RANKING MULTIVARIATE VOLATILITY MODELS: AN APPLICATION TO EMERGING FINANCIAL MARKETS

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Abstract

The increasing number of extensions of the Dynamic Conditional Correlation (DCC) model and computational capability offer various measures for risk management, portfolio selection and the analysis of financial market interdependence or contagion. The success of the TDCC model proposed by Pesaran and Pesaran (2007) in the study of Pesaran *et al* (2009) in analysing the performance of the volatility models (both by finance practitioners and academics) motivates us to check how the TDCC model compares to previously developed multivariate models in the context of the 19 emerging financial markets and the US market. Thus, this study empirically compares the TDCC model with the different specifications of the multivariate GARCH model such as the Riskmetrics model, the Constant Conditional Correlation model (CCC), the Orthogonal-GARCH model, the DCC model, the Asymmetric DCC model (ADCC) and the Consistent DCC model (CDCC). In total, 54 models, categorized into 10 classes, were evaluated for the in-sample performance using the maximized values of the log-likelihood function, the AIC and the SBIC. The out-of-sample evaluation procedure, based on the one-step-ahead forecast of the covariance estimators, utilizes Value-at-Risk analysis to produce the VaR-based diagnostic tests for the models following both the active and the passive risk management manners. Besides, the Kolmogorov-Smirnov (KS) and Kuiper (Ku) tests were also used to give evaluations from different points of view which focuses on the tail distributions. The empirical results using data from the emerging markets give a broader view on the trading strategies and the volatility model selection in the risk management manners. Similar to the study of Pesaran *et al* (2009), the TDCC model dominated the other models in in-sample performance, the DCC-type models were in the top models for both types of evaluations. However, the TDCC and the CDCC models were rejected by the Ku and the KS tests at the 1% significance level while the Riskmetrics filters were reasonably suggested by these two tests. For the data from the emerging markets, the calibrations to select the appropriate values for the risk aversion (δ) and to specify the reasonable range for the evaluation sample are the key to achieve proper statistical results. The Student's $t(6)$-distribution assumption is more relevant than the Gaussian distribution assumption for the volatility models to fit for the emerging markets.

1 Introduction

Over the past two decades, the empirical applications of theoretical models in finance have benefited greatly from developments in financial econometrics. The interrelationship between the theoretical models and the statistical methods in finance which has become dominant trend was a prediction of Pagan (1996). Thus, financial markets have suffered from a lot of structural changes, the behaviours of investors, etc. which cause the market anomalies. Consequently, the theoretical models need to be modified so as to adapt flexibly to the practical changes. The modern financial econometrics, which develops sufficient tools to deal with the anomalies in the financial markets such as non-stationarity, non-normal distributions, heteroskedasticity, etc., is empirically helpful to realize the modified theoretical models in finance.

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The volatility of financial returns, which has been the central focus of financial economics and is known as the unobservable second moment of financial data, shows that the financial returns are not as homoskedastic as assumed in many theoretical models in finance. The introduction of the ARCH model (Autoregressive Conditionally heteroskedasticity) of Engle (1982) puts a rigorous framework to measure the volatility of financial data. Since the introduction of the ARCH model, there have been an incredibly burgeoning number of extensions of it. Bollerslev (2008) gives a glossary of over 100 extensions of the ARCH models, although some extensions are still missing from the list. For reviews of the literature on the univariate ARCH/GARCH models, see Bera and Higgins (1993); Bollerslev, Engle and Nelson (1986); Bollerslev, Chou and Kroner (1992); Diebold and Lopez (1995); Pagan (1996); Palm (1996); Shephard (1996). The most recent review on the univariate GARCH models can be found in Poon and Granger (2003) who performed a broad survey on 93 papers and compared different methods for modelling univariate volatility.

Initially, the GARCH models, which are the generalised version of the ARCH and was proposed by Bollerslev (1986), E-GARCH [Nelson (1991)], GJR [Glosten, Jagannathan and Runkle (1993)], etc., are the univariate extensions of the ARCH model which deal with the movement of a single financial series. However, the applications of the univariate GARCH models are somewhat limited in finance. A multivariate parameterization of the GARCH model, therefore, was expected to have wider applications in financial studies. For example, in the portfolio selection, following the theory of Markowitz (1952, 1959), the weights of financial assets in a portfolio can be optimally chosen with respect to the estimates of conditional volatility and correlations, which can be estimated by a multivariate GARCH model. Specifically, following the coming introduction of the Basel Accord III, as a response to the Global financial crisis, banks are required to calculate the minimum amount of bank capital based on their traded financial assets. In other words, in risk management, the necessary capital amount that a bank is required to maintain is computed based on the Value-at-Risk of the portfolio of financial assets being traded by the bank. A multivariate GARCH model, which delivers a precise forecast of conditional volatilities and correlations of assets, will be a useful tool to obtain correct estimates of the Value-at-Risk of financial assets held by banks. Besides, the pricing method of derivatives can also benefit from the development of the multivariate GARCH models which can be used to estimate the dynamic model of volatility of the underlying asset return, the option price and the contracts traded on volatilities.[1] Last but not least, a multivariate GARCH model can be used to examine the interdependence as well as contagion between financial markets by modelling the conditional covariances and the conditional correlations.

The study of the multivariate GARCH model was initiated in late 1980s by the introduction of the VECH model of Bollerslev, Engle and Wooldridge (1988) and early 1990s by the introduction of the Constant Conditional Correlation (CCC) model of Bollerslev (1990). The two models were proposed by different approaches in the construction of the covariance matrix. The former was constructed based on the direct estimation of the covariance matrix which is regressed on the lagged covariances and past squared errors while the latter was constructed by the recombination of the estimated diagonal matrix of the standard deviations obtained by the univariate GARCH and a time-invariant matrix of the conditional correlations. The BEKK representation proposed by Engle and Kroner (1995) is a modification of the VECH model which ensures the positivity definiteness of the variance matrix. However, both VECH and BEKK parameterizations face the problem that the number of parameters to be estimated rises exponentially with the number of return series, which is known as the 'curse of dimensionality'.[2] Those types of models, therefore, are then difficult to use in practice for the issue of the number of parameters will cause the model to be over-parameterized in even a modestly-sized portfolio of financial assets. The CCC specification was proposed to avoid the curse of dimensionality. In this approach the number of parameters rises linearly with the number of return series.[3] However, conditional correlations are often not constant over time due to different

[1] Nowadays, a volatility contract is designed to be exchangeable or tradeable in a similar way to a futures contract. It relies on the measurements of realized volatility of the underlying instrument. For details, see Krause (2000): *Volatility Contracts - A new alternative.*

[2] The VECH and the full BEKK models involve $\mathcal{O}(k^4)$ parameters, the diagonal VECH and the standard BEKK model involve $\mathcal{O}(k^2)$ parameters.

[3] The CCC model involves $\mathcal{O}(k)$ parameters

degrees of financial integrations or financial crises which cause the correlations of financial assets to vary over time. Therefore, Engle (2002) introduces a new class of the multivariate GARCH model, namely the Dynamic Conditional Correlation model (DCC) which relaxes the assumption of constant conditional correlations to allow for time-varying correlations. Since then, the DCC model has achieved a great success for its popular applications in finance, such as in risk management, asset allocation, derivative pricing and the analysis of interdependence of financial markets. There is also an increasing number of extensions of the DCC model such as the AG-DCC (Asymmetric Generalised DCC) model of Capiello, Engle and Sheppard (2006) which is able to capture the asymmetric properties of the volatilities and correlations, the introduction of the TDCC (Student t-DCC) model of Pesaran and Pesaran (2007) which assumes a multivariate Student's t-distribution for the return series or the CDCC (Consistent DCC) model of Aielli (2011) which is consistent in modelling the portfolios containing a large number of financial assets.

There are several survey papers on the multivariate GARCH models. Laurent, Bauwens and Rombouts (2006) is a comprehensive review of the multivariate GARCH models in terms of model selection, model estimation and the diagnostic checking for model specification. The purpose of their paper is to give a comprehensive background that acts as an indication for appropriate applications of the multivariate GARCH models in financial economics. In this literature, there is a key note that there is a co-existence of both types of the multivariate GARCH model, which are the BEKK model and the DCC model. Hence, there was a need to compare the differences and the similarities between the BEKK model and the DCC model. On this topic, an important review paper is given by Silvennoinen and Terasvirta (2009) who focused on the comparison between the BEKK model and the CCC model or its generalisations. Moreover, Caporin and McAleer (2011) also gave a clear in-depth discussion of where the DCC models are preferred to use in practical applications and BEKK models are mainly mentioned in the theoretical aspects due to their dimensionality curse which makes the model estimation unrealisable in even a portfolio containing a modest number of financial assets. As the DCC models are easier to estimate than the BEKK model, they are mainly used by researchers. Practitioners tend to apply simple models to estimate the covariances and the correlations of financial returns such as the Riskmetrics specifications proposed by J.P.Morgan (1996). However, there has been no research to explain the difference between researchers and practitioners in using the multivariate parameterizations to model the volatility and correlations until Pesaran, Schleicher and Zaffaroni (2009) proposed the average modelling technique which also included the evaluations on the performances of a large number of the multivariate volatility models. The models included in their study range over 9 different classes of models in which the Riskmetrics specifications, such as the EQMA, the EWMA, the MMA, and the GEWMA, used by practitioners are compared with the models used by researchers, such as the CCC model of Bollerslev (1990), the Orthogonal GARCH model of Alexander (2001), the DCC model of Engle (2002), the AG-DCC model of Cappiello *et al* (2006) and the TDCC model of Pesaran and Pesaran (2007). The data set used in this paper comes from developed futures markets, including equity markets, currency markets, bond markets and commodity markets. Hence, the paper is limited to the analysis of the developed markets. In our study, we use data of 19 emerging markets and the US market to re-evaluate the performances of the set of 53 specific models used in the paper of Pesaran *et al* (2009) with the addition of the Consistent DCC model (CDCC) of Aielli (2011) which is the consistent estimator in large-scaled portfolios.

The rest of this paper is organized as follows: Section 2 gives an overview of 10 classes of models to be evaluated in this paper. Section 3 provides the empirical results as well as model evaluations and rankings. The last section provides some concluding remarks on the implications of the empirical results.

2 Specifications for the covariance and correlation models

This section provides an introduction to the specifications of the multivariate GARCH models which are examined in this paper. When the number of financial time series is larger than 5, only a few models are feasible to estimate. Therefore, there are 10 estimation-feasible classes of the multivariate GARCH model selected for the analysis in this paper. Nine of them are used in the study of Pesaran, Schleicher and Zaffaroni (2009):

- The Riskmetrics filters [see J.P. Morgan(1996)], including the Equally-Weighted Moving Average (EQMA), the Exponentially-Weighted Moving Average (EWMA), the Mixed Moving Average (MMA), the Generalised Exponentially-Weighted Moving Average (GWEMA).
- The Orthogonal GARCH (O-GARCH) by Alexander (2001).
- The Constant Conditional Correlation (CCC) by Bollerslev (1990).
- The Dynamic Conditional Correlation (DCC) by Engle(2002).
- The Asymmetric Dynamic Conditional Correlation (ADCC) by Capiello *et al* (2006).
- The Student's t-Dynamic Conditional Correlation (TDCC) by Pesaran and Pesaran (2007).

We included one more model class into the selection of the considered models, the Consistent Dynamic Conditional Correlation (CDCC) proposed by Aielli (2011). This extension of the DCC model is to solve the main problem of the DCC model which has estimated parameters being biased when the dimension of the portfolio is larger. Each class of the models may have more than one representation depending on how many past lags are included. Hence, there are totally 54 different specific volatility models being estimated and evaluated in this study.

Our research is focused on how the various multivariate GARCH models perform both in-sample and out-of-sample using the data from the 19 emerging markets and the US market. So let r_t be $m \times 1$ vector of return series at time t. Without a precise assumption about their distribution, the conditional returns of m financial series, r_t at time t are denoted as $E(r_t|F_{t-1}) = \mu_t$. We specifically assume that the return series follow a first-order autoregressive process, AR(1) characterized as follows

$$r_t = c_0 + c_1 r_{t-1} + \epsilon_t \tag{1}$$

Hence, the $E(r_t|F_{t-1}) = \mu_t = c_0 + c_1 r_{t-1}$. Therefore, we have $\epsilon_t = r_t - \mu_t$ with $\epsilon_t \sim (0,\ H_t)\ \ where\ \ H_t = Var(\epsilon_t|F_{t-1})$ is the conditional covariance matrix at time t of the innovations, ϵ_t. Consequently, the innovation series, ϵ_t can be standardized to satisfy the requirement of the multivariate volatility model estimations and evaluations by using the conditional covariance matrix, H_t as $z_t = \frac{\epsilon_t}{\sqrt{H_t}}$. We denote $\mathcal{H}_M(r_t|F_{t-1})$ as the joint probability distribution of ϵ_t under some model M, which can be specified by the choice of H_t and the specification of the distribution of the standardized returns, z_t. Here, we shall consider both cases of a multivariate Normal distribution and a multivariate Student's t-distribution with ν degrees of freedom. There are many specifications that use parametric methods to estimate the conditional covariance matrix, H_t. Bollerslev, Engle and Wooldridge (1988) put forward the multivariate generalised autoregressive heteroskedasticity model of order (1,1) (which is also known as the VECH specification of the MGARCH(1,1)) as follows

$$VECH(H_t) = C + A_0 VECH(H_{t-1}) + B_0 VECH(\epsilon_{t-1}\epsilon'_{t-1}) \tag{2}$$

where VECH(.) denotes the column stacking operator applied to the upper portion of the symmetric matrix, C is $\frac{m(m+1)}{2} \times 1$ parameter vector, $A_0,\ B_0$ are $\frac{m(m+1)}{2} \times \frac{m(m+1)}{2}$ matrices of unknown parameters. However, the drawback of the MGARCH is that it requires a very large number of parameters as the size of matrices $A_0,\ B_0$ increases quadratically in the number of assets, m, in the portfolio. Hence, the model expressed in Equation 2 is rarely used in practice.

The introduction of the CCC model, the DCC model and its extensions is to realise the estimation of the conditional covariance matrix, H_t. Besides, the simple specifications, introduced by Riskmetrics for practical use in finance, are easy to use in estimating the conditional covariance matrix, H_t. The study of Pesaran, Schleicher and Zaffaroni (2009), aims to compare the performance of the model classes used by researchers with the ones used by practitioners. In their paper, there are 53 specific multivariate volatility models which are used to estimate the conditional covariance matrix, H_t. The 53 models are categorized into 9 different groups which are the CCC group of Bollerslev (1990), the DCC group of Engle (2002), the ADCC group of Cappiello *et al* (2006), the orthogonal GARCH group of Alexander (2001) and the TDCC model of Pesaran *et al* (2007), the group of the equally-weighted moving average models (EQMA), the group of the exponentially-weighted moving average models (EWMA), the group of the mixed moving average models (MMA) and the group of the generalised exponentially-weighted moving average models (GEWMA). Our paper differs from theirs in that we add the newly developed model of Aielli (2011), namely the CDCC and evaluate the performance of the 54 models on a data set from the US market and 19 emerging markets, which are more challenging for both the theoretical models and the practical models initially designed for developed markets.

The procedure for the estimation of each of the 54 specific models is based on the framework suggested by Engle (2002) for the decomposition of the conditional covariance matrix, H_t as in Equation 3 below

$$H_t = D_t R_t D_t \tag{3}$$

where $D_t = diag\left\{\sqrt{\sigma_{kk,t}}\right\}$ is the $m \times m$ diagonal matrix of time-varying standard deviations from the univariate GARCH models with $\sqrt{\sigma_{kk,t}}$ being the k^{th} position on the diagonal; $R_t = \{\rho_{kj,t}\}$ is the $m \times m$ one-step-ahead conditional correlation matrix. This decomposition allows for the feasible estimation of the multivariate volatility models as well as cross-asset correlations regardless of the number of assets in the portfolio. So each of the 54 models is used to estimate and forecast $\sqrt{\sigma_{kk,t}}$ and $\rho_{kj,t}$ by using the $m \times 1$ vector of residuals, ϵ_t obtained from the OLS regressions of the first-order autoregression of return series, r_t.

