since $-\infty < \tilde{g}'(\tau^K) < 0$ then $\partial E_K/\partial \tau^K$ as determined in (28) is always bounded. Hence, the numerator $\left[\frac{z'(E_K)}{z(E_K)}\frac{\partial E_K}{\partial \tau^K} - \frac{1}{1-\tau^K}\right]\left[\alpha_X + (1-\tau^K)\alpha_K\right] + \alpha_K$, which determines the sign of $\partial k_K/\partial \tau^K$, on the right hand side of (30) may be positive when τ^K is sufficiently low (depending on the responsiveness of $z(E_K)$ to the change in E_K), and it approaches $-\infty$ when $\tau^K \to 1^-$. Indeed, under this scheme, two effects on capital accumulation prevail. Firstly, mitigation reduces pollution stock, enhancing the total factor productivity of the economy, and hence improving the labor income. This has the effect of accelerating capital accumulation. Secondly, taxing on capital income causes agents shifts partially their savings to land value ownership, hence lessening the capital accumulation. Therefore, the net effect depends on the responsiveness of the damage function $z(E_K)$ to the change in stock of pollution and on the level of the tax rate τ^K.

4.3 Labor income taxation

Under the labor income taxation scheme only, the system (20) - (21) becomes

$$\frac{\alpha_K + \alpha_X}{\alpha_K} k - \frac{\beta(1-\tau^L)\alpha_L}{1+\beta} z(E) k^{\alpha_K} = 0 \tag{31}$$

$$\frac{\delta E}{z(E)} - (\xi - \gamma \tau^L \alpha_L) k^{\alpha_K} = 0 \tag{32}$$

Proposition 3: *In the overlapping generations economies with* $F(K, L, X) = K^{\alpha_K} L^{\alpha_L} X^{\alpha_X}$ *and* $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$ *set up above:*

(i) For all given $\tau^L \in [0, 1)$*, there always exists a unique competitive steady state.*

(ii) For all $\tau^L \in [0, 1)$*, then* $\partial E_L / \partial \tau^L < 0$*, where* E_L *is the steady state stock of pollution under a labor income taxation scheme.*

Proof: The proofs are quite similar to the proofs of proposition 2.

The effect of labor income taxation scheme on steady state stock of pollution is similar that of capital income taxation. The labor income taxation also has two reinforcing effects on mitigation. First, the tax revenue is used for mitigation hence it directly decreases the stock of pollution. Second, the labor income tax decreases capital accumulation by decreasing the disposable income of households. In this way, it causes the final output to decline and hence reduces emissions. However, the general equilibrium effect of this taxation scheme on capital accumulation is ambiguous because the lower stock of pollution increases the total productivity which enhances capital accumulation by increasing the labor income. Indeed, from (31) we have

$$k_L = \left[\frac{\beta \alpha_K \alpha_L (1-\tau^L)}{(1+\beta)(\alpha_K + \alpha_X)} z(E_L) \right]^{\frac{1}{1-\alpha_K}} \tag{33}$$

So the effect of labor income taxation on steady state capital accumulation depends on the tax rate and the responsiveness of the damage function $z(E)$ to the change in pollution stock. Note that unlike the effects of capital income taxation, the change in portfolio of savings between capital and land is unclear because the labor income taxation has symmetric effects on savings in land and in capital.

4.4 Pigouvian taxation

Now we suppose that the government collects the tax revenue on production, say a Pigouvian tax, in order to invest in mitigation. Under a Pigouvian taxation scheme $\tau^p \in [0, 1)$ alone, the steady state of the economy is characterized by

$$\frac{\alpha_K + \alpha_X}{\alpha_K} k - \frac{\beta \alpha_L (1-\tau^P)}{1+\beta} z(E) k^{\alpha_K} = 0 \tag{34}$$

$$\delta E - (\xi - \gamma\tau^P)z(E)k^{\alpha_K} = 0 \tag{35}$$

From the last equation, for efficiency, the rational government never sets τ^p such that $\xi - \gamma\tau^p < 0$, because if it does, the steady state stock of pollution will be negative. Therefore, $\tau^p \in [0, \xi/\gamma] \subset [0, 1)$.

Proposition 4: *In the overlapping generations economies with $F(K, L, X) = K^{\alpha_K} L^{\alpha_L} X^{\alpha_X}$ and $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$ set up above:*

(i) For all given $\tau^P \in [0, \xi/\gamma]$, there always exists a unique competitive steady state.

(ii) The higher the Pigouvian tax rate $\tau^P \in [0, \xi/\gamma]$ the lower the steady state stock of pollution.

Proof: The proofs are quite similar to the proofs of proposition 2.

In line with the other taxation schemes studied above, the steady state stock of pollution also decreases in the Pigouvian tax scheme. That is because this taxation scheme has at least two reinforcing effects on emission mitigation. First, the tax revenue is used for mitigation, hence the pollution stock is directly reduced. Second, the Pigouvian tax decreases the labor income received by the household, hence reducing the capital accumulation. Therefore, it indirectly decreases the pollution stock. Like the effect of capital income taxation on capital accumulation, we need to be cautious in evaluating the effect of Pigouvian taxation scheme on capital accumulation. Although the Pigouvian tax directly reduces the net labor income of households, hence reducing the savings, the steady state capital per capita is not necessarily lower than that under a *laisserz-faire* economy without any intervention. Indeed, from (34) we have

$$k_P = \left[\frac{\beta\alpha_L\alpha_K(1-\tau^P)}{(1+\beta)(\alpha_K+\alpha_X)} z(E_P) \right]^{\frac{1}{1-\alpha_K}} \tag{36}$$

The net effect of a Pigouvian tax on capital accumulation is ambiguous, depending on the responsiveness of $z(E)$ to the change in E. There are two opposite effects. On the one hand, as stated in proposition 4, this scheme leads to a lower stationary stock of pollution, hence improves the total factor productivity. In this way, this policy increases the labor income of agents, enhancing the capital accumulation. On the other hand, the policy directly decreases the net labor incomes of agents by imposing a tax rate $\tau^P > 0$ on the revenue of the producing firms. Which effect becomes dominant depends on the responsiveness of the damage function $z(E)$ to the change in E and on the level of the tax rate τ^P.

5 Comparison of taxation schemes and welfare analysis

It would be interesting to compare the effects of the four taxation schemes outlined above on capital accumulation, and hence welfare, given a target for mitigation. We suppose that the government has some target E for the long-term stationary pollution stock. This target can be determined through one of the tax and mitigation schemes detailed above. However, these tax schemes may affect the composition of household's savings asymmetrically, and hence asymmetrically affect capital accumulation. In this section, we will compare the capital accumulation under these tax and mitigation schemes.

Proposition 5: *In the overlapping generations economies with* $F(K, L, X) = K^{\alpha_K} L^{\alpha_L} X^{\alpha_X}$ *and* $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$ *set up above, given a target on steady state stock of pollution* $E \in [0, \tilde{E})$*, where* $\tilde{E}$ *is the steady state stock of pollution in case of no mitigation, then:*

(i) $k_X > \max\{k_K, k_P\}$*;*

(ii) $k_P > k_L$*;*

(iii) *if* $1 - \tau^K > (=)(<)\ \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2$ *then* $k_K > (=)(<) k_L$*;*

(iv) *if* $\alpha_X \geq \alpha_K^2/(1 - \alpha_K)$ *then* $k_P > k_K$*.*

Proof: See Appendix A2.

The statements (i) and (ii) of proposition 5 indicate important implications. If the pure competitive economy with environmental externality as described above converges to a steady state with capital under-accumulation, then, for a given target of steady state pollution stock, the land rent taxation scheme may be more efficient than the other taxation schemes in combating climate change and obtaining better social welfare. The results in (i) and (ii) imply that for given target on steady state stock of pollution, the land rent taxation will lead the highest capital accumulation. This result is quite intuitive. That is because, unlike other taxation schemes, taxation of land rent does not decrease the agents' disposable income when young. Moreover, taxation of land rent causes agents shift their savings portfolios towards capital accumulation, whereas capital income taxation has the opposite effect. The Pigouvain and labor income taxation schemes directly decrease the disposable income and hence lessen the capital accumulation compared to the land rent taxation scheme.

Statement (iii) of proposition 5 compares the capital accumulation under capital income taxation and labor income taxation schemes. Which scheme leads to higher capital accumulation depends on the ambition of climate policy. When the steady state stock of pollution is set at a sufficiently low level, requiring the tax rate on capital income (or labor income) to be correspondingly high, in particular $\tau^K > 1 - \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2$, then the labor income taxation scheme leads to higher capital accumulation than the capital income taxation scheme, $k_L > k_K$, and vice versa. This result can be interpreted as follows. When the climate policy is not sufficiently ambitious then both tax rates on capital income and labor income for mitigation corresponding to two taxation schemes will be low. The effect of the low capital income tax in adjusting the savings of the agent's portfolio is not strong enough to cause a greater decrease in capital accumulation than that under the labor income taxation scheme. The latter scheme generates a decrease in capital accumulation through a direct decrease in the disposable incomes of agents. Therefore, in this case, $k_K > k_L$. The situation is reversed when the climate policy is sufficiently ambitious.