2.1 Equally-weighted Moving Average [EQMA(d_0)]

In this specification, the conditional covariance matrix, $H_{t,EQMA}$ can be simply computed by using the rolling moment estimates based on the last d_0 observations as follows

$$H_{t,EQMA} = \frac{1}{d_0}\sum_{s=1}^{d_0} r_{t-s}r'_{t-s} \tag{4}$$

To ensure that H_t is positive definite, the last d_0 observations must be larger than the dimension of returns vector, m. However, setting d_0 too high makes the conditional variance matrix too similar to the unconditional variance. In this application, following the common practice, suggested by the Riskmetrics, four variants of $H_{t,EQMA}$ are considered by setting $d_0 = 50$, 75, 125 and 250. For this class of model, the the conditional covariance matrix, $H_{t,EQMA}$ will behave like the unconditional covariance matrix if the choice of d_0 is too large. The choice of d_0, ranging from 50 to 250, gives us a chance to select a model that best fit to our data set.

2.2 Exponentially-Weighted Moving Average [EWMA(d_0, λ_0, ν_0)]

The EWMA model is essentially the simple extension of the historical average volatility measure which allows the more recent observations to have a stronger impact on the forecast of the volatility than the older observations. This approach gives the EWMA model a practical application where recent events, in practice, are more influential on volatility. The one-parameter EWMA (setting $\lambda_0 = \nu_0$) officially used by Riskmetrics can be expressed in the following recursion:

$$H_{t,EWMA} = \lambda_0 H_{t-1} + \frac{1-\lambda_0}{1-\lambda_0^{d_0}}\epsilon_{t-1}\epsilon'_{t-1} - \frac{1-\lambda_0}{1-\lambda_0^{d_0}}\lambda_0^{d_0-1}\epsilon_{t-d_0-1}\epsilon'_{t-d_0-1} \tag{5}$$

where $0 < \lambda_0 < 1$ a constant parameter, d_0 is the window size. The kj^{th} element in the variance-covariance matrix $H_{t,EWMA}$ can be obtained from

$$\sigma_{kj,t} = \frac{1-\lambda_0}{1-\lambda_0^{d_0}} \sum_{s=1}^{d_0} \lambda_0^{s-1} \epsilon_{k,t-s} \epsilon_{j,t-s} \tag{6}$$

One of the drawbacks of the EWMA model is that when the infinite sum is replaced with a finite sum of observable data as in Equation 6, the weights from the given expression will sum up to less than one. So the parameter $\frac{1-\lambda_0}{1-\lambda_0^{d_0}}$ is added to make the sum of all parameters λs equal to 1. Moreover, the EWMA does not have the property of being mean-reverting. That is, the forecast of the conditional volatility of a series does not converge towards the unconditional variance like for other volatility models. The parameter d_0 is usually fixed as *a priori*. In J.P. Morgan (1996), it is suggested that the decaying factor is $\lambda_0 = 0.94$ (this can be estimated in practice). However, due to the asymptotic properties and the uncertainty over the parameters, we follow the suggestions of Pesaran *et al* (2009) that set the parameters $\lambda_0 = 0.95,\ 0.97$ and $d_0 = 250,\ 125,\ 75$ and 50.

It is documented in both practice and academic research that there are different rates applied to decaying process of transmission of shocks to the conditional volatilities and the conditional correlations. Therefore, it can be assumed that there is some parameter, ν_0 different from λ_0, to construct the conditional covariance dynamic in Equation 6 as follows (normally we have $\nu_0 < \lambda_0$ and those two parameters are set as *a priori*)

$$\sigma_{kj,t} = \frac{1-\nu_0}{1-\nu_0^{d_0}} \sum_{s=1}^{d_0} \nu_0^{s-1} \epsilon_{k,t-s} \epsilon_{j,t-s} \tag{7}$$

Hence, the kj^{th} element of the correlation matrix, R_t in Equation 3 can be computed by using entries from the variance-covariance matrix, $H_{t,EWMA}$ as follows

$$\rho_{kj} = \frac{\sigma_{kj,t}}{\sqrt{\sigma_{kk,t}\sigma_{jj,t}}} \tag{8}$$

2.3 Mixed Moving Average [MMA(d_0, ν_0)]

This specification is actually a generalised version of the equally-weighted moving average model. The k^{th} entry in the standard deviation matrix, D_t is obtained similarly to what we can see in Equation 6 of the EWMA model. The conditional covariance is estimated by using Riskmetrics filter as $\sigma_{kj,t} = \frac{1-\nu_0}{1-\nu_0^{d_0}} \sum_{s=1}^{d_0} \nu_0^{s-1} \epsilon_{k,t-s} \epsilon_{j,t-s}$. The conditional correlation matrix, R_t, is constructed by using the structure in Equation 8. Consequently, the conditional variance matrix, $H_{t,MMA}$ is obtained by combining $D_t,\ R_t$ as in Equation 3.

2.4 Generalised Exponentially-Weighted Moving Average [GEWMA(d_0, p, q, ν_0)]

This specification is a generalisation of the equally-weighted moving average model where there are two different decaying parameters used for the conditional volatilities and correlations. Firstly, we use the univariate GARCH(p,q) models to estimate for each return series in vector of return series, r_t by using the quasi-maximum likelihood to obtain conditional variances to form matrix D_t. The conditional correlation matrix is then computed by using the conditional covariances obtained by applying the Riskmetrics filter in Equation 5 with the conditional correlations computed following Equation 8. Once matrices D_t and R_t are available we can construct the variance-covariance matrix, $H_{t,GWEMA}$ following the structure in Equation 3. The total number of parameters being estimated in the GEWMA model is $m(1+p+q)$ where p, q are the order of the univariate GARCH model.

2.5 Constant Conditional Correlation [CCC(p,q)]

The Constant Conditional Correlation model, proposed by Bollerslev (1990), parameterizes the variance-covariance matrix, H_t, in Equation 3 by assuming that the conditional correlations in R_t are constant over time. The kk^{th}

element, which is the conditional standard deviation in the diagonal matrix D_t, is estimated by using a univariate GARCH(p,q) of Bollerslev (1986)

$$\sigma_{k,t} = c_{0k} + \sum_{i=1}^{q} \lambda_{1,ki}\epsilon_{k,t-i}^{2} + \sum_{j=1}^{p} \lambda_{2,kj}\sigma_{k,t-j} \tag{9}$$

where c_{0k}, $\lambda_{1,ki}$, $\lambda_{2,kj}$ are positive parameters which are sufficient to ensure the positivity of $\sigma_{k,t}$. The conditional correlation matrix, R_t, is comprised of $\dfrac{m(m-1)}{2}$ constant parameters, ρ_{kj} which are computed by using the innovations, $\epsilon_{k,t}$ with k = 1, 2, ..., m as follows

$$\rho_{kj} = \frac{\sigma_{kj}}{\sqrt{\sigma_{kk}\sigma_{jj}}} \tag{10}$$

The entries in the correlation matrix, in fact, are the unconditional correlations estimated by using the quasi-maximum likelihood to give the total estimated parameters which are comprised of $m(p+q+1)$ parameters from the univariate GARCH estimations and $\dfrac{m(m-1)}{2}$ parameters from the constant correlation matrix. The conditional variance-covariance matrix $H_{t,CCC}$ is then constructed by the computed matrices D_t and R_t following Equation 3.

2.6 Orthogonal GARCH [O-GARCH(p,q)]

Alexander (2001) proposed the Orthogonal GARCH model by using a static principle component decomposition of residuals standardized as follows

$$\epsilon_{k,t}^{\star} = \frac{\epsilon_{k,t} - \bar{\epsilon}_k}{\bar{\sigma}_k} \quad \text{with } t\text{=1, 2, ..., } m \text{ and } t\text{=1, 2, ..., } T.$$

where $\bar{\epsilon}_k$ is the sample mean of the return of asset k and $\bar{\sigma}_k$ is the sample standard deviation of the returns of asset k. The standardized returns are used to construct the sample covariance matrix which are, in turn, used to give the eigenvectors, a_k (k = 1, ..., m) and eigenvalues. The sample covariance matrix can be expressed as

$$\bar{S} = \frac{\sum_{t=1}^{T} \epsilon_t^{\star}\epsilon_t^{\star\prime}}{T}$$

In this specification, the time-varying conditional variance-covariance matrix, $H_{t,O-GARCH}$ is defined in the following formula

$$H_{t,O-GARCH} = AD_tA' \tag{11}$$

where $A = U \times a$ is a $m \times m$ matrix of standard deviations normalized by the weighting matrix $a = (a_1,\ a_2,\ ...,\ a_m)$ containing the eigenvectors a_{k_s} with respect to the first m largest eigenvalues of the sample covariance matrix $\bar{S}$ and U is diagonal matrix containing sample standard deviations of the innovations $\epsilon_{k,t}$; D_t is the orthogonal matrix of the conditional variances with $\sigma_{k,t}$, *the* kk^{th} entry on the diagonal being estimated by the univariate GARCH(p,q) with k = 1, 2, ..., m as follows

$$\sigma_{k,t} = c_{0k} + \sum_{i=1}^{q} \lambda_{1,ki}s_{k,t-i}^{2} + \sum_{j=1}^{p} \lambda_{2,kj}\sigma_{k,t-j}$$

where $s_{k,t} = (\epsilon_1, \epsilon_2, ..., \epsilon_T) \times a_k$ is the orthogonalised return series of asset k with k = 1, 2, ..., m. The number of total parameters in the specification of O-GARCH is $m \times (p+q+1)$.

2.7 Dynamic Conditional Correlations [DCC(p,q,M,N)]

This model was proposed by Engle in 2002 based on the modification of the CCC model of Bollerslev (1990) and allows the elements in the conditional correlation matrix, R_t to vary overtime. The matrix of the standard deviations, D_t, is constructed similarly to what is done in the CCC method where the k^{th} element on the diagonal is estimated by a univariate GARCH(p,q) so that

$$\sigma_{k,t} = \omega_{0k} + \sum_{i=1}^{p}\lambda_{1,ki}\epsilon^2_{k,t-i} + \sum_{j=1}^{q}\lambda_{2,kj}\sigma^2_{k,t-j} \tag{12}$$

The restrictions imposed on the parameters in Equation 12 to ensure the non-negativity and the stationarity of the conditional variances are $\omega_{0k} > 0$, $\lambda_{1,ki} > 0$, $\lambda_{2,kj} > 0$ for all i, j, k and $\sum_{i=1}^{p}\lambda_{1,ki} + \sum_{j=1}^{q}\lambda_{2,kj} < 1$. The time-varying dynamic conditional covariance is presented as follows

$$Q_t = (1 - \sum_{i=1}^{M}\alpha_i - \sum_{j=1}^{N}\beta_j)\bar{Q} + \sum_{i=1}^{M}\alpha_i(\epsilon^\star_{t-i}\epsilon^{\star\prime}_{t-i}) + \sum_{j=1}^{N}\beta_j Q_{t-j} \tag{13}$$

where $\bar{Q} = E(\epsilon^\star_t\epsilon^{\star\prime}_t)$ is the unconditional covariance of the standardized residuals, $\epsilon^\star_t = (\epsilon^\star_1, \epsilon^\star_2, ..., \epsilon_m)'$ which are standardized by using the conditional variances estimated in Equation 12 as follows

$$\epsilon^\star_{k,t} = \frac{\epsilon_{k,t}}{\sqrt{\sigma_{k,t}}} \tag{14}$$

Consequently, the time-varying conditional correlation matrix, R_t is formed by

$$R_t = Q^{\star-1}_t Q_t Q^{\star-1}_t \tag{15}$$

where

$$Q^\star_t = \begin{bmatrix} \sqrt{q_{11}} & 0 & 0 & \cdots & 0 \\ 0 & \sqrt{q_{22}} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & \sqrt{q_{mm}} \end{bmatrix}$$

the $\sqrt{q_{kk}}$ of $Q^\star_t$ is the k^{th} diagonal of Q_t. So the kj^{th} entry of R_t is defined as $\rho_{kj} = \frac{q_{kj}}{\sqrt{q_{kk}q_{jj}}}$ giving the correlation matrix positive semi-definite with elements on the diagonal.

In this study, we will use the DCC(p,q,1,1) meaning that there is only one lag of the covariance term and of the standardized residual. Hence, the general structure, as in Equation 13, is reduced as follows

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha(\epsilon^\star_{t-1}\epsilon^{\star\prime}_{t-1}) + \beta Q_{t-1} \tag{16}$$

The covariance matrix, $H_{t,DCC}$ is obtained by recombining D_t and R_t following Equation 3. The DCC is estimated by two-stage quasi-maximum likelihood. The first stage is carried out by estimating the univariate GARCH(p,q) for individual series to compute D_t. In the second stage, the log-likelihood function is set up, using R_t, D_t as in Equation 17, to give estimated parameters $\hat{\alpha}$, $\hat{\beta}$ in the dynamic correlation structure in Equation 16.

$$LLF = -\frac{1}{2}\sum_{t=1}^{T}(klog(2\pi) + 2log(|D_t|) + log(|R_t|) + \epsilon^{\star\prime}_t R^{-1}_t \epsilon^\star_t) \tag{17}$$

The number of total parameters in the DCC(p,q,1,1) specification is equal to $m(1+p+q) + \frac{m(m+1)}{2} + 2$.

2.8 Asymmetric Dynamic Conditional Correlations [ADCC(p,q,q,M,O,N)]

In the standard framework of the DCC, Engle also mentioned the possibility of extending the DCC model by adding an asymmetric term that allows the DCC model to capture the asymmetric behaviour of financial assets.

Cappiello, Engle and Sheppard (2006) proposed a modified specification of the DCC that allows for the estimation of asymmetric properties of financial time series. In the asymmetric DCC framework, the asymmetric term enters both stages of the model estimation. In the first stage of the conditional variance estimation for every single series, the standard univariate GARCH of Bollerslev (1986) is replaced by the asymmetric GARCH such as the GJR-GARCH. Cappiello *et al* (2006) suggested a choice of the 9 asymmetric univariate GARCH models. In this paper, we used the GJR-GARCH model proposed by Glosten, Jagannathan and Runkle (1993) for the first stage of estimation as in the following structure

$$\sigma_{k,t} = \omega_{0k} + \sum_{j=1}^{q} \lambda_{1,kj} \epsilon_{k,t-j}^2 + \sum_{j=1}^{q} \gamma_{kj} d(\epsilon_{k,t-j} < 0) \epsilon_{k,t-j}^2 + \sum_{i=1}^{p} \lambda_{2,ki} \sigma_{k,t-i} \tag{18}$$

where d(ζ) is an indicator function that takes the value of unity if $\zeta < 0$ and zero, otherwise. All parameters in Equation 18 must be positive and $\sum_{j=1}^{q} \lambda_{1,kj} + \sum_{j=1}^{q} \gamma_{kj} + \sum_{i=1}^{q} \lambda_{2,ki} < 1$ to satisfy the positivity and stationary conditions of the conditional variances that will be used to form the matrix of conditional standard deviations, D_t and to obtain the normalized residuals $\epsilon_t^\star$ given by $\epsilon_{k,t}^\star = \frac{\epsilon_{k,t}}{\sqrt{\sigma_{k,t}}}$.