Statement (iv) compares the Pigouvian taxation and capital income taxation schemes under a given condition. When the share of land to final output is large enough compared to that of capital, in particular when $\alpha_x \geq \alpha_k^2/(1 - \alpha_k)$ holds,[7] the Pigouvain taxation scheme always leads to the higher steady state capital accumulation per capita than the capital income taxation scheme, given a target of steady state pollution stock. This is because when the share of capital to final output is relatively low compared to that of land, then under the capital income taxation scheme the tax rate should be high to obtain the given target of pollution stock. Therefore, the price of land under the capital taxation scheme will be relatively high

[7]The condition $\alpha_x \geq \alpha_k^2/(1 - \alpha_k)$ satisfies for $\alpha_k = 1/3$ and $\alpha_x \geq 1/6$ which may hold in the reality.

compared to the income of agents. As a consequence, the capital accumulation is lower than that achieved in the case of Pigouvian taxation.

The heterogenous effects of taxation schemes on capital accumulation may be the cause of the difference in the steady state welfare between schemes. Capital accumulation is highest under tge land rent taxation, which results in the highest stationary final output and stationary flow of pollution. This scheme, however, requires a higher level of mitigation than the other schemes to achieve the target on stationary stock of pollution. So, it may be difficult to evaluate the welfare effects between these taxation schemes. The following proposition provides some comparisons about consumptions which may be useful for welfare analyses.

Proposition 6: *In the overlapping generations economies with $F(K, L, X) = K^{\alpha_K} L^{\alpha_L} X^{\alpha_X}$ and $U(c_t^y, c_{t+1}^o) = \ln c_t^y + \beta \ln c_{t+1}^o$ set up above, given a target on steady state stock of pollution $E \in [0, \tilde{E})$, we have:*

(i) $c_X^y > \max\{c_P^y, c_K^y, c_L^y\}$;

(ii) *if* $\xi/\gamma \leq \alpha_X + \alpha_K$, *then* $c_X^o > c_K^o$;

(iii) *if* $\tau^P \leq 1 - \left[\frac{\alpha_K+\alpha_X}{\alpha_K+\alpha_X(1-\tau^K)^{-1}}\right]^{\alpha_K}$, *then* $c_P^y \geq c_K^y$ *and* $c_P^o > c_K^o$;

(iv) *if* $\tau^P \geq 1 - \left[\frac{\alpha_X(1-\tau^X)+\alpha_K}{\alpha_X+\alpha_K}\right]^{1-2\alpha_K}$, *then* $c_X^o > c_P^o$;

(v) *if* $\tau^L \geq 1 - \left[\frac{\alpha_X(1-\tau^X)+\alpha_K}{\alpha_X+\alpha_K}\right]^{\frac{1-2\alpha_K}{\alpha_K}}$, *then* $c_X^o > c_L^o$.

Proof: See Appendix A3.

Proposition 6 provides interesting results on comparisons of consumption between taxation schemes, given a target of stationary pollution stock. These results straightforwardly concern comparisons of welfare between the taxation schemes. Under given conditions and a given target of pollution stock, the land rent taxation scheme provides a higher welfare than the other schemes. In statement (i) of proposition 6, the consumption when young is highest under the land rent taxation scheme simply because that this scheme leads to the highest capital accumulation, hence the greatest disposable labor income, while the savings rate under logarithm utility is constant.

In statement (ii), when the mitigation technology is effective enough, in particular $\xi/\gamma \leq \alpha_X + \alpha_K$,[8] then the consumption when old under land rent taxation is higher than that under capital income taxation, i.e. $c_X^o > c_K^o$. This can be interpreted as follows. For a given target of stationary pollution stock $E \in [0, \tilde{E})$, the better the mitigation technology, the lower the tax rates imposed on land rent and capital income corresponding to the two taxation schemes. Hence, the relative gap of capital per capita, k_X/k_K is closer to 1,[9] but always greater than 1. The lower relative gap of capital k_X/k_K has two opposite effects on the relative gap of consumption c_X^o/c_K^o. On the one hand, the lower relative gap k_X/k_K leads to a lower relative

[8]Note that this condition is just a sufficient condition under which $c_X^o > c_K^o$ holds.

[9]Indeed, in the proof of proposition 4 we know that

$$\frac{k_X}{k_K} = \left[\frac{\alpha_K + \alpha_X(1-\tau^K)^{-1}}{\alpha_K + \alpha_X(1-\tau^X)}\right]^{\frac{1}{1-\alpha_K}}$$

which is increasing in τ^K and τ^X.

gap of labor income, and hence a lower relative gap of savings between land rent and capital income taxation schemes. On the other hand, the lower k_X/k_K decreases the difference in returns to capital, $[1 + z(E)\alpha_K k_X^{\alpha_K - 1}]/[1 + (1-\tau^K)z(E)\alpha_X k_K^{\alpha_K - 1}]$, between two taxation schemes. When the mitigation technology is effective enough, the difference in returns to capital, under the land rent taxation scheme and capital income taxation scheme, is strictly dominated by the corresponding difference in savings, $(k_X/k_K)^{\alpha_K}$. As a consequence, $c_X^o > c_K^o$.

Statement (iii) provides a sufficient condition relating to Pigouvain tax rate and capital income tax rate in comparing the consumption under these two taxation schemes. Given a target of stationary pollution stock $E \in [0, \tilde{E})$, if the Pigouvian tax rate is relatively low compared to capital income tax rate, in particular if $\tau^P \leq 1 - \left[\frac{\alpha_K + \alpha_X}{\alpha_K + \alpha_X(1-\tau^K)^{-1}}\right]^{\alpha_K}$ holds,[10] then the Pigovian taxation scheme leads to higher stationary consumption for both the young and the old. This result is quite intuitive. As the proof for this statement shows, when the Pigouvian tax rate is sufficiently low compared to the capital income tax rate, then the disposable income of an agent when young and the net return to savings under Pigouvian taxation scheme are higher than those under the capital income taxation scheme. This also results in higher consumption, when young and old, under the Pigouvian taxation. It implies that, with an ambitious target of stationary pollution stock, the steady state welfare under Pigouvian taxation will be higher than that under capital income taxation.

Statements (iv) and (v) state that, for a given target on stationary pollution stock, when the Pigouvian tax rate and labor income tax rate are relatively high compared to land rent tax rate, in particular $\tau^P \geq 1 - \left[\frac{\alpha_X(1-\tau^X)+\alpha_K}{\alpha_X+\alpha_K}\right]^{1-2\alpha_K}$ and $\tau^L \geq 1 - \left[\frac{\alpha_X(1-\tau^X)+\alpha_K}{\alpha_X+\alpha_K}\right]^{\frac{1-2\alpha_K}{\alpha_K}}$, then the steady state consumption when old under land rent taxation scheme are strictly higher than that under Pigouvian and labor income taxation schemes, $c_X^o > c_P^o$ and $c_X^o > c_L^o$. That is because the relatively high Pigouvian tax rate not only reduces disposable income, and hence savings, but also sufficiently lower the net returns to savings. The relatively high labor income tax rate leads to low disposable income, hence low savings. So the steady state consumption when old under these taxation schemes will be lower than that under the land rent taxation scheme. By combining this result with the result in statement (i), we observe that the land rent taxation scheme may provide a strictly higher welfare than Pigouvian taxation and labor income taxation schemes.

6 Social planner's allocation vs. pure competitive allocation

In this section, we consider the optimal allocation from the viewpoint of a benevolent social planner. The social planner's problem is

$$\max_{\left\{c_t^y, c_t^o, k_{t+1}, M_t, E_t\right\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \frac{u(c_t^y) + (1+R)v(c_t^o)}{(1+R)^t}$$

subject to, $\forall t = 0, 1, 2, ...$

[10]This condition may hold for an ambitious target for stationary pollution stock. For instance, suppose that the target for stationary stock of pollution is zero, $E = 0$. For this extreme target, that condition boils down

$$\frac{\xi}{\gamma} \leq 1 - \left[\frac{\alpha_K + \alpha_X}{\alpha_K + \alpha_X \gamma \alpha_K/(\gamma\alpha_K - \xi)}\right]^{\alpha_K}$$

which is satisfied when $\gamma\alpha_K$ is very close to ξ.

$$c_t^y + c_t^o + k_{t+1} + M_t \leq z(E_{t-1})F^t + k_t$$

$$E_t = (1-\delta)E_{t-1} + \xi z(E_{t-1})F^t - \gamma M_t$$

for given initial conditions $k_0 > 0$ and E_{-1}, and $R > 0$ is the subjective discount rate of the social planner.

The Lagrangian for the maximization problem is

$$\mathcal{L} = \sum_{t=0}^{+\infty} \frac{u(c_t^y) + (1+R)v(c_t^o)}{(1+R)^t} + \sum_{t=0}^{+\infty} \frac{\mu_t \left[z(E_{t-1})F^t + k_t - c_t^y - c_t^o - k_{t+1} - M_t\right]}{(1+R)^t}$$
$$+ \sum_{t=0}^{+\infty} \frac{\eta_t \left[E_t - (1-\delta)E_{t-1} - \xi z(E_{t-1})F^t + \gamma M_t\right]}{(1+R)^t}$$

where μ_t and η_t are Lagrangian multipliers (or shadow prices) of constraints. The social planner's allocation $\left\{c_t^{y,s}, c_t^{o,s}, k_{t+1}^s, M_t^s, E_t^s\right\}_{t=0}^{+\infty}$ associated with sequences of Lagrangian multipliers $\{\mu_t, \eta_t\}_{t=0}^{+\infty}$ which are characterized by

$$\frac{u'(c_t^{y,s})}{v'(c_{t+1}^{o,s})} = 1 + \frac{\gamma - \xi}{\gamma} z(E_t^s) F_K^{t+1,s} \tag{37}$$

$$c_t^{y,s} + c_t^{o,s} + k_{t+1}^s + M_t^s - z(E_{t-1}^s)F^{t,s} - k_t^s = 0 \tag{38}$$

$$E_t^s - (1-\delta)E_{t-1}^s - \xi z(E_{t-1}^s)F^{t,s} + \gamma M_t^s = 0 \tag{39}$$

$$\frac{\mu_{t+1} - \eta_{t+1}(1+\xi-\delta)}{1+R} z'(E_t)F^{t+1,s} + \eta_t = 0 \tag{40}$$

$$\mu_t - \eta_t \gamma = 0 \tag{41}$$

$$u'(c_t^{y,s}) = \mu_t \tag{42}$$

$$v'(c_{t+1}^{o,s}) = \mu_{t+1} \tag{43}$$

$\forall t \in \mathbb{N}$, where $E_{-1}^s = E_{-1}$ and $k_0^s = k_0$ are given.

The social planner's choice $(c^{y,s}, c^{o,s}, k^s, M^s, E^s)$ at the steady state is characterized by

$$\frac{u'(c^{y,s})}{v'(c^{o,s})} = 1 + R \tag{44}$$

$$z(E^s)F_K^s = \frac{\gamma R}{\gamma - \xi} \tag{45}$$

$$z'(E^s)F^s = \frac{R + \delta}{\xi - \gamma} \tag{46}$$

$$c^{y,s} + c^{o,s} + M^s = z(E^s)F^s \tag{47}$$

$$\delta E^s = \xi z(E^s)F^s - \gamma M^s \tag{48}$$

The social planner's optimal allocation and steady state allocation are fully derived in Appendix A4.

Without any policy instrument, the competitive allocation $\left\{c_t^{y,c}, c_t^{o,c}, k_{t+1}^c, M_t^c, E_t^c\right\}_{t=0}^{+\infty}$ is characterized as follows[11]

$$\frac{u'(c_t^{y,c})}{v'(c_{t+1}^{o,c})} = 1 + z(E_t^c)F_K^{t+1,c}$$

$$c_t^{y,c} + k_{t+1}^c + p_t = z(E_{t-1}^c)F_L^{t,c}$$

$$c_{t+1}^{o,c} = \left[1 + z(E_t^c)F_K^{t+1,c}\right] k_{t+1}^c + z(E_t^c)F_X^{t+1,c} + p_{t+1}$$

$$E_t^c = (1-\delta)E_{t-1}^c + \xi z(E_{t-1}^c)F^{t,c}$$

$$M_t^c = 0$$

given $k_0^c = k_0$, $E_{-1}^c = E_{-1}$, and price sequence of land in which each pair (p_t, p_{t+1}) satisfies the following no-arbitrage condition

$$[1 + z(E_t^c)F_K^{t+1,c}]p_t = z(E_t^c)F_X^{t+1,c} + p_{t+1}$$

At the steady state, the competitive allocation $(c^{y,c}, c^{o,c}, k^c, M^c, E^c)$ is characterized by

$$\frac{u'(c^{y,c})}{v'(c^{o,c})} = 1 + z(E^c)F_K^c \tag{49}$$

$$c^{y,c} + k^c + \frac{F_X^c}{F_K^c} = z(E^c)F_L^c \tag{50}$$

$$c^{o,c} = [1 + z(E^c)F_K^c]\,(k^c + \frac{F_X^c}{F_K^c}) \tag{51}$$

$$\delta E^c - \xi z(E^c)F^c = 0 \tag{52}$$

$$M^c = 0 \tag{53}$$

Let's compare the competitive steady state described by the system of equations from (49) to (53) with the social planner's steady state described by the system of equations from (44) to (48). Indeed, these two steady states are different. This difference not only comes from the imperfect altruism in the competitive overlapping generations model compared to the social planner's Ramsey model, but also from the ability of the social planner to internalize the negative effect of pollution on production. In this case the competitive steady state allocation is a sub-optimal allocation. In the next section we study the policy intervention of the social planner in order to decentralize the social planner's allocation as a competitive outcome.

[11] For convenience in denoting variables, we use upper scripts "s" and "c" to denote variables and functions under the social planner's allocation and pure competitive allocation, respectively.

7 Implementation of social planner's optimal allocation

As metioned in the previous section, the difference between competitive allocation and social planner's allocation comes from two sources of inefficiency in the pure competitive economy. In the pure competitive economy, the mitigation level is zero because each household is negligible within its generation and hence the household has no incentive to invest in environment. This has adverse effect on the total factor productivity of the economy. In addition, each household in the pure competitive economy just considers its own welfare in its *finite* life, hence it chooses a sub-optimal of capital accumulation. In order to eliminate the inefficiency in the competitive economy above, we need, in general, no less than two policy instruments. In this section we introduce taxes on land rent and capital income as well as investment in mitigating pollution to obtain the social planner's allocation. The mitigation is always set at the optimal choice of social planner. The land rent tax and capital income tax are designed such that the household chooses the correct saving level to have social planner's capital accumulation. These taxes and mitigation are balanced by the intergenerational lump-sum tax and transfer.

One of the difficulties in decentralizing the planner's allocation is how to determine the whole sequence prices of land. These prices are linked together via the no-arbitrage conditions. In this way, the price of land today depends on that in the future and vice versa. In the pure competitive economy, it is necessary to assume that the price sequence of land is given, and an individual in any period t has perfect foresight of the price of land in $t+1$. This price information affects the individual's decisions in allocating his/her savings portfolio in terms of capital and land, and hence affects the capital accumulation for production. When a policy is introduced, the whole sequence of land prices may be affected. So, under the policy of the social planner, the sequence of land prices should be well determined. In this section, we propose a policy under which the land prices are not only determined but also unchanged compared to the sequence in the pure competitive economy.

Along with taxes on capital income, τ^K_{t+1}, and land rent, τ^X_{t+1}, the social planner introduces T^y_t as a lump-sum tax (if negative), levied on individual t's income when young and T^o_{t+1} as a lump-sum transfer (if positive) to the same individual when old in period $t+1$. The planner invests $M_t = M^s_t$ for mitigation. The problem of the individual t is then

$$\max_{c^y_t, k_{t+1}, c^o_{t+1}} u(c^y_t) + v(c^o_{t+1})$$

subject to

$$c^y_t + k_{t+1} + p_t \leq z(E_{t-1})F^t_L + T^y_t$$

$$c^o_{t+1} \leq [1 + (1-\tau^K_{t+1})z(E_t)F^{t+1}_K]k_{t+1} + (1-\tau^X_{t+1})z(E_t)F^{t+1}_X + p_{t+1} + T^o_{t+1}$$

where

$$E_t = (1-\delta)E_{t-1} + \xi z(E_{t-1})F^t - \gamma M^s_t$$

The balanced budget constraint in each period $t+1$ implies

$$T^y_t + T^o_t + \tau^K_t z(E_{t-1})F^t_K k_t + \tau^X_t z(E_{t-1})F^t_X = M_t$$

Under the balanced policy $(T^y_t, T^o_{t+1}, \tau^K_{t+1}, \tau^X_{t+1}, M_t)$, the competitive equilibrium is characterized by

$$\frac{u'(c_t^y)}{v'(c_{t+1}^o)} = 1 + (1-\tau_{t+1}^K) z(E_t) F_K^{t+1,c}$$

$$c_t^y + k_{t+1} + p_t = z(E_{t-1}) F_L^t + T_t^y$$

$$c_{t+1}^o = [1 + (1-\tau_{t+1}^K) z(E_t) F_K^{t+1}] k_{t+1} + (1-\tau_{t+1}^X) z(E_t) F_X^{t+1} + p_{t+1} + T_{t+1}^o$$

$$[1 + (1-\tau_{t+1}^K) z(E_t) F_K^{t+1}] p_t = (1-\tau_{t+1}^X) z(E_t) F_X^{t+1} + p_{t+1}$$

$$E_t = (1-\delta) E_{t-1} + \xi z(E_{t-1}) F^t - \gamma M_t$$

For consistency in notations and to distinguish between the pure competitive allocation and social planner's allocation, we use the upper subscript "s" to denote variables and functions under the social planner's allocation, and we use the upper subscript "c" to denote corresponding variables and functions of pure competitive allocation.