In the second stage, the standardized residuals are used to compute the time-varying correlation dynamic as presented below

$$q_t = (1 - \alpha' \bar{\rho} \alpha - \beta' \bar{\rho} \beta - \delta' \bar{\eta} \delta) + \sum_{j=1}^{N} \alpha_j (\epsilon_{t-j}^\star \epsilon_{t-j}^{\star\prime}) + \sum_{j=1}^{O} \delta_j d(\epsilon_{t-j}^\star < 0)(\epsilon_{t-j}^\star \epsilon_{t-j}^{\star\prime}) + \sum_{i=1}^{M} \beta_i q_{t-i} \tag{19}$$

where d(ζ) is an indicator function that takes the value of unity if $\zeta < 0$ and zero, otherwise; $\bar{\rho}$, $\bar{\eta}$ are the unconditional covariance matrices given by

$$\bar{\rho} = \frac{1}{T} \sum_{t=1}^{T} \epsilon_t^\star \epsilon_t^{\star\prime}$$

$$\bar{\eta} = \frac{1}{T} \sum_{t=1}^{T} d(\epsilon_t^\star < 0) \epsilon_t^\star \epsilon_t^{\star\prime}$$

The condition necessary to hold such that the q_t in Equation 19 is positive and stationary is $\sum_{j=1}^{N} \alpha_j^2 + \sum_{j=1}^{O} \delta_j^2 + \sum_{i=1}^{M} \beta_i^2 < 1$, the intercept $(1 - \alpha' \bar{\rho} \alpha - \beta' \bar{\rho} \beta - \delta' \bar{\eta} \delta)$ is positive semi-definite with the initial value of the covariance matrix, Q_0 is positive definite. These conditions are sufficient to make all realisations of ADCC possible.

It can be noticed that the DCC representation is a special case of the ADCC model. In the scalar version of the ADCC, the correlation dynamic structure can be expressed as

$$q_t = (1 - \alpha^2 \bar{\rho} - \beta^2 \bar{\rho} - \delta^2 \bar{\eta}) + \alpha^2 (\epsilon_{t-1}^\star \epsilon_{t-1}^{\star\prime}) + \delta^2 d(\epsilon_{t-1}^\star < 0)(\epsilon_{t-1}^\star \epsilon_{t-1}^{\star\prime}) + \beta^2 q_{t-1} \tag{20}$$

with the GJR-GARCH(1,1,1) as below

$$\sigma_{k,t} = \omega_{0k} + \lambda_{1,k} \epsilon_{k,t-1}^2 + \gamma_k d(\epsilon_{k,t-1} < 0) \epsilon_{k,t-1}^2 + \lambda_{2,k} \sigma_{k,t-1} \tag{21}$$

The sufficient condition to secure the positivity of q_t is that the intercept in Equation 20 must be positive semi-definite. Hence, it is necessary and sufficient to derive the condition that makes the model estimations feasible in any realisations is that $\alpha^2 + \beta^2 + \kappa \delta^2 < 1$ where κ is the maximum eigenvalue of matrix $\left[\bar{\rho}^{-1/2} \bar{\eta} \bar{\rho}^{-1/2}\right]$. This nonlinear constraint will be imposed on the maximization process of the log-likelihood function which can be written in a similar form to that of the DCC specification in Equation 17. The conditional correlation matrix R_t is then computed by

$$R_t = q_t^{\star -1} q_t q_t^{\star -1} \tag{22}$$

where $q_t^\star = diag\left(\sqrt{q_{kk,t}}\right)$ with $q_{kk,t}$ is the kk^{th} element of matrix q_t meaning that it is on the k^{th} diagonal position of q_t.

The variance-covariance matrix $H_{t,ADCC}$ is recombined as in Equation 3 by using D_t which contains the conditional variances from Equation 18 and R_t given by Equation 22. Quasi-maximum likelihood is also used for the estimation of the ADCC model. The total number of parameters in the specification of the ADCC(p,p,q,1,1,1) is $m(1+2p+q)+\dfrac{m(m+1)}{2}+3$ which gives $m \times p + 1$ parameters more than those of the DCC(p,q,1,1) due to the inclusion of the asymmetric terms. Thus, the p asymmetric terms are included in the m univariate-GARCH processes and 1 asymmetric term added in correlation dynamic structure as in Equation 20.

2.9 Consistent Dynamic Conditional Correlation [CDCC(1,1,1,1)]

The DCC-type estimators have been shown in Aielli (2011) to be biased and inconsistent in large systems of financial assets. Hence, the consistent DCC model (CDCC) has been proposed by Aielli (2011) to solve the problem of inconsistency of the DCC models. The main modification of the CDCC model is the correction in the dynamic correlation structure in Equation 16 as follows

$$Q_t = (1-\alpha-\beta)\tilde{Q} + \alpha(\mathcal{E}_{t-1}\mathcal{E}'_{t-1}) + \beta Q_{t-1} \tag{23}$$

where

$$\mathcal{E}_{t-1} = \left(\mathcal{E}_{1,t-1},\ \mathcal{E}_{2,t-1},\ ...,\ \mathcal{E}_{m,t-1}\right),\ where\ \mathcal{E}_{i,t-1} = \epsilon^\star_{i,t-1}\sqrt{q_{i,t-1}}$$

$$\tilde{Q} = E(\mathcal{E}_t\mathcal{E}'_t) \text{ is the sample correlation matrix of } \mathcal{E}_t$$

For the small and medium portfolios, the DCC and the CDCC models have similar performances. However, the CDCC model performs better with large portfolios. Hence, it allows for a wider range of applications in practice. The number of parameters in the CDCC parameterization is similar to those of the DCC.

2.10 *t* - Dynamic Conditional Correlation [TDCC(1,1,1,1)]

All previous DCC-type frameworks are based on the use of residuals normalized by using the conditional variances estimated in the first stage while the second stage is to estimate the dynamic correlation process. The 2-stage estimation procedure of the DCC-type specifications is realised by assuming that the distribution of innovations is multivariate Gaussian. However, financial time series show fat-tailed behaviour which can be better approximated by the assumption of the Student's t-distribution with ν different degrees of freedom. Moreover, the 2-stage estimation procedure of the DCC-type framework is proven to be inefficient even though it is still consistent. The TDCC model of Pesaran and Pesaran (2007) is fitted to returns series which are assumed to have a multivariate Student's t-distribution with ν degrees of freedom.

To improve the performance of the DCC-type models, Pesaran *et al* (2007) introduced a crucial modification to the standard framework of the DCC model of Engle (2002). In the TDCC specification, the method used to standardize the residuals, ϵ_t, is replaced by the devolatization method that uses a realised variance to get the series of residuals devolatized as in the following computations

$$\sigma^{2,realized}_{k,t}(p) = \frac{\sum\limits_{j=0}^{p-1}\epsilon^2_{k,t-s}}{p}\ \ with\ \ k=1,2,...,m;\ \ p \text{ is lag order} \tag{24}$$

The lag order p, indicating p latest observations being included to compute realised volatility, needs to be chosen so as to give the most appropriate variances to render almost Gaussian series of innovation as computed in the following formula

$$\tilde{\epsilon}_{k,t} = \frac{\epsilon_{k,t}}{\sigma_{k,t}^{realised}} \tag{25}$$

Pesaran *et al* (2007) indicated that p should be calibratedly equal to 20 to make the devolatized series of residuals nearly Gaussian. The noted difference of the devolatization process from the standardization technique used in the DCC of Engle (2001) is that the devolatization technique includes the contemporaneous residuals while the technique of the DCC does not. This feature works better in the case of intradaily data and also reduces the data-driven effects on estimated parameters for daily data or data of higher frequencies. It also helps when dealing with jumps in data that cause the financial data to have a non-Gaussian distribution that can be observed more often in emerging markets where shocks usually occur.

The devolatized returns that have approximate Gaussian distribution are utilized for the construction of the time-varying conditional correlations, in similar fashion to the previous structures of the DCC family. However, the TDCC is offered with an option of two types of the dynamic conditional correlation which are non-mean reverting and mean reverting.

The non-mean reverting structure is expressed in Equation 26

$$q_{kj,t} = \phi q_{kj,t-1} + (1-\phi)\tilde{\epsilon}_{k,t-1}\tilde{\epsilon}_{j,t-1} \tag{26}$$

The mean reverting structure is given by

$$q_{kj,t} = \bar{\rho}_{kj}(1-\phi_1-\phi_2) + \phi_1 q_{kj,t-1} + \phi_2\tilde{\epsilon}_{k,t-1}\tilde{\epsilon}_{j,t-1} \tag{27}$$

where the $\bar{\rho}_{kj}$ is the sample correlation of residuals $\tilde{\epsilon}_{k,t}$ and $\tilde{\epsilon}_{j,t}$. For the mean-reverting case, the condition for all possible realisations of the TDCC model is $\phi_1 + \phi_2 < 1$. It can be seen that the non-reverting case is a special case of the mean-reverting case when $\phi_1 + \phi_2 = 1$. The conditional correlation $\tilde{\rho}_{kj,t}$ of residuals $\tilde{\epsilon}_{k,t}$ and $\tilde{\epsilon}_{j,t}$ is computed following Engle (2002) as

$$\tilde{\rho}_{kj,t} = \frac{q_{kj,t}}{\sqrt{q_{kk,t}q_{jj,t}}} \tag{28}$$

And $\tilde{\rho}_{kj,t}$ is also the kj^{th} entry of conditional correlation matrix $\tilde{R}_t$. Hence, the conditional variance-covariance matrix, $H_{t,TDCC}$ is obtained by recombining D_t and $\tilde{R}_t$ based on Equation 3. It is noted that $\sigma_{k,t}$, the kk^{th} diagonal element of the diagonal matrix, D_t is given by the univariate GARCH(1,1) model as follows

$$\sigma_{k,t} = \bar{\sigma}_k(1-\lambda_{1k}-\lambda_{2k}) + \lambda_{1k}\sigma_{k,t-1} + \lambda_{2k}\epsilon_{k,t-1}^2 \tag{29}$$

where $\bar{\sigma}_k$ is the unconditional variance of k^{th} return series.

The estimation procedure of the TDCC is also a modified version of those of the DCC when all parameters are estimated in one stage that helps to improve the efficiency of the TDCC and the return series are assumed to follow Student's t-distribution with ν degrees of freedom. The structure of the log-likelihood function is given in the following equation

$$\begin{cases} LLF_\tau(\theta) = & -\frac{m}{2}ln(\pi) - \frac{1}{2}ln \mid \tilde{R}_{\tau-1}(\theta) \mid - ln \mid D_{\tau-1}(\lambda_1,\lambda_2) \mid + ln\left[\frac{\Gamma\left(\frac{m+\nu}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\right] \\ & -\frac{m}{2}ln(\nu-2) - \left(\frac{m+\nu}{2}\right) ln\left[1 + \frac{e_\tau' D_{\tau-1}^{-1}(\lambda_1,\lambda_2)\tilde{R}_{\tau-1}^{-1}(\theta)D_{\tau-1}^{-1}(\lambda_1,\lambda_2)e_\tau}{\nu-2}\right] \\ & with \;\; e_\tau = r_\tau - \mu_{\tau-1} \end{cases} \tag{30}$$

where $\theta = (\lambda_1,\ \lambda_2,\ \phi_1,\ \phi_2,\ \upsilon)'$ is the vector of parameters with $\lambda_1 = (\lambda_{11}, \lambda_{12}, ..., \lambda_{1m}),\ \lambda_2 = (\lambda_{21}, \lambda_{22}, ..., \lambda_{2m})$ denoted as parameters obtained from the univariate GARCH(1,1) model; $\phi_1,\ \phi_2$ are parameters that drive the dynamic correlation process and ν is the degrees of freedom of the multivariate Student's t-distribution. The total of parameters in θ for TDCC(1,1,1,1) model is $2m+3$ including $2m$ parameters from the univariate GARCH, 2 parameters from the correlation dynamic process and the degrees of freedom of the Student's t-distribution.

3 Empirical results and discussion

3.1 Data Analysis

The data set, used in this paper, contains the market indices of 19 emerging financial markets and the US market. The 19 emerging markets under consideration are eight markets in Asia (China, India, Indonesia, Korea, Malaysia, Philippines, Taiwan and Thailand), five markets in Latin America (Brazil, Chile, Colombia, Mexico, and Peru) and six markets in Europe (Czech, Hungary, Israel, Poland, Russia and Turkey). The criterion for a country to be selected into the portfolio as an emerging market is suggested by the Morgan Stanley Capital International (MSCI). The MSCI indices, used in this paper, are free float-adjusted market capitalization weighted indices that are designed to measure the equity market performance.

The Free-float market capitalization takes into consideration only those shares issued by the company that are readily available for trading in the market. It generally excludes holding of promoters, government holding, strategic holding and other locked-in shares that will not come to the market for trading in the normal course. In other words, the market capitalization of each company in a Free-float index is reduced to the extent of its readily available shares in the market

The whole sample covers over the period from 15/05/1995 to 07/05/2010, with a total of 3910 observations. Hence, price indices of the emerging markets are obtained from Datastream following source of MSCI and quoted in US dollar. The market index for each country is computed by incorporating all listed and investible securities within the country. Then daily returns are calculated from the market indices as follows

$$r_{k,t} = \frac{100 \times (P_{k,t} - P_{k,t-1})}{P_{k,t-1}} \text{ with } k\text{=1, 2, ..., 20 and } P_{k,t} \text{ is the index of } k^{th}\text{market}$$

Figures 1, 2 and 3 show the plots of daily returns of the 20 market indices. From the first graphical inspection, the daily returns show substantial volatility clustering, which can be noticed for most emerging markets. This gives motivation for the case of GARCH models to explain the volatility behaviour of each individual market, and the multivariate parameterization allows us to estimate the conditional correlations among emerging markets as well as between an individual emerging market and the US. At the availability of various multivariate volatility models, we need to answer a research question of how to select the best model that fit well to the emerging data. This will be discussed in more details in the next chapter, which explains the model choice for the case of emerging financial markets.

Table 1 gives a primary descriptive statistics of the return series with the unconditional means, the standard deviations, the skewness, the kurtosis and the estimates of the univariate t-GARCH(1,1) for each return series which is individually assumed to follow univariate Student's t-distribution with $\hat{\nu}$ degrees of freedom. All observations are used and the results indicate that all countries except Philippines have a positive mean. The European and Latin American countries generally have higher mean in returns than the countries from Asia. The significant degrees of skewness of some markets such as China, Korea, Malaysia is positive and significantly different from zero and all markets show a clear excessive kurtosis from the lowest level at 5.497 for Taiwan to the highest being 59.259 for Malaysia and the mean of the kurtosis centres around 15. This suggests that the use of the volatility models with the normality assumption is likely to be too restrictive.