Suppose that in some period $T > 0$, the social planner starts to introduce the policy in order to implement her/his allocation $\left\{c_t^{y,s}, c_t^{o,s}, k_{t+1}^s, M_t^s, E_t^s\right\}_{t=T}^{+\infty}$ given $k_{T+1}^s = k_T$ and $E_{T-1}^s = E_{T-1}$. At the time that the policy is first introduced, the price of land under the pure competitive economy is p_T. Suppose that this price p_T is predetermined between generations T and $T-1$. The price p_T links to p_{T-1} via the no-arbitrage condition under the pure competitive regime. Introducing policy $(T_T^y, T_{T+1}^o, \tau_{T+1}^k, \tau_{T+1}^x, M_T)$ to generation T will affect the land price in period $T+1$, p_{T+1}. We now first identify the relationship between τ_{t+1}^K and τ_{t+1}^X for all $t \geq T$, under perfect foresight on the return to land and capital, the price of land p_{t+1}^s, under the policy $(T_t^y, T_{t+1}^o, \tau_{t+1}^k, \tau_{t+1}^x, M_t)_{t\geq T}$ which decentralizes the social planner's allocation $\left\{c_t^{y,s}, c_t^{o,s}, k_{t+1}^s, M_t^s, E_t^s\right\}_{t\geq T}$, is also the price of land without any policy intervention (i.e. under pure competitive regime), i.e. $p_{t+1}^s = p_{t+1}$ $\forall t \geq T$. The following lemma presents this relation.

Lemma 7: *Under the policy $(T_t^y, T_{t+1}^o, \tau_{t+1}^K, \tau_{t+1}^X, M_t)_{t\geq T}$ in decentralizing the social planner's allocation $\left\{c_t^{y,s}, c_t^{o,s}, k_{t+1}^s, M_t^s, E_t^s\right\}_{t=T}^{+\infty}$, in order to the sequence of land prices $(p_{t+1})_{t=T}^{+\infty}$ unchanged compared to the scenario of no policy intervention, then following relationship between τ_{t+1}^K and τ_{t+1}^X must holds for all $t \geq T$, given p_T.*

$$1 - \tau_{t+1}^X = \frac{z(E_t^c) F_X^{t+1,c} - [z(E_t^c) F_K^{t+1,c} - (1-\tau_{t+1}^K) z(E_t^s) F_K^{t+1,s}] p_t}{z(E_t^s) F_X^{t+1,s}} \tag{54}$$

Proof:

When $t = T$, without any policy intervention the no-arbitrage condition is

$$[1 + z(E_T^c) F_K^{T+1,c}] p_T = z(E_T^c) F_X^{T+1,c} + p_{T+1} \tag{55}$$

With policy intervention, the corresponding no-arbitrage condition becomes

$$[1 + (1-\tau_{T+1}^K) z(E_T^s) F_K^{T+1,s}] p_T^s = (1-\tau_{T+1}^X) z(E_T^s) F_X^{T+1,s} + p_{T+1}^s \tag{56}$$

Since $p_T^s = p_T$ then for $p_{T+1}^s = p_{T+1}$, it must hold from (55) and (56) that

$$1-\tau^X_{T+1}=\frac{z(E^c_T)F^{T+1,c}_X-[z(E^c_T)F^{T+1,c}_K-(1-\tau^K_{T+1})z(E^s_T)F^{T+1,s}_K]p_T}{z(E^s_T)F^{T+1,s}_X}$$

By the induction method, we argue that the condition (54) holds for all $t\geq T$.
Q.E.D.

Indeed, the capital income tax and land rent tax introduced in lemma 7 have two opposite effects which completely offset each other and keep the sequence of land prices unchanged compared to that in the scenario of no policy intervention. This construction of taxation helps us to control the land price in decentralizing the social planner's allocation. The following proposition states the strategies of decentralizing such an allocation.

Proposition 8: *In the overlapping generations set up above, the social planner's optimal allocation from any period $T\geq 0$ onward can be obtained as a competitive outcome by following a period-by-period balanced budget tax and transfer policy and mitigation: announcing in any period $t\geq T$ the policy instruments $(T^y_t, T^o_{t+1}, \tau^K_{t+1}, \tau^X_{t+1}, M_t)_{t\geq T}$ which is characterized by*

$$\tau^K_{t+1}=\frac{\xi}{\gamma}\equiv\tau^K_*$$

$$\tau^X_{t+1}=1-\frac{z(E^c_t)F^{t+1,c}_X-[z(E^c_t)F^{t+1,c}_K-(1-\tau^K_*)z(E^s_t)F^{t+1,s}_K]p_t}{z(E^s_t)F^{t+1,s}_X}$$

$$T^y_t=c^{y,s}_t+k^s_{t+1}+p_t-z(E^s_{t-1})F^{t,s}_L$$

$$T^o_{t+1}=c^{o,s}_{t+1}-[1+(1-\tau^k_*)z(E^s_t)F^{t+1,s}_K](k^s_{t+1}+p_t)$$

$$M_t=M^s_t$$

given $k^s_T=k_T$ and $E^s_{T-1}=E_{T-1}$, will be implemented. Note that in period T, the lump-sum transfer (or tax) to the old is $T^o_T=M^s_T-T^y_T$, which guarantees the government budget in period T to be balanced.

Proof:

Indeed, in period T, by choosing $M_T=M^s_T$, we have $E_T=E^s_T$. Under the policy $(T^y_T, T^o_{T+1}, \tau^K_*, \tau^X_{T+1}, M^s_T)$, the Euler equation is

$$\frac{u'(z(E_{T-1})F^T_L+T^y_t-k_{T+1}-p_T)}{v'([1+\frac{\gamma-\xi}{\gamma}z(E^s_T)F^{T+1}_K](k_{T+1}+p_T)+T^o_{T+1})}=1+\frac{\gamma-\xi}{\gamma}z(E^s_T)F^{T+1}_K \qquad (57)$$

Note that, by construction, in period T we have $F^{T,s}_L=F^T_L$ and $E^s_{T-1}=E_{T-1}$. It is obvious that if $k_{T+1}=k^s_{T+1}$ then by construction, $c^y_T=c^{y,s}_T$ and $c^o_{T+1}=c^{o,s}_{T+1}$ and hence the Euler equation (57) obviously holds because it exactly coincides with the Euler equation (37) under the allocation of the social planner. We now prove that, under the policy $(T^y_T, T^o_{T+1}, \tau^K_*, \tau^X_{T+1}, M^s_T)$, the optimal choice of household savings in terms of capital $k_{T+1}=k^s_{T+1}$ is unique. In effect, suppose that there existed $\hat{k}_{T+1}\neq k^s_{T+1}$ to be also optimal choice of capital saving. Without loss of generality, suppose that $\hat{k}_{T+1} > k^s_{T+1}$, hence the corresponding consumptions $\hat{c}^y_T$ and $\hat{c}^o_{T+1}$ will be

$$\hat{c}^y_T = z(E^s_{T-1})F^{T,s}_L + T^y_T - \hat{k}_{T+1} - p_T < c^{y,s}_T$$

$$\hat{c}^o_{T+1} = \left[1 + (1-\tau^K_*)z(E^s_{T-1})\hat{F}^{T+1}_K\right]\hat{k}_{T+1} + (1-\tau^X_{T+1})z(E^s_{T-1})\hat{F}^{T+1}_X + p_{T+1} > \hat{c}^{o,s}_{T+1}$$

Therefore,

$$\frac{u'(\hat{c}^y_T)}{v'(\hat{c}^o_{T+1})} > \frac{u'(c^y_T)}{v'(c^o_{T+1})} = 1 + \frac{\gamma-\xi}{\gamma}z(E^s_T)F^{T+1,s}_K > 1 + \frac{\gamma-\xi}{\gamma}z(E^s_T)\hat{F}^{T+1}_K$$

i.e. the Euler equation is no longer satisfied, contradicting $(\hat{c}^y_T, \hat{k}_{T+1}, \hat{c}^o_{T+1})$ is the optimal choice of the household. So $(c^{y,s}_T, k^s_{T+1}, c^{o,s}_s)$ is the unique optimal choice of a household under the policy $(T^y_T, T^o_{T+1}, \tau^K_*, \tau^X_{T+1}, M^s_T)$.

We next prove that, from period $T+1$ onwards, the policy described in proposition 8 is period-and-period budget balanced and will implement the social planner's allocation. Without of loss any generality, we consider the budget constraint and allocation in period $T+1$. Under the policy $(T^y_T, T^o_{T+1}, \tau^k_*, \tau^x_{T+1}, M^s_T)$ applied for generation T, we have $k_{T+1} = k^s_{T+1}$ and $E_T = E^s_T$ are given for the generation $T+1$. Therefore, the balanced government budget constraint under the policy $(T^y_{T+1}, T^o_{T+2}, \tau^K_*, \tau^X_{T+2}, M^s_{T+1})$, applied for the generation $T+1$, requires that it holds

$$T^y_{T+1} + T^o_{t+1} + M^s_{T+1} = \tau^K_* z(E^s_T)F^{T+1,s}_K k^s_{T+1} + \tau^X_{T+1} z(E^s_T)F^{T+1,s}_X$$

Indeed, using the no arbitrage condition, the last equation is equivalent to

$$c^{y,s}_{T+1} + k^s_{T+2} + p_{T+1} - z(E^s_T)F^{T+1,s}_L + c^{o,s}_{T+1} - [1 + (1-\tau^K_*)z(E^s_T)F^{T+1,s}_K](k^s_{t+1} + p_t) + M^s_{T+1}$$

$$= \tau^K_* z(E^s_T)F^{T+1,s}_K k^s_{T+1} + z(E^s_T)F^{T+1,s}_X - [1 + (1-\tau^K_*)z(E^s_T)F^{T+1,s}_K]p_T + p_{T+1}$$

which boils down

$$c^{y,s}_{T+1} + k^s_{T+2} + c^{o,s}_{T+1} + M^s_{T+1} = z(E^s_T)F^{T+1} + k^s_{T+1}$$

i.e., the feasibility constraint of the social planner.

Similar to the proof about optimal choice of generation T above, we also prove that the optimal choice of generation $T+1$ under the policy $(T^y_{T+1}, T^o_{T+2}, \tau^K_*, \tau^X_{T+2}, M^s_{T+1})$ is $(c^y_{T+1}, k_{T+2}, c^o_{T+2})$. This argument can be induced for all $t \geq T+1$.

Q.E.D.

Under the policy $(T^y_t, T^o_{t+1}, \tau^K_{t+1}, \tau^X_{t+1}, M_t)_{t\geq T}$ introduced in proposition 8, the environmental externality on production, and the imperfect altruism between generations are fully corrected during the entire transitional phase. In each period $t \geq T$, the tax rate on capital income is fixed at $\tau^K_* = \xi/\gamma$, which is derived by comparing the social planner's Euler equation and the Euler equation under decentralized allocation. The tax rate on land rent alters over time depending on the state of the economy so as to keep the sequence of land prices unchanged compared to pure competitive allocation. In the meantime, the mitigation and intergenerational transfers are set such that the budget is balanced, in which the agents will choose the savings in terms of capital to be the capital level under the social planner's allocation and the pollution stock evolves according to the social planner's choice. As a consequence, the allocation under the policy exactly coincides with the social planner's optimal allocation, i.e. the social planner's optimal allocation is implemented.

In this implementation, the generation born in $T-1$ may not be worse off when the stock of pollution E_{T-1} is low and capital accumulation k_T is sufficiently high so that the labor income of an agent is sufficiently high and the transfer T_T^y is strictly positive and dominates the mitigation M_T^s. Hence, in this case the transfer to the agent $T-1$, T_T^o will be non-negative. The consumption when young, and savings of generation T, will decrease in this case but the welfare of this generation does not necessarily decrease because the positive effect of lower stock of pollution on productivity may improve the return to capital and hence increase the consumption when old, offsetting the welfare loss from the decrease in consumption when young. These results depend on the state of the economy, capital accumulation level and environmental quality, at the timing of the implementation and responsiveness of damage function $z(E)$ to the stock of pollution E. They imply that the timing for triggering implementation is important to avoid the conflict between current and future generations.

8 Conclusion

In this paper we set up an overlapping generations economy with fixed a factor of production and environmental externality to study the heterogenous effects on capital accumulation and welfare of alternatives tax and mitigation policies. Four taxation schemes are examined: land rent taxation, capital income, labor income, and Pigouvian taxation. In section 5, we show that, given plausible mitigation technology in relation to other parameters of the model, and a given target of steady state stock of pollution, the land rent taxation scheme leads to highest capital accumulation, highest consumption and hence highest welfare of all the schemes. This result implies that land rent taxation should be an efficient instrument for combating climate change when the mitigation technology is appropriate.

In addition, in section 7, we described the period-by-period budget balanced policy to decentralize the socially optimal allocation during the entire transitional phase to the social planner's steady state allocation. One of the difficulties of decentralizing the socially optimal allocation is how to determine the whole sequence of land prices. In this paper, we propose a policy in which the land prices are not only determined but also unchanged compared to those under a purely competitive economy. In this policy, the tax rates on capital income and land rent are set so as to keep the land prices unchanged and make the Euler equation, under a competitive economy with taxation, coincide with that of the social planner. In the meantime the intergenerational transfers accompanied by the tax revenue from land rent and capital income taxation are used for mitigation and to keep the government budget balanced.

The model in this paper could be extended in at least two ways. Firstly, the current paper solely considers the model of the homogenous households, so it would be interesting to extend the model to incorporate the issue of inequality. In other words, the households are heterogenous in land ownership and incomes, and hence the land rent taxation may generate double dividends in combating climate change and reducing inequality. Secondly, we could consider the directed technical change by introducing multiple sectors of intermediate production which include clean and dirty sectors. The aggregate land rent tax may be used along with the tax imposed on dirty sectors in order to subsidize clean innovations. These ideas are on our research agenda.

Appendix

A1. Proof of proposition 1

(i) In effect, from (22) and (23) we have

$$k = \left[\frac{\beta\alpha_L\alpha_K z(E)}{(1+\beta)\left[\alpha_K + (1-\tau^X)\alpha_X\right]}\right]^{\frac{1}{1-\alpha_K}} \tag{58}$$

$$\frac{\delta E}{z(E)} = (\xi - \gamma\tau^X\alpha_X)k^{\alpha_K} \tag{59}$$

Substitute (58) into (59), and with a simple transformation, we have

$$z(E)^{\frac{1}{\alpha_K-1}}E = g(\tau^X) \tag{60}$$

where

$$g(\tau^X) = \frac{\xi - \gamma\tau^X\alpha_X}{\delta}\left[\frac{\beta\alpha_L\alpha_K z(E)}{(1+\beta)\left[\alpha_K + (1-\tau^X)\alpha_X\right]}\right]^{\frac{\alpha_K}{1-\alpha_K}}$$

It is straightforward that for all $\tau^X \in [0,1]$, $g(\tau^X)$ is bounded, while, from the assumptions on $z(E)$, the left hand side of (60) is continuous and monotonically increasing in E and

$$\lim_{E\to-\infty}\left(z(E)^{\frac{1}{\alpha_K-1}}E\right) = -\infty \qquad \text{and} \qquad \lim_{E\to+\infty}\left(z(E)^{\frac{1}{\alpha_K-1}}E\right) = +\infty$$

Therefore, for all $\tau^X \in [0,1]$, there always exists a unique solution E to equation (60), which implies that there always exists a unique steady state.

For graphical representation of the existence and uniqueness of the steady state, we rewrite (60) as

$$E = g(\tau^X)z(E)^{\frac{1}{1-\alpha_K}} \tag{61}$$

Since $z(E) > 0\ \forall E$ then the sign of the right hand side of (61) is the sign of $g(\tau^X)$, while the left hand side of (61) is the 45° line. From the assumptions of $z(E)$, the existence and uniqueness of the steady state is straightforwardly depicted by the following graph.