The maximum likelihood estimates of the univariate t-GARCH model also give evidence of high volatility persistence and the parameters show similar behaviour of the conditional volatility across emerging markets. The similarity can also be found in the estimates of the degrees of freedom of the Student's t-distribution, which range from the highest of 8.530 for Chile to the lowest of 4.085 for Indonesia with the mean for all markets of 5.744. This result suggests again that the normality assumption should be replaced by that of Student's t-distribution. However,

Table 1: Descriptive Statistics and Univariate GARCH under Student's *t*-distribution Assumption (3910 observations)

Countries	**Mean**	**St.Dev.**	**Skewness**	**Kurtosis**	***t*-GARCH(1,1)**					
	(× 100%)				$\hat{\lambda}_{2i}$	**S.E.**	$\hat{\lambda}_{1i}$	**S.E.**	$\hat{\nu}_i$	**S.E.**
Asia										
CHINA	1.809	2.126	0.270	08.374	0.095	0.0125	0.894	0.0108	5.900	0.4744
INDIA	5.143	1.823	0.186	11.076	0.108	0.0160	0.868	0.0123	6.221	0.4557
INDONESIA	5.077	2.922	-0.003	22.400	0.101	0.0130	0.890	0.0116	4.085	0.2193
KOREA	4.989	2.609	0.809	18.417	0.061	0.0079	0.934	0.0072	5.892	0.4883
MALAYSIA	1.786	1.890	0.892	59.259	0.094	0.0123	0.901	0.0115	4.452	0.1964
PHILIPPINES	-0.390	1.812	0.960	18.993	0.164	0.0248	0.784	0.0173	5.012	0.3180
TAIWAN	1.381	1.757	0.045	05.497	0.052	0.0098	0.939	0.0077	5.630	0.3322
THAILAND	0.310	2.245	0.894	13.987	0.091	0.0151	0.893	0.0118	4.680	0.2511
Mean	*2.513*	*2.148*	*0.507*	*19.751*	*0.096*		*0.888*		*5.234*	
Latin America										
BRAZIL	6.967	2.413	0.144	10.600	0.094	0.0150	0.893	0.0126	7.045	0.7294
CHILE	2.492	1.324	0.135	16.820	0.111	0.0170	0.861	0.0123	8.530	1.0621
COLOMBIA	6.139	1.670	0.170	14.088	0.236	0.0270	0.714	0.0220	5.050	0.5649
MEXICO	6.417	1.915	0.134	12.540	0.103	0.0156	0.873	0.0116	5.851	0.4874
PERU	6.048	1.792	0.026	10.347	0.080	0.0129	0.909	0.0107	4.833	0.2641
Mean	*5.613*	*1.823*	*0.122*	*12.879*	*0.125*		*0.850*		*6.262*	
Europe										
CZECH	6.194	1.786	0.181	15.209	0.100	0.0134	0.884	0.0108	6.634	0.6201
HUNGARY	4.126	1.449	-0.223	07.767	0.043	0.0088	0.948	0.0069	4.230	0.2145
ISRAEL	7.468	2.259	0.088	12.674	0.109	0.0177	0.868	0.0138	5.727	0.4796
POLAND	3.613	2.086	-0.024	06.687	0.075	0.0122	0.906	0.0091	7.201	0.4330
RUSSIA	11.278	3.243	0.189	12.382	0.126	0.0137	0.861	0.0117	4.965	0.3077
TURKEY	7.914	3.162	0.250	09.230	0.110	0.0229	0.855	0.0155	5.412	0.3880
UNITED STATES	2.733	1.267	-0.022	11.414	0.068	0.0084	0.926	0.0076	7.533	0.8012
Mean	*6.190*	*2.179*	*0.063*	*10.766*	*0.090*		*0.893*		*5.957*	
Mean(all countries)	*4.575*	*2.078*	*0.255*	*14.888*	*0.101*		*0.880*		*5.744*	

as presented by Pesaran *et al* (2009), the estimation of the multivariate volatility models has a significant technical difficulty when assuming Student's t-distributions due to the fact the Quasi Maximum Likelihood Estimation (QMLE) gives the estimates of the multivariate volatility models using normally distributed errors. To overcome this difficulty, the authors used the assumption of a Student's t-distribution with generic degrees of freedom, which was fixed at 7 to estimate all 52 volatility models. Thus, in Pesaran *et al* (2009), based on the descriptive analysis of data from developed markets, the mean of estimated degrees of freedom ($\hat{\nu}$), given by the univariate t-GARCH model for 18 developed markets, is 6.7. Therefore, those authors chose the 7 degrees of freedom for the Student's t-distribution assumption for the volatility models, which are designed to work under the Normal distribution. Following our descriptive statistics of return series of 20 financial markets, where the mean of $\hat{\nu}$ for 20 markets is 5.7, we are going to estimate the 53 volatility models assuming that the errors follow a Student's t-distribution with 6 degrees of freedom and the TDCC model was estimated using the t-distributed errors with the endogenous degrees of freedom, ν_t which was estimated at every sub-sample.

The difference in $\hat{\nu}$ between emerging data and developed data showed that the distribution of emerging data, which has the smaller $\hat{\nu}$, has fatter tails than those of the distribution of developed data. This indicates that the emerging financial markets experience larger positive and negative returns than the developed markets. It means that the emerging markets are more volatile than the developed markets. Therefore, the data analysis helps to choose the appropriate degrees of freedom for the Student's t-distribution assumption used in the model estimations and evaluations in this study to give empirical results relevant to the emerging data.

To estimate the 54 volatility models, which belong to 10 different model types, we followed the method introduced by Pesaran *et al* (2009). Firstly, we used the AR(1) model defined in Equation 1 to generate one-day-ahead forecast of the conditional mean, $\hat{\mu}_{k,t+1}$ which is denoted as follows

$$\hat{\mu}_{k,t+1} = E(r_{k,t+1}|F_t) = \hat{c}_{0,k} + \hat{c}_{1,k} r_{k,t} \text{ with } k\text{=1, 2, ..., 20} \tag{31}$$

The AR(1) model presented in Equation 1 was fitted to individual return series using a window of 800 observations which was rolling forward by each day when the AR(1) model was re-estimated, then the forecasted error of the conditional mean, $\hat{\epsilon}_{k,t+1} = r_{k,t+1} - \hat{\mu}_{k,t+1}$ was generated recursively. So our 54 models were estimated using the one-day-ahead forecast errors, $\hat{\epsilon}_{k,t+1}$ and a rolling window of 800 observations. All models were re-estimated at the frequency of 25 days indicating that the risk updates for the parameters of the volatility models is monthly, which is considered as reasonable in terms of risk management. A daily risk update was considered, which requires the 54 models to be estimated every day. This means that the total times of model estimations is 167,949 (=54 x (3910-800)). With the current ability of computation, it is imposible to realise this type of risk update. Therefore, monthly risk update is more reasonable, where each of the 54 models was estimated 125 times over the whole estimation sample and the total times of model estimations is 6750. For the data from 19 emerging financial markets and the US market, no model failed to converge.

3.2 Model Ranking

3.2.1 In-sample evaluations

3.2.1.1 The methodology of in-sample evaluations

This section will give a brief discussion of how the 54 volatility models can be evaluated in this study. In this study, we used the two popular methods in financial econometrics for the evaluations of model performance, which are the in-sample and out-of-sample evaluations.

The method of in-sample evaluation is to evaluate how well a model fits to a data set. In this study, all multivariate volatility models were estimated by the method of maximum likelihood so the in-sample evaluation was performed by using the maximized value of the log-likelihood (LL) function. The best-fitting model will have a highest LL value. Moreover, one can argue that models with larger number of parameters are more likely to have high LL values. Hence, AIC and SBIC are introduced to give robust in-sample evaluations, which use a penalty for the number of parameters used by a volatility model.

Based on the estimation results of the 54 volatility models, we use the maximized log-likelihood values to compute the AIC and the SBIC as follows

$$AIC_{i,t} = LL_{i,t} - \kappa_i \; and \; SBIC_{i,t} = LL_{i,t} - \frac{\kappa_i}{2} ln(W) \tag{32}$$

where $LL_{i,t}$ is the maximized log-likelihood value of model i at time t; κ_i is the total number of parameters used by model i; W is the size of estimation window which is set to 800 observations. The information criteria were computed based on the estimates of the volatility models using the Gaussian errors and the Student's t-distributed errors with 6 degrees of freedom.

3.2.1.2 Result and discussion

Tables 2 and 3 display the maximized log-likelihood values of the estimated volatility models using the Gaussian and the Student's t-distribution with 6 degrees of freedom, respectively. These tables deliver the values of $LL_{k,t}$ for the first sub-sample starting from 15/05/1995 to 16/06/98, for the last sub-sample starting from 03/04/2007 to 07/05/2010 and for the average of 125 sub-samples for each individual volatility model. For the Gaussian assumption, the log-likelihood values are between the lowest of -38,386 for MMA(50,0.95) model to the highest of -28,299 for the TDCC model. Also, these values vary among the different model types while they are similar among the models in the same model type. For the family of the Riskmetrics filters, none of the averages of the LL values is above -30,000. The best model of this type is the GEWMA(2,2,0.97) model with the LL value of -30,566. The O-GARCH models which the LL values centering around -29,390 performed better than the Riskmetrics specifications. However, the DCC-type models showed the best performance with the average values of the LL centering around -28,600 for the DCC models, around -28,620 for the CCC models, at -29,717 for the CDCC model and the highest value of -28,299 for the TDCC model. The ADCC models except the ADCC(1,1) provided the poorest performance in this model type with the average values of the LL being between -35,886 for the ADCC(2,2) and -30,111 for the ADCC(2,1). The ADCC(1,1), in contrast, was the second best model after the TDCC model. The reason for the differences in the performance of all estimated models is that the Gaussian assumption is not relevant for the data of emerging markets which show the fat-tailed behaviour. That is, the TDCC model, which was estimated using t-distributed innovations with the degrees of freedom, ν_t which was estimated at every sub-sample, showed the best in-sample performance while the poor-performing ADCC models, which allows for the asymmetric shocks causing the fat-tailed behaviour of the returns series, did not fit the data well under the Gaussian assumption. However, by assuming the Student's t-distribution with 6 degrees of freedom for the returns series, all models, except the TDCC, showed a significant increase in the maximized LL values. The TDCC model, which was fitted to the t-distributed innovations with the endogenous degrees of freedom, was still the best model with the highest LL value of -28,299 while all models of the ADCC type showed the biggest increase in the LL values, for example the LL value for the ADCC(2,2) increased by -7,256 from -35,886 to -28,630. The DCC-type models also fitted best under the assumption of the Student's t innovations with the LL value of the DCC models, ranging from -28,418 for the DCC(1,1) to -28,403 for the DCC(2,2); the LL value of the CCC model being between -28,435 for the CCC(1,1) and -28,418 for CCC(2,2). The Riskmetrics filters under the Student assumption were still the worst-performing models with the LL values ranging from -32,519 for to the MMA(50,0.95) to -29,584 for the EQMA(250) while the O-GARCH models continued to be ranked in the middle between the Riskmetrics and the DCC-type models.

Tables 4 and 5 display the AIC values for the 54 models under the Gaussian assumption and the Student's t-distribution with 6 degrees of freedom, respectively while Tables 6 and 7 deliver the SBIC values for the 54 models under the Gaussian assumption and the Student's t-distribution with 6 degrees of freedom, respectively. In these tables, models were ranked by using the information criteria and the ranking results were reported in parentheses, where 1 means the best model and 54 means the worst model, for the first sub-sample (15/05/1995 to 16/06/98), the last sub-sample (03/04/2007 to 07/05/2010), and for the average values of 125 sub-samples for each of 54 models.

Table 2: Maximized Values of Log-Likelihood for 54 Multivariate Volatility Models under Normal Distribution Assumption

Model type	Sample periods				Sample periods		
	16-Jun-98	07-May-10	Average		16-Jun-98	07-May-10	Average
	(1)	(2)	(3)		(4)	(5)	(6)
EQMA				(1,1,0.97)	-31216	-31413	-30586
(n_0)				(1,2,0.97)	-31134	-31400	-30572
(250)	-32562	-32680	-30700	(2,1,0.97)	-31210	-31484	-30592
(125)	-31599	-32033	-30870	(2,2,0.97)	-31124	-31460	-30566
(75)	-32288	-32479	-31813				
(50)	-34622	-34427	-34186	OGARCH			
				(p,q)			
EWMA				(1,1)	-29573	-29235	-29397
(n_0,λ_0,ν_0)				(1,2)	-29555	-29220	-29382
(250,0.95,0.95)	-35014	-34056	-33694	(2,1)	-29573	-29235	-29408
(250,0.97,0.95)	-35685	-34604	-33977	(2,2)	-29552	-29221	-29386
(250,0.95,0.97)	-32111	-31673	-31197				
(250,0.97,0.97)	-32496	-32019	-31337	CCC			
(125,0.95,0.95)	-34410	-34220	-33659	(p,q)			
(125,0.97,0.95)	-35015	-34790	-33966	(1,1)	-28006	-28693	-28636
(125,0.95,0.97)	-31740	-31899	-31331	(1,2)	-27973	-28692	-28626
(125,0.97,0.97)	-32081	-32228	-31485	(2,1)	-28001	-28699	-28628
(75,0.95,0.95)	-34850	-34786	-34342	(2,2)	-27961	-28684	-28610
(75,0.97,0.95)	-35461	-35397	-34700				
(75,0.95,0.97)	-32534	-32689	-32317	DCC			
(75,0.97,0.97)	-32897	-33030	-32508	(p,q)			
(50,0.95,0.95)	-37580	-37124	-36980	(1,1)	-27995	-28651	-28614
(50,0.97,0.95)	-38285	-37699	-37398	(1,2)	-27962	-28649	-28604
(50,0.95,0.97)	-35447	-35211	-35136	(2,1)	-27990	-28661	-28607
(50,0.97,0.97)	-35917	-35557	-35385	(2,2)	-27950	-28646	-28589
MMA				ADCC			
(n_0,ν_0)				(p,q)			
(250,0.95)	-42570	-40983	-36871	(1,1)	-27924	-28513	-28516
(250,0.97)	-36990	-36687	-33343	(1,2)	-27900	-28516	-34136
(125,0.95)	-38208	-38485	-35847	(2,1)	-27904	-28540	-30111
(125,0.97)	-34183	-34735	-32756	(2,2)	-27860	-28502	-35886
(75,0.95)	-37409	-37598	-36001				
(75,0.97)	-34217	-34495	-33388	TDCC			
(50,0.95)	-39849	-39081	-38386	(p,q)			
(50,0.97)	-37026	-36496	-36057	(1,1)	-27733	-28252	-28299
GEWMA				CDCC			
(p,q,ν_0)				(p,q)			
(1,1,0.95)	-33391	-33366	-32593	(1,1)	-28495	-35528	-29717
(1,2,0.95)	-33286	-33350	-32582				
(2,1,0.95)	-33381	-33442	-32602				
(2,2,0.95)	-33267	-33416	-32580				

Table 3: Maximized Values of Log-Likelihood for 54 Multivariate Volatility Models under Student's t-distribution Assumption

Model type	Sample periods				Sample periods		
	16-Jun-98	07-May-10	Average		16-Jun-98	07-May-10	Average
	(1)	(2)	(3)		(4)	(5)	(6)
EQMA				(1,1,0.97)	-30195	-30574	-29897
(n_0)				(1,2,0.97)	-30164	-30573	-29892
(250)	-30436	-30264	-29584	(2,1,0.97)	-30190	-30593	-29894
(125)	-29980	-30201	-29768	(2,2,0.97)	-30156	-30595	-29885
(75)	-30271	-30559	-30322				
(50)	-31064	-31457	-31382	OGARCH			
				(p,q)			
EWMA				(1,1)	-29182	-28771	-29052
(n_0,λ_0,ν_0)				(1,2)	-29168	-28770	-29044
(250,0.95,0.95)	-31568	-31569	-31160	(2,1)	-29181	-28770	-29061
(250,0.97,0.95)	-31767	-31568	-31221	(2,2)	-29168	-28771	-29051
(250,0.95,0.97)	-30482	-30558	-30075				
(250,0.97,0.97)	-30632	-30543	-30104	CCC			
(125,0.95,0.95)	-31091	-31490	-31102	(p,q)			
(125,0.97,0.95)	-31270	-31515	-31173	(1,1)	-27791	-28374	-28435
(125,0.95,0.97)	-30075	-30559	-30112	(1,2)	-27773	-28376	-28428
(125,0.97,0.97)	-30205	-30560	-30151	(2,1)	-27790	-28376	-28428
(75,0.95,0.95)	-31108	-31574	-31352	(2,2)	-27769	-28374	-28418
(75,0.97,0.95)	-31296	-31626	-31436				
(75,0.95,0.97)	-30305	-30849	-30582	DCC			
(75,0.97,0.97)	-30447	-30872	-30634	(p,q)			
(50,0.95,0.95)	-31885	-32303	-32236	(1,1)	-27778	-28336	-28418
(50,0.97,0.95)	-32048	-32363	-32320	(1,2)	-27761	-28337	-28412
(50,0.95,0.97)	-31301	-31798	-31698	(2,1)	-27776	-28350	-28413
(50,0.97,0.97)	-31428	-31834	-31755	(2,2)	-27757	-28349	-28403
MMA				ADCC			
(n_0,ν_0)				(p,q)			
(250,0.95)	-32817	-31952	-31744	(1,1)	-27740	-28253	-28353
(250,0.97)	-31601	-31006	-30604	(1,2)	-27726	-28261	-28626
(125,0.95)	-31943	-31784	-31543	(2,1)	-27711	-28272	-28486
(125,0.97)	-30805	-30811	-30482	(2,2)	-27703	-28261	-28630
(75,0.95)	-31811	-31842	-31719				
(75,0.97)	-30896	-31049	-30873	TDCC			
(50,0.95)	-32396	-32514	-32519	(p,q)			
(50,0.97)	-31725	-31951	-31915	(1,1)	-27733	-28252	-28299
GEWMA				CDCC			
(p,q,ν_0)				(p,q)			
(1,1,0.95)	-31186	-31518	-30915	(1,1)	-28200	-30476	-28932
(1,2,0.95)	-31158	-31519	-30912				
(2,1,0.95)	-31180	-31539	-30911				
(2,2,0.95)	-31146	-31540	-30905				