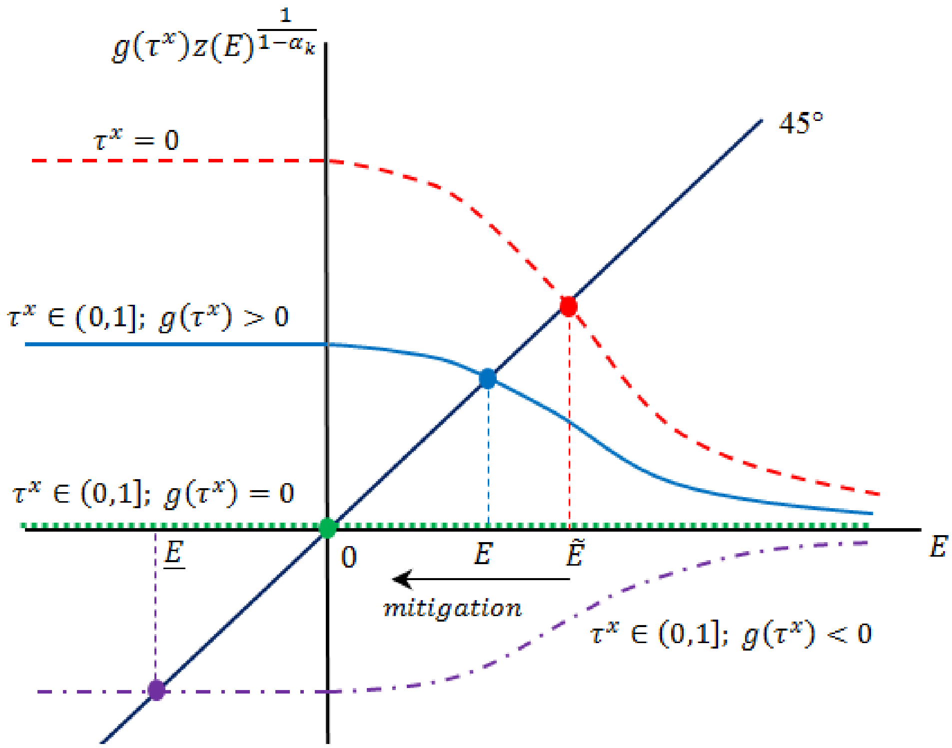

Fig 1. The existence and uniqueness of the steady state

(ii) Applying the implicit function theorem for (60) we have

$$\frac{\partial E}{\partial \tau^X} = \frac{(1-\alpha_K)z(E)^{\frac{2-\alpha_K}{1-\alpha_K}} g'(\tau^X)}{(1-\alpha_K)z(E) - z'(E)E} \tag{62}$$

It is obvious that

$$\text{sign}\left(\frac{\partial E}{\partial \tau^X}\right) \equiv \text{sign}\, g'(\tau^X) \tag{63}$$

We have

$$g'(\tau^X) = \left(\frac{\beta\alpha_L\alpha_K}{(1+\beta)\left[\alpha_K + (1-\tau^X)\alpha_X\right]}\right)^{\frac{\alpha_k}{1-\alpha_k}} \frac{\alpha_X}{\delta}\left[\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma\right] \tag{64}$$

where

$$\hat{g}(\tau^X) = \frac{\xi - \gamma\tau^X\alpha_X}{\alpha_K + (1-\tau^X)\alpha_X}$$

Hence, for all $\tau^X \in [0,1]$,

$$\text{sign}\, g'(\tau^X) \equiv \text{sign}\left[\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma\right] \tag{65}$$

We have

$$\hat{g}'(\tau^X) = \frac{\xi - \gamma(\alpha_K + \alpha_X)}{[\alpha_K + (1-\tau^X)\alpha_X]^2}\alpha_x > (=)(<)\, 0 \quad \Leftrightarrow \quad \frac{\xi}{\gamma} > (=)(<)\, \alpha_K + \alpha_K$$

- If $\frac{\xi}{\gamma} > \alpha_K + \alpha_X$ then $\hat{g}'(\tau^X) > 0$, hence $\forall \tau^X \in [0,1]$, $\hat{g}(\tau^X) < \hat{g}(1) = \frac{\xi - \gamma\alpha_X}{\alpha_K}$, therefore

$$\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma < \frac{\alpha_K}{1-\alpha_K}\frac{\xi - \gamma\alpha_X}{\alpha_K} - \gamma = \frac{\xi - \gamma(1-\alpha_K+\alpha_X)}{1-\alpha_K} < 0 \tag{66}$$

because of the assumption $\frac{\xi}{\gamma} < 1 - \alpha_K + \alpha_X$.

So, from (63), (65), and (66) we have $\frac{\partial E}{\partial \tau^X} < 0$.

- If $\frac{\xi}{\gamma} = \alpha_K + \alpha_X$ then $\hat{g}'(\tau^X) = 0$, hence $\hat{g}(\tau^X) = \frac{\gamma(\alpha_K+\alpha_X) - \gamma\tau^X\alpha_X}{\alpha_K + (1-\tau^X)\alpha_X} = \gamma$, therefore (because $\alpha_K < \frac{1}{2}$)

$$\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma = \gamma\left(\frac{\alpha_K}{1-\alpha_K} - 1\right) < 0 \tag{67}$$

So, from (63), (65), and (67) we have $\frac{\partial E}{\partial \tau^X} < 0$.

- If $\frac{\xi}{\gamma} < \alpha_K + \alpha_X$ then $\hat{g}'(\tau^X) < 0$, hence $\forall \tau^X \in [0,1]$, $\hat{g}(\tau^X) < \hat{g}(0) = \frac{\xi}{\alpha_K + \alpha_X} < \gamma$, therefore (because $\alpha_K < \frac{1}{2}$)

$$\frac{\alpha_K}{1-\alpha_K}\hat{g}(\tau^X) - \gamma < \gamma\left(\frac{\alpha_K}{1-\alpha_K} - 1\right) < 0 \tag{68}$$

So, from (63), (65), and (68) we have $\frac{\partial E}{\partial \tau^X} < 0$.

In summary, we have

$$\frac{\partial E}{\partial \tau^X} < 0$$

We know from (58) that $k_X = \left[\frac{\beta\alpha_L\alpha_K z(E)}{(1+\beta)[\alpha_K + (1-\tau^X)\alpha_X]}\right]^{\frac{1}{1-\alpha_K}}$, where as in (60), E can be represented as a function of τ^X only, then $\forall \tau^X \in (0,1)$, it holds

$$\frac{\partial k_X}{\partial \tau^X} = \left[\frac{\beta\alpha_L\alpha_K z(E)}{(1+\beta)\left[\alpha_K + (1-\tau^X)\alpha_X\right]}\right]^{\frac{1}{1-\alpha_K}} \frac{\frac{z'(E)}{z(E)}\frac{\partial E}{\partial \tau^X}\left[\alpha_K + (1-\tau^X)\alpha_X\right] + \alpha_X}{(1-\alpha_K)\left[\alpha_K + (1-\tau^X)\alpha_X\right]} > 0 \tag{69}$$

Q.E.D.

A2. Proof of proposition 5

(i) We compare (29) to (58) by equalizing the stock of pollution in both equations. So, we have

$$\frac{k_K}{k_X} = \left[\frac{\alpha_K + \alpha_X(1-\tau^X)}{\alpha_K + \alpha_X(1-\tau^K)^{-1}}\right]^{\frac{1}{1-\alpha_K}} \tag{70}$$

where k_K and k_X are stationary capital per capita under capital income taxation and land rent taxation schemes as determined in (29) and (58), respectively. Since $\tau^X, \tau^K \in (0,1)$ then it is straightforward that $k_K/k_X < 1$, i.e.

$$k_K < k_X$$

Similarly, we also compare (36) to (58) by equalizing the stock of pollution in both equations. We have

$$\frac{k_P}{k_X} = \left[\frac{(1-\tau^P)\left[\alpha_K + (1-\tau^X)\alpha_X\right]}{\alpha_K + \alpha_X}\right]^{\frac{1}{1-\alpha_K}} \tag{71}$$

where k_P is the steady state capital per capita under a Pigouvian tax scheme as determined in (36). Since $\tau^X, \tau^P \in (0,1)$ then it is obvious that $k_P/k_X < 1$, i.e.

$$k_P < k_X$$

(ii) We have from (33) and (36), and setting $E_L = E_P$, that

$$\frac{k_P}{k_L} = \left(\frac{1-\tau^P}{1-\tau^L}\right)^{\frac{1}{1-\alpha_K}}$$

Suppose that $k_P \leq k_L$ then from the last equation we had $\tau^P \geq \tau^L$. Since $E_L = E_P$ then it holds that

$$(\xi - \gamma\tau^L\alpha_L)k_L^{\alpha_K} = (\xi - \gamma\tau^P\alpha_P)k_P^{\alpha_K} = \frac{\delta E}{z(E)}$$

But for $k_P \leq k_L$ and $\tau^P \geq \tau^L$ we obtain a contradiction to the last equality that

$$(\xi - \gamma\tau^L\alpha_L)k_L^{\alpha_K} > (\xi - \gamma\tau^P\alpha_P)k_P^{\alpha_K}$$

Therefore,

$$k_P > k_L$$

(iii) We have

$$\frac{k_L}{k_K} = \left[\frac{1-\tau^L}{1-\tau^K}\frac{\alpha_X + (1-\tau^K)\alpha_K}{\alpha_X + \alpha_K}\right]^{\frac{1}{1-\alpha_K}} \tag{72}$$

Hence, $k_L > (=)(<)\, k_K$ is equivalent to

$$\frac{1-\tau^L}{1-\tau^K}\frac{\alpha_X + (1-\tau^K)\alpha_K}{\alpha_X + \alpha_K} > (=)(<)\, 1 \quad \Leftrightarrow \quad \tau^L\left[1 + (1-\tau^K)\frac{\alpha_K}{\alpha_X}\right] < (=)(>)\, \tau^K$$

- First, we show that if $1-\tau^K = \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2$ then $k_K = k_L$. Suppose a contradiction that $k_K \neq k_L$. Without lost of generality, we assume that $k_K < k_L$. Hence,

$$\tau^L > \frac{\alpha_K}{\alpha_L}\tau^K \qquad \text{since } (\xi - \gamma\tau^L\alpha_L)k_L^{\alpha_K} = (\xi - \gamma\tau^K\alpha_K)k_K^{\alpha_K}$$

i.e.

$$\tau^L > \frac{\alpha_K}{\alpha_L}\left[1 - \frac{\alpha_X(\alpha_L - \alpha_K)}{\alpha_K^2}\right] \tag{73}$$

So from (72), (73), and given that $1 - \tau^K = \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2$, we have

$$\begin{aligned}\left(\frac{k_L}{k_K}\right)^{1-\alpha_K} &= (1-\tau^L)\frac{\alpha_X(1-\tau^K)^{-1} + \alpha_K}{\alpha_X + \alpha_K} \\ &< \left(1 - \frac{\alpha_K}{\alpha_L}\left[1 - \frac{\alpha_X(\alpha_L - \alpha_K)}{\alpha_K^2}\right]\right)\frac{\alpha_K^2/(\alpha_L - \alpha_K) + \alpha_K}{\alpha_X + \alpha_K} = 1\end{aligned}$$

i.e. $k_L < k_K$, which contradicts the initial assumption that $k_L > k_K$. An analogous logic will applied in the case we assume initially that $k_K > k_L$. Therefore, it holds $k_L = k_K$.