As the AIC and the SBIC are computed based on the LL values, the ranking of the 54 models is also similar to the one above obtained using the LL values. Following the average of the AIC and the SBIC of 125 sub-samples, the DCC-type models were ranked in the top 10 in which the TDCC model was the best model. The reason why the TDCC was the best model based on either the AIC or the SBIC is that the degrees of freedom were estimated for every sub-sample and the devolatization technique made the innovations approximately Gaussian rather than standardization technique used in the DCC model. There is not much difference in the ranking between the AIC and the SBIC under the same distribution assumption. However, a significant difference can be noted between the different distribution assumptions of the same information criterion. Specifically, the ADCC models were ranked in the top 10 models based on the SBIC and in the top 15 models based on the AIC under the assumption of Student's t-distribution while only one of the ADCC models (the ADCC(1,1)) was in the top 10 models based on both the information criteria under the Gaussian distribution. Moreover, under the assumption of the Gaussian distribution, 2 out of 4 ADCC models (the ADCC(1,2) and the ADCC(2,2)) were ranked in the bottom 10 models. The DCC models were consistently in the top 10 models regardless of the distribution assumptions while the CDCC model, which was newly developed by Aielli (2011), was ranging from a rank of 10 to 15 following the different information criteria under different the distributional assumptions. This result indicates that using the medium-scaled data containing 20 returns series the DCC model still performs consistently as the CDCC model is suggested by Aielli to show the consistency for large-scaled data set, i.e. data set containing over 100 return series. The next best model after the DCC model was the CCC ranking from 7 to 18. For the CCC models, it is suggested that these were fitted better using the assumption of normally distributed emerging data rather than the t-distributed emerging data. For example, the CCC models were ranked from 7 to 10 for the SBIC under the Gaussian assumption while they were ranked by the same information criterion from 12 to 18 under the Student's t-distribution assumption. The models following the CCC models in the ranking table were the O-GARCH models with the ranks ranging 11 to 18. The family of the Riskmetrics models was ranked in the bottom of all models, which indicates that the Riskmetrics filters are not relevant to fit to the data from emerging markets. Thus, following Pesaran *et al* (2009), one member of the Riskmetrics filters (the EQMA(250)) was ranked in the top 10 model for the data from developed financial markets. However, in our study, no Riskmetrics models were ranked even in the top 15 models. Among the Riskmetrics model, the simplest specification, the EQMA(250), with the ranks ranging from 17 to 21, and the most advanced filter, the GEWMA(1,1,0.97), with the ranks being between 17 and 21, performed considerably better than the other filters in the family.

For in-sample evaluation, the TDCC model continued to be the best model when being fitted to the emerging data. The noticeable difference is that the group of the ADCC models which used to be the second best group in the study of Pesaran *et al* (2009) has now 3 out of 4 models having the worst performance under the Gaussian assumption. For the medium-scaled data of 20 return series, the rank of the CDCC(1,1) model was lower than those of the DCC models, which is also consistent with the conclusion proposed by Aielli (2011) that the CDCC model will only be more consistent when used with large-scaled data. The DCC models consistently performed better than the ADCC models. However, the ADCC(1,1) shows exceptional performance being ranked in the top 3 by any in-sample criteria. This suggests that it is necessary to consider the asymmetric property of the financial emerging data. The poor performance of the Riskmetrics models indicates that these specifications maybe are designed to fit to the data from more integrated markets. The dominance of the DCC-type models in the in-sample performance when they are fitted to both developed and developing data shows that the DCC-type models are more reliable in general and can be applied to various types of financial markets and assets.

3.2.2 Out-of-sample evaluations

The in-sample evaluation can only tell us the goodness of fit of a model while it cannot tell how well a model is in forecast. That is why we also need to have out-of-sample evaluation, which is based on the forecast of a model to evaluate its performance. In econometrics, the popular method to evaluate the out-of-sample performance of a volatility model is to use some standard statistics such as MSE (Mean Squared Error), Mean Absolute Error

Table 4: AIC Values for 54 Multivariate Volatility Models under Normal Distribution Assumption

Model type	Sample periods 16-Jun-98 (1)		07-May-10 (2)		Average (3)	
EQMA						
(n_0)						
(250)	-32562	(30)	-32680	(28)	-30700	(21)
(125)	-31599	(23)	-32033	(25)	-30870	(22)
(75)	-32288	(27)	-32479	(27)	-31813	(27)
(50)	-34622	(39)	-34427	(37)	-34186	(41)
EWMA						
(n_0, λ_0, ν_0)						
(250,0.95,0.95)	-35014	(41)	-34056	(35)	-33694	(38)
(250,0.97,0.95)	-35685	(45)	-34604	(39)	-33977	(40)
(250,0.95,0.97)	-32111	(26)	-31673	(22)	-31197	(23)
(250,0.97,0.97)	-32496	(28)	-32019	(24)	-31337	(25)
(125,0.95,0.95)	-34410	(38)	-34220	(36)	-33659	(37)
(125,0.97,0.95)	-35015	(42)	-34790	(42)	-33966	(39)
(125,0.95,0.97)	-31740	(24)	-31899	(23)	-31331	(24)
(125,0.97,0.97)	-32081	(25)	-32228	(26)	-31485	(26)
(75,0.95,0.95)	-34850	(40)	-34786	(41)	-34342	(43)
(75,0.97,0.95)	-35461	(44)	-35397	(44)	-34700	(44)
(75,0.95,0.97)	-32534	(29)	-32689	(29)	-32317	(28)
(75,0.97,0.97)	-32897	(31)	-33030	(30)	-32508	(29)
(50,0.95,0.95)	-37580	(50)	-37124	(49)	-36980	(52)
(50,0.97,0.95)	-38285	(52)	-37699	(51)	-37398	(53)
(50,0.95,0.97)	-35447	(43)	-35211	(43)	-35136	(45)
(50,0.97,0.97)	-35917	(46)	-35557	(45)	-35385	(46)
MMA						
(n_0, ν_0)						
(250,0.95)	-42570	(54)	-40983	(54)	-36871	(51)
(250,0.97)	-36990	(47)	-36687	(48)	-33343	(35)
(125,0.95)	-38208	(51)	-38485	(52)	-35847	(47)
(125,0.97)	-34183	(36)	-34735	(40)	-32756	(34)
(75,0.95)	-37409	(49)	-37598	(50)	-36001	(48)
(75,0.97)	-34217	(37)	-34495	(38)	-33388	(36)
(50,0.95)	-39849	(53)	-39081	(53)	-38386	(54)
(50,0.97)	-37026	(48)	-36496	(47)	-36057	(50)
GEWMA						
(p, q, ν_0)						
(1,1,0.95)	-33451	(34)	-33426	(31)	-32653	(30)
(1,2,0.95)	-33366	(32)	-33430	(32)	-32662	(31)
(2,1,0.95)	-33461	(35)	-33522	(34)	-32682	(33)
(2,2,0.95)	-33367	(33)	-33516	(33)	-32680	(32)

Model type	Sample periods 16-Jun-98 (4)		07-May-10 (5)		Average (6)	
(1,1,0.97)	-31276	(21)	-31473	(18)	-30646	(17)
(1,2,0.97)	-31214	(19)	-31480	(19)	-30652	(18)
(2,1,0.97)	-31290	(22)	-31564	(21)	-30672	(20)
(2,2,0.97)	-31224	(20)	-31560	(20)	-30666	(19)
OGARCH						
(p, q)						
(1,1)	-29633	(15)	-29295	(14)	-29457	(11)
(1,2)	-29635	(16)	-29300	(15)	-29462	(12)
(2,1)	-29653	(18)	-29315	(16)	-29488	(14)
(2,2)	-29652	(17)	-29321	(17)	-29486	(13)
CCC						
(p, q)						
(1,1)	-28256	(12)	-28943	(10)	-28886	(7)
(1,2)	-28243	(10)	-28962	(11)	-28896	(8)
(2,1)	-28271	(13)	-28969	(12)	-28898	(9)
(2,2)	-28251	(11)	-28974	(13)	-28900	(10)
DCC						
(p, q)						
(1,1)	-28057	(8)	-28713	(6)	-28676	(3)
(1,2)	-28044	(6)	-28731	(7)	-28686	(4)
(2,1)	-28072	(9)	-28743	(8)	-28689	(5)
(2,2)	-28052	(7)	-28748	(9)	-28691	(6)
ADCC						
(p, q)						
(1,1)	-28007	(4)	-28596	(2)	-28599	(2)
(1,2)	-28003	(3)	-28619	(3)	-34239	(42)
(2,1)	-28027	(5)	-28663	(5)	-30234	(16)
(2,2)	-28003	(2)	-28645	(4)	-36029	(49)
TDCC						
(p, q)						
(1,1)	-27776	(1)	-28295	(1)	-28342	(1)
CDCC						
(p, q)						
(1,1)	-28557	(14)	-35590	(46)	-29779	(15)

Table 5: AIC Values for 54 Multivariate Volatility Models under Student's t-distribution Assumption

Model type	16-Jun-98 (1)		07-May-10 (2)		Average (3)			16-Jun-98 (4)		07-May-10 (5)		Average (6)	
EQMA							(1,1,0.97)	-30255	(23)	-30634	(26)	-29957	(21)
(n_0)							(1,2,0.97)	-30244	(22)	-30653	(27)	-29972	(22)
(250)	-30436	(28)	-30264	(19)	-29584	(19)	(2,1,0.97)	-30270	(25)	-30673	(28)	-29974	(23)
(125)	-29980	(19)	-30201	(18)	-29768	(20)	(2,2,0.97)	-30256	(24)	-30695	(29)	-29985	(24)
(75)	-30271	(26)	-30559	(23)	-30322	(29)							
(50)	-31064	(34)	-31457	(35)	-31382	(44)	OGARCH						
							(p,q)						
EWMA							(1,1)	-29242	(15)	-28831	(14)	-29112	(15)
(n_0,λ_0,ν_0)							(1,2)	-29248	(16)	-28850	(16)	-29124	(16)
(250,0.95,0.95)	-31568	(45)	-31569	(39)	-31160	(40)	(2,1)	-29261	(17)	-28850	(15)	-29141	(17)
(250,0.97,0.95)	-31767	(48)	-31568	(38)	-31221	(42)	(2,2)	-29268	(18)	-28871	(17)	-29151	(18)
(250,0.95,0.97)	-30482	(30)	-30558	(22)	-30075	(25)							
(250,0.97,0.97)	-30632	(31)	-30543	(21)	-30104	(26)	CCC						
(125,0.95,0.95)	-31091	(35)	-31490	(36)	-31102	(39)	(p,q)						
(125,0.97,0.95)	-31270	(41)	-31515	(37)	-31173	(41)	(1,1)	-28041	(10)	-28624	(10)	-28685	(8)
(125,0.95,0.97)	-30075	(20)	-30559	(24)	-30112	(27)	(1,2)	-28043	(11)	-28646	(12)	-28698	(9)
(125,0.97,0.97)	-30205	(21)	-30560	(25)	-30151	(28)	(2,1)	-28060	(13)	-28646	(11)	-28698	(10)
(75,0.95,0.95)	-31108	(36)	-31574	(40)	-31352	(43)	(2,2)	-28059	(12)	-28664	(13)	-28708	(11)
(75,0.97,0.95)	-31296	(42)	-31626	(44)	-31436	(45)							
(75,0.95,0.97)	-30305	(27)	-30849	(31)	-30582	(31)	DCC						
(75,0.97,0.97)	-30447	(29)	-30872	(32)	-30634	(33)	(p,q)						
(50,0.95,0.95)	-31885	(50)	-32303	(52)	-32236	(52)	(1,1)	-27840	(5)	-28398	(5)	-28480	(3)
(50,0.97,0.95)	-32048	(52)	-32363	(53)	-32320	(53)	(1,2)	-27843	(6)	-28419	(7)	-28494	(4)
(50,0.95,0.97)	-31301	(43)	-31798	(47)	-31698	(47)	(2,1)	-27858	(8)	-28432	(8)	-28495	(5)
(50,0.97,0.97)	-31428	(44)	-31834	(48)	-31755	(50)	(2,2)	-27859	(9)	-28451	(9)	-28505	(6)
MMA							ADCC						
(n_0,ν_0)							(p,q)						
(250,0.95)	-32817	(54)	-31952	(51)	-31744	(49)	(1,1)	-27823	(2)	-28336	(2)	-28436	(2)
(250,0.97)	-31601	(46)	-31006	(33)	-30604	(32)	(1,2)	-27829	(3)	-28364	(3)	-28729	(12)
(125,0.95)	-31943	(51)	-31784	(46)	-31543	(46)	(2,1)	-27834	(4)	-28395	(4)	-28609	(7)
(125,0.97)	-30805	(32)	-30811	(30)	-30482	(30)	(2,2)	-27846	(7)	-28404	(6)	-28773	(13)
(75,0.95)	-31811	(49)	-31842	(49)	-31719	(48)							
(75,0.97)	-30896	(33)	-31049	(34)	-30873	(34)	TDCC						
(50,0.95)	-32396	(53)	-32514	(54)	-32519	(54)	(p,q)						
(50,0.97)	-31725	(47)	-31951	(50)	-31915	(51)	(20)	-27776	(1)	-28295	(1)	-28342	(1)
GEWMA							CDCC						
(p,q,ν_0)							(p,q)						
(1,1,0.95)	-31246	(38)	-31578	(41)	-30975	(35)	(1,1)	-28262	(14)	-30538	(1,1)	-28994	(14)
(1,2,0.95)	-31238	(37)	-31599	(42)	-30992	(37)							
(2,1,0.95)	-31260	(40)	-31619	(43)	-30991	(36)							
(2,2,0.95)	-31246	(39)	-31640	(45)	-31005	(38)							

Table 6: SBIC Values for 54 Multivariate Volatility Models under Normal Distribution Assumption