- Second, we show that if $1 - \tau^K > \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2$ then $k_K > k_L$. Indeed, suppose that $k_K \le k_L$ then $\tau^L \ge \alpha_K\tau^K/\alpha_L$, and from (72), it would hold

$$\psi(\tau^K) = \frac{1 - \alpha_K\tau^K/\alpha_L}{1-\tau^K}\frac{\alpha_X + (1-\tau^K)\alpha_K}{\alpha_X + \alpha_K} \ge \frac{1-\tau^L}{1-\tau^K}\frac{\alpha_X + (1-\tau^K)\alpha_K}{\alpha_X + \alpha_K} \ge 1$$

We will show a contradiction that

$$\psi(\tau^K) = \left(1 - \frac{\alpha_K}{\alpha_L}\tau^K\right)\frac{\alpha_X(1-\tau^K)^{-1} + \alpha_K}{\alpha_X + \alpha_K} < 1$$

Indeed

$$\psi'(\tau^K) = -\frac{\alpha_K}{\alpha_L}\frac{\alpha_X(1-\tau^K)^{-1} + \alpha_K}{\alpha_X + \alpha_K} + \left(1 - \frac{\alpha_K}{\alpha_L}\tau^K\right)\frac{\alpha_X}{(\alpha_X + \alpha_K)(1-\tau^K)^2}$$

and, by simple transformation,

$$\text{sign } \psi'(\tau^K) \equiv \text{sign } \left[\frac{\alpha_X(\alpha_L - \alpha_K)}{\alpha_K^2} - (1-\tau^K)^2\right]$$

So for all $\tau^K \in (0,1)$ we have

$$\psi'(\tau^K) > (=)(<)\, 0 \quad \Longleftrightarrow \quad 1 - \tau^K < (=)(>)\, \frac{\sqrt{\alpha_X(\alpha_L - \alpha_K)}}{\alpha_K}$$

Hence the function $\psi(\tau^K)$ get unique minimum at $\tau^K = 1 - \frac{\sqrt{\alpha_X(\alpha_L - \alpha_K)}}{\alpha_K} \in (0, 1 - \frac{\alpha_X(\alpha_L - \alpha_K)}{\alpha_K^2})$. In addition, we have

$$\psi(0) = \psi\left(1 - \frac{\alpha_X(\alpha_L - \alpha_K)}{\alpha_K^2}\right) = 1$$

Therefore, for all $\tau^K \in (0, 1 - \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2)$, i.e. $1 - \tau^K > \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2$, we have $\psi(\tau^K) < 1$ which contradicts the result $\psi(\tau^K) \ge 1$. Hence, if $1 - \tau^K > \alpha_X(\alpha_L - \alpha_K)/\alpha_K^2$ then $k_K > k_L$.

- Finally, it is quite similar to the previous proof, we have if $1-\tau^K < \alpha_X(\alpha_L-\alpha_K)/\alpha_K^2$ then $k_K < k_L$.

(iv) When both capital taxation and Pigouvian taxation schemes leads to some given target on steady state stock of pollution, it holds

$$\frac{\delta E}{z(E)} = (\xi - \gamma\tau^P)k_P^{\alpha_K} = (\xi - \gamma\alpha_K\tau^K)k_K^{\alpha_K} \tag{74}$$

On the other hand we have from (70) and (71) that

$$\left(\frac{k_K}{k_P}\right)^{1-\alpha_K} = \frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\tau^P)[\alpha_K(1-\tau^K)+\alpha_X]} \tag{75}$$

Suppose that $k_K \geq k_P$ then from (74) we have

$$\tau^P \leq \alpha_K\tau^K \tag{76}$$

From (75) we have

$$\frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\tau^P)[\alpha_K(1-\tau^K)+\alpha_X]} \geq 1$$

In fact, from (76) we have

$$\frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\tau^P)[\alpha_K(1-\tau^K)+\alpha_X]} \leq \frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\alpha_K\tau^K)[\alpha_K(1-\tau^K)+\alpha_X]}$$

We will complete the proof with a contradiction through proving that

$$\frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\alpha_K\tau^K)[\alpha_K(1-\tau^K)+\alpha_X]} < 1$$

Indeed, the last inequality is equivalent to

$$(1-\tau^K)(\alpha_K+\alpha_X) < (1-\alpha_K\tau^K)[\alpha_K(1-\tau^K)+\alpha_X] \qquad \Longleftrightarrow \qquad 1-\tau^K < \frac{\alpha_X(1-\alpha_K)}{\alpha_K^2}$$

which always holds because $\tau^K \in (0,1)$ and $\alpha_X \geq \alpha_K^2/(1-\alpha_K)$. Therefore, we have

$$k_K < k_P$$

Q.E.D.

A3. Proof of proposition 6

(i) It is fairly straightforward from logarithm utility function that

$$c_X^y = \frac{1}{1+\beta}z(E)F_L^X > \frac{1}{1+\beta}z(E)F_L^K = c_K^y$$

$$c_X^y = \frac{1}{1+\beta} z(E) F_L^X > \frac{1-\tau^P}{1+\beta} z(E) F_L^P = c_P^y$$

$$c_X^y = \frac{1}{1+\beta} z(E) F_L^X > \frac{1-\tau^L}{1+\beta} z(E) F_L^L = c_L^y$$

since, from proposition 5, $k_X > \max\{k_K, k_P, k_L\}$.

(ii) Note that under land rent taxation and capital income taxation, the stationary land prices are $p_X = (1-\tau^X) F_X^X / F_K^X$ and $p_K = F_X^K / [(1-\tau^K) F_K^K]$ respectively. So the consumption when old under these schemes are respectively

$$c_X^o = \left[1 + z(E) F_K^X\right] \left[k_X + \frac{(1-\tau^X) F_X^X}{F_K^X}\right]$$

and

$$c_K^o = \left[1 + (1-\tau^K) z(E) F_K^K\right] \left[k_K + \frac{F_X^K}{(1-\tau^K) F_K^K}\right]$$

With the logarithm utility function, the saving rate is constant and the savings under these schemes are $\frac{\beta}{1+\beta} F_L^x$ and $\frac{\beta}{1+\beta} F_L^k$. We have

$$k_X + \frac{(1-\tau^X) F_X^X}{F_K^X} = \frac{\beta}{1+\beta} F_L^X > \frac{\beta}{1+\beta} F_L^K = k_K + \frac{F_X^K}{(1-\tau^K) F_K^K}$$

We complete the proof by showing that

$$F_K^X k_X + (1-\tau^X) F_X^X \geq (1-\tau^K) F_K^K k_K + F_X^K \tag{77}$$

$$\iff \quad \left(\frac{k_X}{k_K}\right)^{\alpha_K} \geq \frac{(1-\tau^K)\alpha_K + \alpha_X}{\alpha_K + (1-\tau^X)\alpha_X} = \frac{\alpha_K + \alpha_X - \tau^K \alpha_K}{\alpha_K + \alpha_X - \tau^X \alpha_X}$$

For a given target on steady state stock of pollution $E \in [0, \tilde{E})$, we have

$$\frac{\delta E}{z(E)} = (\xi - \gamma \tau^X \alpha_X) k_X^{\alpha_K} = (\xi - \gamma \tau^K \alpha_K) k_K^{\alpha_K}$$

$$\Longrightarrow \left(\frac{k_X}{k_K}\right)^{\alpha_K} = \frac{\xi - \gamma \tau^K \alpha_K}{\xi - \gamma \tau^X \alpha_X} = \frac{\xi/\gamma - \tau^K \alpha_K}{\xi/\gamma - \tau^X \alpha_X}$$