Model type	Sample periods 16-Jun-98 (1)		07-May-10 (2)		Average (3)	
EQMA						
(n_0)						
(250)	-32562	(30)	-32680	(28)	-30700	(17)
(125)	-31599	(23)	-32033	(25)	-30870	(21)
(75)	-32288	(27)	-32479	(27)	-31813	(27)
(50)	-34622	(39)	-34427	(37)	-34186	(41)
EWMA						
(n_0, λ_0, ν_0)						
(250,0.95,0.95)	-35014	(41)	-34056	(35)	-33694	(38)
(250,0.97,0.95)	-35685	(45)	-34604	(39)	-33977	(40)
(250,0.95,0.97)	-32111	(26)	-31673	(20)	-31197	(23)
(250,0.97,0.97)	-32496	(28)	-32019	(24)	-31337	(25)
(125,0.95,0.95)	-34410	(38)	-34220	(36)	-33659	(37)
(125,0.97,0.95)	-35015	(42)	-34790	(42)	-33966	(39)
(125,0.95,0.97)	-31740	(24)	-31899	(23)	-31331	(24)
(125,0.97,0.97)	-32081	(25)	-32228	(26)	-31485	(26)
(75,0.95,0.95)	-34850	(40)	-34786	(41)	-34342	(42)
(75,0.97,0.95)	-35461	(44)	-35397	(44)	-34700	(44)
(75,0.95,0.97)	-32534	(29)	-32689	(29)	-32317	(28)
(75,0.97,0.97)	-32897	(31)	-33030	(30)	-32508	(29)
(50,0.95,0.95)	-37580	(50)	-37124	(49)	-36980	(52)
(50,0.97,0.95)	-38285	(52)	-37699	(51)	-37398	(53)
(50,0.95,0.97)	-35447	(43)	-35211	(43)	-35136	(45)
(50,0.97,0.97)	-35917	(46)	-35557	(45)	-35385	(46)
MMA						
(n_0, ν_0)						
(250,0.95)	-42570	(54)	-40983	(54)	-36871	(51)
(250,0.97)	-36990	(47)	-36687	(48)	-33343	(35)
(125,0.95)	-38208	(51)	-38485	(52)	-35847	(47)
(125,0.97)	-34183	(36)	-34735	(40)	-32756	(30)
(75,0.95)	-37409	(49)	-37598	(50)	-36001	(48)
(75,0.97)	-34217	(37)	-34495	(38)	-33388	(36)
(50,0.95)	-39849	(53)	-39081	(53)	-38386	(54)
(50,0.97)	-37026	(48)	-36496	(47)	-36057	(49)
GEWMA						
(p, q, ν_0)						
(1,1,0.95)	-33592	(33)	-33567	(31)	-32794	(31)
(1,2,0.95)	-33553	(32)	-33618	(32)	-32849	(32)
(2,1,0.95)	-33648	(35)	-33710	(33)	-32869	(33)
(2,2,0.95)	-33601	(34)	-33750	(34)	-32914	(34)

Model type	Sample periods 16-Jun-98 (4)		07-May-10 (5)		Average (6)	
(1,1,0.97)	-31417	(20)	-31613	(18)	-30787	(18)
(1,2,0.97)	-31402	(19)	-31668	(19)	-30839	(19)
(2,1,0.97)	-31478	(22)	-31751	(21)	-30859	(20)
(2,2,0.97)	-31458	(21)	-31794	(22)	-30901	(22)
OGARCH						
(p, q)						
(1,1)	-29774	(15)	-29436	(10)	-29598	(11)
(1,2)	-29823	(16)	-29487	(11)	-29649	(12)
(2,1)	-29840	(17)	-29503	(12)	-29676	(13)
(2,2)	-29886	(18)	-29555	(14)	-29721	(14)
CCC						
(p, q)						
(1,1)	-28841	(11)	-29528	(13)	-29471	(7)
(1,2)	-28875	(12)	-29594	(15)	-29528	(8)
(2,1)	-28903	(13)	-29602	(16)	-29530	(9)
(2,2)	-28930	(14)	-29653	(17)	-29580	(10)
DCC						
(p, q)						
(1,1)	-28202	(3)	-28858	(3)	-28821	(3)
(1,2)	-28237	(4)	-28923	(5)	-28878	(4)
(2,1)	-28264	(6)	-28935	(6)	-28881	(5)
(2,2)	-28291	(7)	-28987	(9)	-28930	(6)
ADCC						
(p, q)						
(1,1)	-28201	(2)	-28791	(2)	-28794	(2)
(1,2)	-28244	(5)	-28860	(4)	-34480	(43)
(2,1)	-28315	(8)	-28951	(7)	-30522	(16)
(2,2)	-28338	(9)	-28980	(8)	-36364	(50)
TDCC						
(p, q)						
(1,1)	-27876	(1)	-28396	(1)	-28443	(1)
CDCC						
(p, q)						
(1,1)	-28702	(10)	-35735	(46)	-29924	(15)

Table 7: SBIC Values for Multivariate Volatility Models under Student's empht-distribution Assumption

Model type	Sample periods							Sample periods					
	16-Jun-98 (1)		07-May-10 (2)		Average (3)			16-Jun-98 (4)		07-May-10 (5)		Average (6)	
EQMA							(1,1,0.97)	-30395	(24)	-30775	(26)	-30098	(22)
(n_0)							(1,2,0.97)	-30432	(25)	-30840	(28)	-30160	(26)
(250)	-30436	(26)	-30264	(19)	-29584	(19)	(2,1,0.97)	-30457	(28)	-30860	(30)	-30161	(27)
(125)	-29980	(19)	-30201	(18)	-29768	(20)	(2,2,0.97)	-30490	(30)	-30929	(32)	-30219	(28)
(75)	-30271	(22)	-30559	(22)	-30322	(29)							
(50)	-31064	(34)	-31457	(35)	-31382	(44)	OGARCH						
							(p,q)						
EWMA							(1,1)	-29382	(15)	-28971	(10)	-29252	(11)
(n_0,λ_0,ν_0)							(1,2)	-29435	(16)	-29037	(12)	-29312	(13)
(250,0.95,0.95)	-31568	(45)	-31569	(39)	-31160	(37)	(2,1)	-29449	(17)	-29037	(11)	-29329	(14)
(250,0.97,0.95)	-31767	(48)	-31568	(38)	-31221	(41)	(2,2)	-29502	(18)	-29105	(13)	-29385	(17)
(250,0.95,0.97)	-30482	(29)	-30558	(21)	-30075	(21)							
(250,0.97,0.97)	-30632	(31)	-30543	(20)	-30104	(23)	CCC						
(125,0.95,0.95)	-31091	(35)	-31490	(36)	-31102	(35)	(p,q)						
(125,0.97,0.95)	-31270	(37)	-31515	(37)	-31173	(38)	(1,1)	-28627	(11)	-29209	(14)	-29270	(12)
(125,0.95,0.97)	-30075	(20)	-30559	(23)	-30112	(24)	(1,2)	-28675	(12)	-29278	(16)	-29331	(15)
(125,0.97,0.97)	-30205	(21)	-30560	(24)	-30151	(25)	(2,1)	-28692	(13)	-29278	(15)	-29331	(16)
(75,0.95,0.95)	-31108	(36)	-31574	(40)	-31352	(43)	(2,2)	-28739	(14)	-29343	(17)	-29387	(18)
(75,0.97,0.95)	-31296	(38)	-31626	(41)	-31436	(45)							
(75,0.95,0.97)	-30305	(23)	-30849	(29)	-30582	(31)	DCC						
(75,0.97,0.97)	-30447	(27)	-30872	(31)	-30634	(33)	(p,q)						
(50,0.95,0.95)	-31885	(50)	-32303	(52)	-32236	(52)	(1,1)	-27985	(2)	-28543	(3)	-28625	(2)
(50,0.97,0.95)	-32048	(52)	-32363	(53)	-32320	(53)	(1,2)	-28035	(4)	-28611	(5)	-28686	(4)
(50,0.95,0.97)	-31301	(39)	-31798	(45)	-31698	(47)	(2,1)	-28050	(5)	-28624	(6)	-28687	(5)
(50,0.97,0.97)	-31428	(42)	-31834	(47)	-31755	(50)	(2,2)	-28098	(7)	-28690	(8)	-28744	(6)
MMA							ADCC						
(n_0,ν_0)							(p,q)						
(250,0.95)	-32817	(54)	-31952	(51)	-31744	(49)	(1,1)	-28018	(3)	-28531	(2)	-28630	(3)
(250,0.97)	-31601	(46)	-31006	(33)	-30604	(32)	(1,2)	-28070	(6)	-28605	(4)	-28970	(8)
(125,0.95)	-31943	(51)	-31784	(43)	-31543	(46)	(2,1)	-28122	(8)	-28683	(7)	-28897	(7)
(125,0.97)	-30805	(32)	-30811	(27)	-30482	(30)	(2,2)	-28181	(9)	-28739	(9)	-29108	(9)
(75,0.95)	-31811	(49)	-31842	(48)	-31719	(48)							
(75,0.97)	-30896	(33)	-31049	(34)	-30873	(34)	TDCC						
(50,0.95)	-32396	(53)	-32514	(54)	-32519	(54)	(p,q)						
(50,0.97)	-31725	(47)	-31951	(50)	-31915	(51)	(1,1)	-27876	(1)	-28396	(1)	-28443	(1)
GEWMA							CDCC						
(p,q,ν_0)							(p,q)						
(1,1,0.95)	-31386	(40)	-31718	(42)	-31116	(36)	(1,1)	-28407	(10)	-30683	(25)	-29139	(10)
(1,2,0.95)	-31426	(41)	-31786	(44)	-31180	(40)							
(2,1,0.95)	-31448	(43)	-31806	(46)	-31179	(39)							
(2,2,0.95)	-31480	(44)	-31875	(49)	-31239	(42)							

(MAE), etc. However, it is difficult to apply this traditional method to evaluate a large number of multivariate volatility models from different families. Moreover, a major drawback of MSE method is that it relies on the fourth moment (squares of squares) of realised return. We can see in the following formula

$$MSE = \frac{1}{T}\sum_{t=1}^{T}(\bar{\sigma}_t - \hat{\sigma}_t)^2 \tag{33}$$

where $\bar{\sigma}_i$ is the realised covariance of a portfolio return; $\hat{\sigma}_i$ is the forecast of covariance of a portfolio. Both of them are considered as the second moment of return. If the distribution of return has a fat-tailed behaviour, the MSE criterion will be heavily biased due to effect of large shocks, thereby is less reliable in measuring a portfolio performance in finance.

Hence, another method, recently suggested in the literature, is the use of VaR-based diagnostic tests to check the performance of volatility models. This method differs from the traditional one by evaluating a volatility model based on decisions on how it performs in trading and risk management. The core of this method is the application of the Value at Risk theory in financial econometrics.

3.2.2.1 Value at Risk Theory in finance

Following its introduction in October, 1994 by JP Morgan, VaR is widely accepted by portfolio managers as a reliable method of quantifying market risk and by financial regulators as a milestone in the revolution of risk management. The role of risk measures, using VaR, in risk management are the interest of both academia and practitioners. VaR is defined as the maximum loss, which can occur with possibility of $X\%$ over a holding period of t days. It means that VaR gives an estimate of downside risk of a portfolio. Therefore, the VaR of a portfolio at time t can be defined in the following formula

$$Pr\left[p_t < VaR_t(\alpha)\right] = \alpha \tag{34}$$

This means that there is a possibility of $\alpha\%$ for a portfolio return, p_t, at time t to fall below $VaR_t(\alpha)$. The advantage of VaR is that it can summarize risk in a single number, which is a loss of a portfolio over a period from t-1 to t. For example, if a daily VaR is stated as 1% to a 95% level of confidence, this means that during the day there is a only a 5% chance that the portfolio return (the loss) will fall below 1%. The VaR measures a potential loss in market value of a portfolio using the estimated volatility and correlations. Therefore, the Value at Risk theory is now popular in risk management in finance. Thus, it is applied in the risk management of financial assets such as fixed-income instruments, options and stocks. Moreover, the VaR theory can be applied in credit risk management.[4]

To estimate VaR, there are three methods: Historical Method, Variance-Covariance Method and Monte-Carlo Simulation Method. The Historical Method simply rearranges actual portfolio returns, putting them into a histogram. It then assumes that history will repeat itself, from a risk perspective. The Variance-Covariance method assumes that portfolio return has Normal distribution. We then need to estimate the expected portfolio return and its standard deviation, which allow for plotting a Normal distribution curve along the actual return. The normal curve will help to locate where the worst 5% or 1% of actual portfolio return is. The Monte-Carlo Simulation approach uses a model to predict the future portfolio return and randomly runs hypothetical trials for this model. The predicted returns, generated by performing a number of trials, are now re-organised from the worst to the best. Looking at the 5% or 1% from the left tail, we can tell the maximum loss of a portfolio, which is the VaR of portfolio.

In financial econometrics, a multivariate volatility model can be used to estimate the conditional volatilities and correlations of a number of financial assets. So it can estimate the VaR of a portfolio constructed from those financial assets. The Variance-Covariance approach is appropriate to estimate the VaR of portfolio in this case.

[4] For the details of application of VaR in finance, see Introduction to Value at Risk, 4th ed., Choudhry (2006).

However, this approach assumes a multivariate Normal distribution for the return of a portfolio and linearity in the dependence structure between financial assets in portfolio. In fact, the distribution of financial return is more likely to have fatter tails than Normal distribution due to the presence of extremes in financial data. Moreover, the asymmetric property of financial return may cause a non-linearity in correlation between financial asset. Hence, a multivariate volatility model may not give a precise estimate VaR of a portfolio due to those reasons. It, therefore, initiate an idea that an estimate of the VaR of a portfolio can be use to evaluate the performance of a multivariate volatility model. A well-performing model will have to deliver an adequate VaR of a portfolio.

Using VaR method, there are two manners of model evaluation: one is from the point of view of financial authorities who monitor the trading behaviour of investors; the other is from the point of view of investors themselves. The manner for financial authorities is known as passive risk management, which assumes equal weights for assets in a portfolio, as they do not know the structure of portfolio. For investors, who know the weights of assets in a portfolio, the manner is called active risk management. Therefore, the passive risk management uses pre-determined weights for assets in a portfolio and the active risk management employs flexible weights for assets in a portfolio, which are optimally computed by using the popular approach of mean-variance analysis.

3.2.2.2 VaR-based diagnostic tests

The Value-at-Risk theory is used to focus on the estimate of a portfolio return based on the risk represented by the covariance matrix, H_t estimated by a volatility model. In our study, this technique is also appropriate for the out-of-sample evaluations of the 54 volatility models. The first step of the diagnostic test is to construct a portfolio based on $m \times 1$ vector of returns, $r_t \sim (\mu_t,\ H_t|\Omega_{t-1})$.

Let $\rho_{i,t}$ be the return on a portfolio comprised of m assets with weights, $w_{i,t-1}$ which can be pre-determined weights, $w^p_{i,t-1}$ for the passive risk management or the optimal weights, $w^a_{i,t-1}$ of model i for the active risk management. In this study, for the passive risk management, the weights for assets in the portfolio, $w^p_{i,t-1}$ are equally set to $\frac{1}{20}$ to compute the portfolio return, $\hat{\rho}^p_{i,t}$. For portfolio return, $\hat{\rho}^a_{i,t}$ in the active risk management, the optimal portfolio weights, $w^a_{i,t-1}$ were computed using the forecast of the conditional mean, $\mu_{i,t}$ given by Equation 31 and the one-step-ahead recursive forecast of the $H_{i,t}$ of each of the 54 volatility models under the assumption of the Gaussian distribution or the Student's t-distribution with 6 degrees of freedom. So the portfolio return can be generally expressed as

$$\rho_{i,t} = w'_{i,t-1} r_t \tag{35}$$

In risk management, managers always need a benchmark for the loss of their portfolios. For example, $\rho^\star_{i,t-1}$ is considered as the maximum daily loss so managers would expect a probability, α for their portfolio return, $\rho_{i,t}$ at time t to fall below the benchmark, $\rho^\star_{i,t-1}$ conditionally specified by all available information up to time t-1. Hence, α is the risk tolerance of managers which could be set at 1% or 5%. This constraint in risk management can be expressed as follows

$$Pr(\rho_{i,t} < -\rho^\star_{i,t-1}|\Omega_{t-1}) \leq \alpha \tag{36}$$

The main idea of this diagnostic test to check for the validity of a volatility model is to count the number of times this VaR constraint is violated using a count function, I_t as follows

$$I_t(\rho_{i,t} + \rho^\star_{i,t-1}) \begin{cases} = 1 & \text{if } \rho_{i,t} + \rho^\star_{i,t-1} < 0 \text{ or VaR constraint in } Equation 36 \text{ is violated} \\ = 0 & \text{otherwise} \end{cases} \tag{37}$$

For the evaluation for each model, the whole sample is divided into two sub-samples $T_{est}(t = 1 : T)\ and\ T_{eval}(t = T + 1 : T + N)$ where T_{est} is for model estimation and T_{eval} is for model evaluation. The VaR indicator, I_t is recursively computed by using N observations in the evaluation period. We can count the number of days when the VaR constraint is violated by using the VaR indicators for model i as follows

$$\hat{\pi}_i = \frac{1}{N}\sum_{t=T+1}^{T+N} \hat{I}_t \tag{38}$$

Hence, under the specification of a volatility model, $\hat{\pi}_i$ will have mean α and variance $\frac{\alpha(1-\alpha)}{N}$. Moreover, a standardized test statistic can be obtained based on the result from Equation 38 as follows

$$z_{\hat{\pi}_i} = \frac{\sqrt{N}(\hat{\pi}_i - \alpha)}{\sqrt{\alpha(1-\alpha)}} \tag{39}$$

For a sufficiently large sample of evaluation, the above test statistic is asymptotically normally distributed with zero mean and unit variance. The standardized test statistic is used to test the null hypothesis under which volatility model i is correctly specified

$$H_0: \quad H_t = H_t\left(\hat{\theta}_{S_{est}}\right) \ or \ \hat{\pi}_i = \alpha \tag{40}$$

3.2.2.3 Passive risk management

In passive risk management, the VaR constraint in Equation 36 becomes

$$Pr(\rho_t < -\bar{\rho}_{i,t-1}|\Omega_{t-1}) \leq \alpha \tag{41}$$

where ρ_t is constructed with no need of a volatility model i and weights are equally set to $\frac{1}{20}$. A volatility model i is only used to compute the benchmark for the maximum daily loss, $\bar{\rho}_{i,t-1}$.[5]

The first 800 observations were used for model estimation and the last 3104 observations were used for the recursive computations of the above statistics of model performance with the update frequency is 25 days or monthly update. By setting the risk tolerance probability $\alpha = 1\%$ and $\alpha = 5\%$ and based on the equally-weighted portfolio returns, we can compute the VaR exceedance ratio ($\hat{\pi}_i$) and the standardized test statistic ($z_{\hat{\pi}_i}$) for each model.

Tables 8 and 9 display the results for VaR-based diagnostic tests following the passive risk management with $\alpha = 1\%$ *and* $\alpha = 5\%$, respectively. These two tables give the estimated VaR exceedance ratio ($\hat{\pi}_i$, in percent) defined as the percentage of the days in the evaluation sample when the VaR constraint in Equation 36 was violated and the standardized test statistic ($z_{\hat{\pi}_i}$) described in Equation 39 following the Gaussian distribution and the Student's t-distribution with 6 degrees of freedom.

In Table 8, the risk tolerance, the violation rates for all models under the Gaussian distribution assumption, which range from a low of 1.90% for 6 out of the 8 GEWMA models to a high of 2.58% for the EQMA(250) and the EQMA(125) models, are consistently higher than those of all models under the Student's t-distribution assumption, which are between a low of 1.16% for the GEWMA(1,1,0.97) and a high of 2.03 for the EQMA(250) and the MMA(125,0.97) models. Therefore, the volatility models in this study performed better under the Student's t-distribution assumption. Under the Student's $t(6)$-distribution assumption, the rate of VaR constraint exceedance, $\hat{\pi}_i$ centres around 1.5% and does not vary markedly across the 54 models. The VaR exceedance rate of three Riskmetrics filters of the GEWMA(1,1,0.97), the GEWMA(1,2,0.97) and the GEWMA(2,1,0.97) are 1.16%, 1.19% and 1.19%, respectively. These are the lowest rates among the 54 model and marginally close to 1% with the test statistic, $z_{\hat{\pi}_i}$ of the GEWMA(1,1,0.97), the GEWMA(1,2,0.97), the GEWMA(2,1,0.97) and the GEWMA(2,2,0.97) models are 0.90, 1.08, 1.08, and 1.26, respectively. This indicates that the 3 models are correctly specified as the null hypothesis in Equation 40 cannot be rejected at 99% significance levels. The GEWMA(2,2,0.97) and O-GARCH(1,1) models also passed this test with the value of $\hat{\pi}_i$ and $z_{\hat{\pi}_i}$ being 1.22% and 1.26, respectively. The

[5] For the derivation of $\bar{\rho}_{i,t-1}$, see Appendix in [25]

Table 8: VaR-based Diagnostic Tests under Passive Risk Management Using 54 Multivariate Volatility Models ($\alpha = 1\%$)

Model type	normal		$t6$		Model type	normal		$t6$	
	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$		$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$
EQMA					(1,1,0.97)	1.90	5.05	1.16	0.90
(n_0)					(1,2,0.97)	1.90	5.05	1.19	1.08
(250)	2.26	7.03	1.87	4.87	(2,1,0.97)	1.93	5.23	1.19	1.08
(125)	2.58	8.84	2.03	5.77	(2,2,0.97)	1.90	5.05	1.22	1.26
(75)	2.58	8.84	2.00	5.59					
(50)	2.48	8.29	1.93	5.23	OGARCH				
					(p,q)				
EWMA					(1,1)	1.97	5.41	1.22	1.26
(n_0, λ_0, ν_0)					(1,2)	1.97	5.41	1.29	1.62
(250,0.95,0.95)	2.26	7.03	1.58	3.24	(2,1)	1.97	5.41	1.26	1.44
(250,0.97,0.95)	2.16	6.49	1.58	3.24	(2,2)	1.97	5.41	1.35	1.98
(250,0.95,0.97)	2.16	6.49	1.61	3.42					
(250,0.97,0.97)	2.16	6.49	1.68	3.78	CCC				
(125,0.95,0.95)	2.26	7.03	1.58	3.24	(p,q)				
(125,0.97,0.95)	2.19	6.67	1.58	3.24	(1,1)	2.19	6.67	1.64	3.60
(125,0.95,0.97)	2.19	6.67	1.61	3.42	(1,2)	2.26	7.03	1.58	3.24
(125,0.97,0.97)	2.16	6.49	1.71	3.96	(2,1)	2.13	6.31	1.61	3.42
(75,0.95,0.95)	2.26	7.03	1.64	3.60	(2,2)	2.19	6.67	1.61	3.42
(75,0.97,0.95)	2.19	6.67	1.61	3.42					
(75,0.95,0.97)	2.19	6.67	1.68	3.78	DCC				
(75,0.97,0.97)	2.22	6.85	1.68	3.78	(p,q)				
(50,0.95,0.95)	2.29	7.21	1.64	3.60	(1,1)	2.09	6.13	1.55	3.06
(50,0.97,0.95)	2.45	8.11	1.74	4.14	(1,2)	2.13	6.31	1.55	3.06
(50,0.95,0.97)	2.35	7.57	1.64	3.60	(2,1)	2.06	5.95	1.58	3.24
(50,0.97,0.97)	2.32	7.39	1.84	4.69	(2,2)	2.09	6.13	1.55	3.06
MMA					ADCC				
(n_0, ν_0)					(p,q)				
(250,0.95)	2.16	6.49	1.68	3.78	(1,1)	2.16	6.49	1.48	2.70
(250,0.97)	2.22	6.85	1.71	3.96	(1,2)	2.26	7.03	1.55	3.06
(125,0.95)	2.35	7.57	1.97	5.41	(2,1)	2.29	7.21	1.48	2.70
(125,0.97)	2.48	8.29	2.03	5.77	(2,2)	2.26	7.03	1.68	3.78
(75,0.95)	2.38	7.75	1.93	5.23					
(75,0.97)	2.51	8.47	1.93	5.23	TDCC				
(50,0.95)	2.48	8.29	1.80	4.51	(p,q)				
(50,0.97)	2.58	8.84	1.84	4.69	(1,1)	-	-	1.80	4.51
GEWMA					CDCC				
(p, q, ν_0)					(p,q)				
(1,1,0.95)	1.90	5.05	1.26	1.44	(1,1)	2.22	6.85	1.77	4.32
(1,2,0.95)	1.90	5.05	1.29	1.62					
(2,1,0.95)	1.97	5.41	1.29	1.62					
(2,2,0.95)	1.90	5.05	1.32	1.80					

Table 9: VaR-based Diagnostic Tests under Passive Risk Management Using 54 Multivariate Volatility Models ($\alpha = 5\%$)

Model type	normal		$t6$		Model type	normal		$t6$	
	$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$		$\hat{\pi}$	$z_{\hat{\pi}}$	$\hat{\pi}$	$z_{\hat{\pi}}$
EQMA					(1,1,0.97)	5.22	0.56	5.77	1.96
(n_0)					(1,2,0.97)	5.19	0.48	5.80	2.05
(250)	5.38	0.98	5.77	1.96	(2,1,0.97)	5.22	0.56	5.77	1.96
(125)	5.93	2.38	6.45	3.69	(2,2,0.97)	5.16	0.40	5.80	2.05
(75)	6.09	2.79	6.48	3.78					
(50)	6.12	2.87	6.48	3.78	OGARCH				
					(p,q)				
EWMA					(1,1)	5.00	-0.01	5.64	1.64
(n_0, λ_0, ν_0)					(1,2)	5.16	0.40	5.67	1.72
(250,0.95,0.95)	5.77	1.96	6.12	2.87	(2,1)	5.03	0.07	5.67	1.72
(250,0.97,0.95)	5.48	1.22	6.16	2.95	(2,2)	5.06	0.15	5.67	1.72
(250,0.95,0.97)	5.74	1.88	6.28	3.28					
(250,0.97,0.97)	5.45	1.14	6.09	2.79	CCC				
(125,0.95,0.95)	5.74	1.88	6.12	2.87	(p,q)				
(125,0.97,0.95)	5.58	1.47	6.19	3.04	(1,1)	5.93	2.38	6.57	4.02
(125,0.95,0.97)	5.74	1.88	6.28	3.28	(1,2)	5.96	2.46	6.54	3.94
(125,0.97,0.97)	5.45	1.14	6.22	3.12	(2,1)	5.90	2.29	6.51	3.86
(75,0.95,0.95)	5.70	1.80	6.35	3.45	(2,2)	5.96	2.46	6.48	3.78
(75,0.97,0.95)	5.70	1.80	6.25	3.20					
(75,0.95,0.97)	5.64	1.64	6.38	3.53	DCC				
(75,0.97,0.97)	5.61	1.55	6.38	3.53	(p,q)				
(50,0.95,0.95)	5.83	2.13	6.54	3.94	(1,1)	5.70	1.80	6.48	3.78
(50,0.97,0.95)	5.77	1.96	6.64	4.19	(1,2)	5.74	1.88	6.54	3.94
(50,0.95,0.97)	5.87	2.21	6.51	3.86	(2,1)	5.80	2.05	6.48	3.78
(50,0.97,0.97)	5.96	2.46	6.61	4.11	(2,2)	5.70	1.80	6.54	3.94
MMA					ADCC				
(n_0, ν_0)					(p,q)				
(250,0.95)	5.38	0.98	5.83	2.13	(1,1)	5.83	2.13	6.48	3.78
(250,0.97)	5.45	1.14	5.83	2.13	(1,2)	6.09	2.79	6.67	4.27
(125,0.95)	5.87	2.21	6.51	3.86	(2,1)	5.96	2.46	6.41	3.61
(125,0.97)	6.03	2.62	6.51	3.86	(2,2)	5.99	2.54	6.51	3.86
(75,0.95)	5.74	1.88	6.32	3.36					
(75,0.97)	5.90	2.29	6.48	3.78	TDCC				
(50,0.95)	5.93	2.38	6.54	3.94	(p,q)				
(50,0.97)	5.87	2.21	6.57	4.02	(1,1)	-	-	5.61	1.55
GEWMA					CDCC				
(p, q, ν_0)					(p,q)				
(1,1,0.95)	5.22	0.56	5.87	2.21	(1,1)	6.51	3.86	7.12	5.42
(1,2,0.95)	5.22	0.56	5.83	2.13					
(2,1,0.95)	5.25	0.65	5.90	2.29					
(2,2,0.95)	5.22	0.56	5.90	2.29					

group of the DCC-type models performed well in this test with the low estimates of $\hat{\pi}_i$ ranging from 1.48% for the ADCC(1,1) model to 1.80% for the TDCC model under the Student's t-distribution assumption. However, no models in this group passed the test.

Under the Normal distribution assumption, the VaR violation rates of all model classes, excepts the GEWMA, OGARCH classes, are well above 2% and the values of test statistic for all model are significantly larger than 5. This shows that all models are clearly rejected by this test under the Gaussian assumption. So the Gaussian assumption used for innovations is not relevant as it cannot capture the fat-tailed behaviour of the financial returns.

The results presented in Table 9 consider larger risk tolerance probability, $\alpha = 5\%$. The majority of the volatility models had the estimated VaR violation rates around 5.5% following the Gaussian distribution assumption and around 6.5% following the Student's t-distribution assumption. According to this result, the Gaussian distribution assumption is more relevant than the Student's t-distribution when all GEWMA models with $z_{\hat{\pi}_i}$ ranging from 0.40 for the GEWMA(2,2,0.97) to 0.65 for GEWMA(2,1,0.95) were significant at the 1% level under the Gaussian distribution assumption. Similarly, all the O-GARCH models passed the test under the Gaussian assumption with $\hat{\pi}_i$ ranging from 5% to 5.16% and $z_{\hat{\pi}_i}$ being between -0.01 and 0.4. The TDCC model also passed this test at the 5% significance level while all other members of the DCC family failed to pass this test regardless to any distribution assumption. Interestingly, the TDCC model is the only model that managed to pass the test under the Student's t-distribution with the lowest value of $\hat{\pi}_i$ being at 5.61 and the lowest value of $z_{\hat{\pi}_i}$ being at 1.55.

Under the Student's $t(6)$-distribution, the VaR exceedance rates of all models, being above 6%, are clearly larger than the hypothesized level of 5%. This suggests that the use of the Student's t-distribution assumption is not necessary when the risk tolerance of managers is getting higher. In the passive risk management manner, the use of the distribution assumption depends on the choice of the risk tolerance parameter to choose the best volatility model to fit to the emerging data.