Since $k_X > k_K$ then $\xi/\gamma - \tau^K \alpha_K > \xi/\gamma - \tau^X \alpha_X > 0$. Hence, it is straightforward that, when $\xi/\gamma \leq \alpha_X + \alpha_K$ then

$$\frac{\xi/\gamma - \tau^K \alpha_K}{\xi/\gamma - \tau^X \alpha_X} \geq \frac{\alpha_K + \alpha_X - \tau^K \alpha_K}{\alpha_K + \alpha_X - \tau^X \alpha_X}$$

which implies that (77) always holds. Therefore, we have

$$c_X^o > c_K^o$$

(iii) We have

$$c_K^y = \frac{z(E)\alpha_L k_K^{\alpha_K}}{1+\beta} \qquad \text{and} \qquad c_P^y = \frac{(1-\tau^P)z(E)\alpha_L k_P^{\alpha_K}}{1+\beta}$$

$$c_P^y \geq c_K^y \quad \Longleftrightarrow \quad 1-\tau^P \geq \left(\frac{k_K}{k_P}\right)^{\alpha_k}$$

By substituting k_K/k_P determined in (75) into the last inequality we have

$$1-\tau^P \geq \left[\frac{(1-\tau^K)(\alpha_K+\alpha_X)}{(1-\tau^P)[\alpha_K(1-\tau^K)+\alpha_X]}\right]^{\frac{\alpha_K}{1-\alpha_K}} \qquad \Longleftrightarrow \qquad 1-\tau^P \geq \left[\frac{(1-\tau^K)(\alpha_K+\alpha_X)}{\alpha_K(1-\tau^K)+\alpha_X}\right]^{\alpha_K},$$

$$\text{i.e.} \quad \tau^P \leq 1 - \left[\frac{\alpha_K+\alpha_X}{\alpha_K+\alpha_X(1-\tau^K)^{-1}}\right]^{\alpha_K}$$

which holds by construction.

Similarly, we have

$$c_K^o = \frac{\beta\alpha_L z(E)k_K^{\alpha_K}\left[1+(1-\tau^K)\alpha_K z(E)k_K^{\alpha_K-1}\right]}{1+\beta}$$

and

$$c_P^o = \frac{\beta(1-\tau^P)\alpha_L z(E)k_P^{\alpha_K}\left[1+(1-\tau^P)\alpha_K z(E)k_P^{\alpha_K-1}\right]}{1+\beta}$$

In order to prove $c_P^o > c_K^o$, we prove first that

$$(1-\tau^P)k_P^{\alpha_K-1} > (1-\tau^K)k_K^{\alpha_K-1} \quad \Longleftrightarrow \quad \left(\frac{k_K}{k_P}\right)^{1-\alpha_K} > \frac{1-\tau^K}{1-\tau^P}$$

Substitute k_K/k_P determined in (75) into the last inequality we have

$$\frac{\alpha_K+\alpha_X}{\alpha_K(1-\tau^K)+\alpha_X} > 1$$

which trivially holds because $\tau^K \in (0,1]$.

We also have

$$\frac{c_P^o}{c_K^o} = \frac{(1-\tau^P)k_P^{\alpha_K}}{k_K^{\alpha_K}} \times \frac{1+(1-\tau^P)\alpha_K z(E)k_P^{\alpha_K-1}}{1+(1-\tau^K)\alpha_K z(E)k_K^{\alpha_K-1}}$$

where the first fraction $(1-\tau^P)k_P^{\alpha_K}/k_K^{\alpha_K} = c_P^y/c_K^y \geq 1$ as proved above. We complete by proving that the second fraction in the last equation is strictly greater than 1. That is straightforward because from the proof above we have

$$(1-\tau^P)k_P^{\alpha_K-1} > (1-\tau^K)k_K^{\alpha_K-1}$$

Therefore,

$$c_P^o > c_K^o$$

(iv) We have

$$\frac{c_X^o}{c_P^o} = \frac{k_X^{\alpha_K}}{(1-\tau^P)k_P^{\alpha_K}} \times \frac{1+\alpha_K z(E)k_X^{\alpha_K-1}}{1+(1-\tau^P)\alpha_K z(E)k_P^{\alpha_K-1}}$$

The second fraction in the right hand side of the last satisfies

$$\frac{1+\alpha_K z(E)k_X^{\alpha_K-1}}{1+(1-\tau^P)\alpha_K z(E)k_P^{\alpha_K-1}} > \frac{k_X^{\alpha_K-1}}{(1-\tau^P)k_P^{\alpha_K-1}}$$

because, from (71) we have

$$\frac{\alpha_K z(E)k_X^{\alpha_K-1}}{(1-\tau^P)\alpha_K z(E)k_P^{\alpha_K-1}} = \frac{k_X^{\alpha_K-1}}{(1-\tau^P)k_P^{\alpha_K-1}} = \frac{\alpha_K+(1-\tau^X)\alpha_X}{\alpha_K+\alpha_X} < 1$$

Hence we have

$$\frac{c_X^o}{c_P^o} > \frac{k_X^{\alpha_K}}{(1-\tau^P)k_P^{\alpha_K}} \times \frac{k_X^{\alpha_K-1}}{(1-\tau^P)k_P^{\alpha_K-1}} = \left(\frac{1}{1-\tau^P}\right)^{\frac{1}{1-\alpha_K}} \left[\frac{\alpha_K+(1-\tau^X)\alpha_X}{\alpha_K+\alpha_X}\right]^{\frac{1-2\alpha_K}{1-\alpha_K}}$$

By the condition $\tau^P \geq 1 - \left[\frac{\alpha_K+(1-\tau^X)\alpha_X}{\alpha_K+\alpha_X}\right]^{1-2\alpha_K}$, we have $\frac{1}{1-\tau^p}\left[\frac{\alpha_K+(1-\tau^X)\alpha_X}{\alpha_K+\alpha_X}\right]^{1-2\alpha_K} \geq 1$. Therefore, $c_X^o/c_P^o > 1$, i.e.

$$c_X^o > c_P^o$$

(v) The proof for this statement is quite similar to the proof in statement (iv) above.

Q.E.D.

A.4. Social planner's optimal allocation

The Lagrangian is

$$\mathcal{L} = \sum_{t=0}^{+\infty} \frac{u(c_t^y)+(1+R)v(c_t^o)}{(1+R)^t} + \sum_{t=0}^{+\infty} \frac{\mu_t\left[z(E_{t-1})F^t + k_t - c_t^y - c_t^o - k_{t+1} - M_t\right]}{(1+R)^t}$$
$$+ \sum_{t=0}^{+\infty} \frac{\eta_t\left[E_t - (1-\delta)E_{t-1} - \xi z(E_{t-1})F^t + \gamma M_t\right]}{(1+R)^t}$$

The FOCs are

$$\frac{\partial \mathcal{L}}{\partial c_t^y} = \frac{u'(c_t^y)}{(1+R)^t} - \frac{\mu_t}{(1+R)^t} = 0$$
$$\frac{\partial \mathcal{L}}{\partial c_t^o} = \frac{v'(c_t^o)}{(1+R)^{t-1}} - \frac{\mu_t}{(1+R)^t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\frac{\mu_t}{(1+R)^t} + \frac{\mu_{t+1}}{(1+R)^{t+1}}\left[1+z(E_t)F_K^{t+1}\right] - \frac{\eta_{t+1}}{(1+R)^{t+1}}\xi z(E_t)F_K^{t+1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial M_t} = -\frac{\mu_t}{(1+R)^t} + \frac{\eta_t \gamma}{(1+R)^t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial E_t} = \frac{\mu_{t+1} - \eta_{t+1}\xi}{(1+R)^{t+1}} z'(E_t) F^{t+1} + \frac{\eta_t}{(1+R)^t} - \frac{\eta_{t+1}(1-\delta)}{(1+R)^{t+1}} = 0$$

$$z(E_{t-1})F^t + k_t - c_t^y - c_t^o - k_{t+1} - M_t = 0$$

$$E_t - (1-\delta)E_{t-1} - \xi z(E_{t-1})F^t + \gamma M_t = 0$$

At the steady state we have

$$u'(c^y) = \mu$$

$$v'(c^o) = \frac{\mu}{1+R}$$

$$\frac{\mu - \eta\xi}{R} z(E) F_K = \mu$$

$$\mu = \eta\gamma$$

$$\frac{\mu - \eta\xi}{1+R} z'(E) F + \eta - \frac{\eta(1-\delta)}{1+R} = 0$$

$$c^y + c^o + M = z(E)F$$

$$\delta E = \xi z(E) F - \gamma M$$

i.e.

$$\frac{u'(c^y)}{v'(c_t^o)} = 1 + R$$

$$z(E) F_K = \frac{\gamma R}{\gamma - \xi}$$

$$z'(E) F = \frac{R + \delta}{\xi - \gamma}$$

$$c^y + c^o + M = z(E)F$$

$$\delta E = \xi z(E) F - \gamma M$$

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