3.2.2.4 Active risk management

The use of VaR constrain, $\bar{\rho}_{i,t-1}$ in Equation 41, which is given by a model i, is relatively efficient in controlling the violation rate with respect to the risk tolerance. However, the limit of this test is that the portfolio return is computed with no need for a multivariate volatility model. So we are not sure if the portfolio is optimally computed with respect to the variance estimated by a multivariate volatility model. The limit of the VaR-based diagnostic test for equal weights is indicated by Pesaran *et al* (2009) in which the power of the test is dependent on the weights, $w_{i,t-1}$ and the correlations between the assets in portfolio are likely to be time-varying due to shocks during the time of the recent financial crises and the financial integration of the emerging markets. Therefore, the VaR-based diagnostic test is more likely to be biased in practice. Hence, in the active risk management, the portfolio return is directly constructed by the complete use of a volatility model to compute optimal portfolio weights, $w^a_{i,t-1}$ as follows

$$\begin{cases} w^a_{i,t-1} = \dfrac{1}{\delta}\hat{H}^{-1}_{i,t-1}\hat{\mu}_{i,t-1} & \text{if the VaR constraint in Equation 44 does not bind} \\ w^a_{i,t-1} = \dfrac{1}{\delta^\star_{i,t-1}}\hat{H}^{-1}_{i,t-1}\hat{\mu}_{i,t-1} & \text{if the VaR constraint in Equation 44 binds} \end{cases} \tag{42}$$

where δ is the risk aversion coefficient representing the attitude of managers to risk; $\delta^\star_{i,t-1}$[6] is the risk aversion at time t-1 which is chosen when the VaR binds for the optimal weights; $\hat{\mu}_{i,t-1}$ is the conditional mean of portfolio given by model i; $\hat{H}_{i,t-1}$ is the one-step-ahead forecast of the conditional variance-covariance matrix given by model i.

Once the portfolio return is obtained based on the use of a volatility model, the maximum daily loss can be pre-specified based on the preference of risk managers. So the VaR constraint in Equation 36 now becomes

$$Pr(\rho_{i,t} < -L_{t-1}|\Omega_{t-1}) \leq \alpha \tag{43}$$

[6] For the derivation of $\delta^\star_{i,t-1}$, see Appendix in [25]

Table 10 provides the results of the VaR-based diagnostic test for the 54 models following the active risk management manner mentioned above. Using the same risk aversion, $\delta = 75$ as in Pesaran *et al* (2009), we obtained the VaR binding rate of the models for the optimal weights with the data from the emerging markets, which is well above 80%, is significantly higher than those in the original research, which focused on more integrated and developed financial markets. This is because the risk aversion was set too small relative to the evaluation sample that covers the whole period of the global financial crisis with large unexpected shortfalls in the emerging markets. To obtain a reasonable VaR binding rate for the optimal weights, the risk aversion coefficient is changed to 105. Following the results in these two table, the VaR constraint bound more often in the case of the $t(6)$-distributed returns and for the Riskmetrics specifications. This indicated that the VaR constraint is more reasonable if a t-distribution is used. Under the Student's t-distribution assumption, the TDCC model has the lowest VaR binding rate which is 35% while the DCC(1,2) has the lowest VaR binding rate of 22% under the Gaussian assumption. Besides, a volatility model can be simply evaluated using the Information Ratio (IR) obtained by dividing the optimal portfolio return by it standard deviation. A model that performs well is expected to have a positive and high value of IR.

For the evaluations on the trading performance by using the Information Ratio (IR), all values of the IR are positive and between the lowest of 0.45 for the ADCC(2,2) model under the Student's $t(6)$-distribution assumption to the highest of 4.71 for the DCC(2,1) model under both of the assumptions. The DCC models perform best with the IR ranging from 4.60 to 4.71. The TDCC model with the IR of 4.41 and the ADCC(1,1) model are also in the top models. The CCC models are the second best models after the DCC models. Amongst the Riskmetrics filters, the GEWMA models have the highest IRs which are in the range from 3.29 for the GEWMA(1,2,0.95) to 3.98 for the GEWMA(2,1,0.97).

With the risk tolerance probability $\alpha = 1\%$ and the maximum daily loss $L_{t-1} = 1\%$, the estimated rate of VaR exceedance for each model under the Normal distribution is higher than that under the Student's t-distribution. Under the Gaussian assumption, those rates for all Riskmetrics filters are ranging from the lowest of 4.25% for the GEWMA(1,2,0.97) model to the highest of 13.21% for the MMA(50,0.95) model while under the Student's t-distribution those rates are between the lowest of 3.22% for the GEWMA(1,1,0.97) model and the highest of 11.25% for the EWMA(50,0.97,0.95) model. The O-GARCH models perform better than the Riskmetrics filters with the rates centering around 3% for Gaussian innovations and around 2.5% for $t(6)$-distributed innovations. The DCC-type models are ranked in the top models with the violation rates close to the hypothesized level of 1%. Specifically, under the Student's t-distribution assumption, the DCC models are the best models with the value of $\hat{\pi}_i$ being from 1% for the DCC(1,2) model to 1.13% for the DCC(2,2) model and with the values of $z_{\hat{\pi}_i}$ being from -0.01 for the DCC(1,2) model to 0.72 the DCC(2,2) model. This result indicates that the standard DCC models are correctly specified by the test. Following this test, under the t-distribution assumption: the ADCC(1,1) model is significant at the 1% level with the estimated rate of VaR violation being 1.22%; the CCC(1,1) model is also significant at the 5% level with the estimated rate of VaR violation being 1.26%; the CDCC model is not significant with the violation rate of 2% and the value of test statistic 5.59. However, for the Gaussian assumption under which the models were actually estimated, there is only the DCC(1,1) model which managed to pass the test with $\hat{\pi}_i$ and $z_{\hat{\pi}_i}$ being 1.22% and 1.26, respectively. The TDCC model, which is the only model with the degrees of freedom being endogenously estimated, has the values of $\hat{\pi}_i$ and $z_{\hat{\pi}_i}$ being at 1.47% and 2.70, respectively. In term of practical estimations of the 54 models, the TDCC model, therefore, is considered as the second best model in this test while the DCC(1,1) is the best model.

In the two approaches for the VaR-based test, the DCC-type models showed that it is more relevant in modelling volatilities and correlations of the emerging financial markets while this model type consistently remained in the top models in both of the tests. The Riskmetrics filters failed the VaR-based test in the active risk management where the complete knowledge of a multivariate volatility model is utilized.

3.2.2.5 The Kolmogorov-Smirnov and the Kuiper tests

Table 10: Information Ratios and VaR-based Diagnostic Tests under Active Risk Management using 54 Multivariate Volatility Models ($\alpha = 1\%$)

Model type		Normal					Student-$t(6)$				
		mean return	IR	$\hat{\pi}$	$z_{\hat{\pi}}$	% VaR binds	mean return	IR	$\hat{\pi}$	$z_{\hat{\pi}}$	% VaR binds
EQMA	(n_0)										
	(250)	44.16	2.83	4.38	18.94	36	40.62	2.95	3.64	14.79	56
	(125)	48.63	2.92	5.51	25.25	42	43.88	3.04	4.35	18.76	64
	(75)	56.52	3.17	6.90	33.01	51	50.04	3.26	5.48	25.07	71
	(50)	69.87	3.11	10.70	54.30	60	61.02	3.16	9.02	44.92	78
EWMA	(n_0, λ_0, ν_0)										
	(250,0.95,0.95)	64.48	3.14	8.48	41.85	52	57.67	3.15	7.22	34.82	73
	(250,0.97,0.95)	63.77	2.64	8.25	40.59	50	57.44	2.65	7.12	34.27	70
	(250,0.95,0.97)	54.30	3.83	5.45	24.89	45	48.07	3.90	4.54	19.84	67
	(250,0.97,0.97)	52.77	3.36	5.35	24.35	44	47.30	3.46	4.51	19.66	65
	(125,0.95,0.95)	64.72	3.13	8.54	42.21	53	57.86	3.15	7.28	35.18	73
	(125,0.97,0.95)	64.09	2.65	8.41	41.49	51	57.62	2.66	7.25	35.00	71
	(125,0.95,0.97)	55.31	3.76	6.06	28.32	47	48.86	3.83	4.74	20.92	69
	(125,0.97,0.97)	53.95	3.32	5.77	26.70	45	48.31	3.40	4.80	21.28	66
	(75,0.95,0.95)	68.01	3.09	9.35	46.72	55	60.53	3.11	7.96	38.97	75
	(75,0.97,0.95)	68.42	2.73	9.15	45.64	54	60.95	2.73	8.06	39.51	74
	(75,0.95,0.97)	60.24	3.61	7.12	34.27	51	53.00	3.68	5.99	27.96	73
	(75,0.97,0.97)	59.77	3.30	7.44	36.08	51	53.19	3.36	6.16	28.86	71
	(50,0.95,0.95)	81.24	2.83	12.28	63.14	62	71.48	2.81	10.89	55.38	80
	(50,0.97,0.95)	83.87	2.61	12.79	66.03	61	73.86	2.57	11.25	57.37	80
	(50,0.95,0.97)	73.10	3.18	11.31	57.73	61	64.06	3.23	9.47	47.45	79
	(50,0.97,0.97)	74.59	2.99	11.41	58.27	60	65.65	2.99	9.76	49.07	78
MMA	(n_0, ν_0)										
	(250,0.95)	73.04	1.42	8.96	44.56	45	67.32	1.35	7.80	38.06	64
	(250,0.97)	55.05	1.72	6.25	29.40	39	51.40	1.71	5.48	25.07	59
	(125,0.95)	68.30	1.57	9.28	46.36	48	62.36	1.49	8.28	40.77	67
	(125,0.97)	54.50	2.00	6.83	32.65	43	50.00	1.96	5.61	25.80	63
	(75,0.95)	71.48	2.03	10.02	50.51	53	63.36	1.92	8.73	43.30	72
	(75,0.97)	60.30	2.49	7.86	38.42	50	54.20	2.48	6.41	30.31	70
	(50,0.95)	89.28	2.24	13.21	68.37	60	78.36	2.11	11.70	59.89	77
	(50,0.97)	77.66	2.59	11.70	59.89	59	68.59	2.55	9.93	49.97	77
GEWMA	(p, q, ν_0)										
	(1,1,0.95)	56.48	3.35	7.03	33.73	45	50.80	3.33	5.80	26.88	68
	(1,2,0.95)	55.19	3.31	6.86	32.83	45	49.68	3.29	5.54	25.43	67
	(2,1,0.95)	60.63	3.39	6.96	33.37	46	54.36	3.36	5.74	26.52	69
	(2,2,0.95)	58.96	3.41	6.90	33.01	46	52.86	3.40	5.83	27.06	69
	(1,1,0.97)	46.80	3.89	4.42	19.12	37	42.24	3.96	3.22	12.44	62
	(1,2,0.97)	45.54	3.82	4.25	18.22	36	41.21	3.89	3.35	13.17	61
	(2,1,0.97)	49.70	3.92	4.45	19.30	38	44.81	3.98	3.38	13.35	63
	(2,2,0.97)	48.36	3.91	4.61	20.20	38	43.66	3.96	3.51	14.07	62
OGARCH	(p, q)										
	(1,1)	43.40	3.61	3.09	11.72	32	39.80	3.76	2.51	8.47	51
	(1,2)	42.73	3.60	2.96	11.00	31	39.22	3.74	2.42	7.93	52
	(2,1)	42.55	3.39	3.13	11.90	32	39.11	3.57	2.55	8.65	52
	(2,2)	41.86	3.38	3.06	11.54	32	38.58	3.58	2.48	8.29	52
CCC	(p, q)										
	(1,1)	40.41	4.56	1.51	2.88	26	37.12	4.57	1.26	1.44	49
	(1,2)	39.38	4.50	1.58	3.24	26	36.25	4.51	1.35	1.98	48
	(2,1)	42.08	4.62	1.55	3.06	28	38.53	4.63	1.32	1.80	50
	(2,2)	41.36	4.58	1.77	4.32	27	37.92	4.59	1.48	2.70	50
DCC	(p, q)										
	(1,1)	38.39	4.66	1.22	1.26	23	35.53	4.66	1.03	0.18	45
	(1,2)	37.38	4.61	1.29	1.62	22	34.68	4.60	1.00	-0.01	44
	(2,1)	40.07	4.71	1.39	2.16	25	36.92	4.71	1.06	0.36	47
	(2,2)	39.39	4.67	1.45	2.52	24	36.37	4.67	1.13	0.72	46
ADCC	(p, q)										
	(1,1)	41.21	4.60	1.71	3.96	27	37.75	4.60	1.22	1.26	49
	(1,2)	87.09	0.50	2.32	7.39	26	82.80	0.48	1.97	5.41	48
	(2,1)	62.92	0.59	2.38	7.75	29	58.84	0.55	1.97	5.41	51
	(2,2)	138.10	0.46	2.58	8.84	28	134.08	0.45	2.26	7.03	50
TDCC	(p, q)										
	(1,1)	-	-	-	-	-	37.39	4.41	1.48	2.70	35
CDCC	(p, q)										
	(1,1)	42.74	1.82	2.22	6.85	24	40.18	1.83	2.00	5.59	45

Another diagnostic test based on Berkowitz (2001), as proposed in Pesaran and Pesaran (2007), is the probability integral transforms (PITs) as follows

$$\hat{U}_{i,t} = F_{\nu}\left(\frac{w'_{t-1}r_t - w'_{t-1}\hat{\mu}_{i,t-1}}{\sqrt{\frac{\nu-2}{\nu}w'_{t-1}\hat{H}_{i,t-1}w_{t-1}}}\right),\ t \in T_1 = \{T+1,\ T+2,\ ...,\ T+N\} \tag{44}$$

where w_{t-1} is the $m \times 1$ matrix of portfolio weights which are equally set to $\frac{1}{20}$; $\hat{\mu}_{i,t-1}$ is the conditional mean of the portfolio given by model i; $\hat{H}_{i,t-1}$ is the one-step-ahead forecast of the conditional variance-covariance matrix given by model i; ν is the degrees of freedom of the portfolio, which takes the endogenous value if model i is the TDCC or is equal to 6 degrees of freedom, otherwise.

Under the null hypothesis that the model i is correctly specified, the estimates of $\hat{U}_{i,t}$ are uniformly distributed within the range (0,1). To test if $\hat{U}_{i,t}$ is uniformly distributed over time t ranging over the evaluation period, the Kolmogorov-Smirnov Test and the Kuiper Test are suggested with KS and Ku statistics, respectively. These two test statistics are defined by

$$\mathrm{KS}_N = \max_{T+1\le j\le T+N}\left|\frac{j}{N} - \hat{U}^{\star}_j\right| \tag{45}$$

$$\mathrm{Ku} = \max_{T+1\le j\le T+N}\left(\frac{j}{N} - \hat{U}^{\star}_j\right) + \max_{T+1\le j\le T+N}\left(\hat{U}^{\star}_j - \frac{j}{N}\right) \tag{46}$$

where $\hat{U}^{\star}_1 \le \hat{U}^{\star}_2 \le ... \le \hat{U}^{\star}_N$ are ordered values of $\hat{U}_t(x)$ for t ranging in the evaluation period, $S_{eval} = \{T{+}1,\ T{+}2,\ ...,\ T{+}N\}$.

In this study, the choice of the evaluation range significantly affects the statistical results of the two tests due to the fact that the Global financial crisis in 2007-2008 included in the evaluation period, which caused all models to fail to be well specified because of the large unexpected jumps during the time of the financial turmoil. Table 11 delivers the p-values of these tests for the 54 models with the evaluation period expanding from 17/06/1998 to 07/05/2010. Consequently, all 54 models were rejected by the two tests at the 1% significance level with all p-values being equal to zero. However, the test results are clearly different after the evaluation was changed so as to rule out the period of the financial crisis. Thus, these tests focus on the tail behaviour of the distribution. During the crisis period, we can observe a fatter left tail of the distribution, which causes the models to fail the tests. These results displayed in Table 12 indicates that under the Gaussian distribution assumption all models were rejected by both tests at the 1% significance level. However, when assuming the Student's $t(6)$-distributed innovations, there are a considerable number of models which cannot be rejected by the tests at the 1% or even at the 5% significance level. Thus, all of the ADCC, the CCC and the O-GARCH models were only rejected by the Ku test at the 10% significance level and by the KS test at the 5% level. The DCC models were rejected by both of the tests at the 5% significance level. However, the CDCC and TDCC models were both rejected by the two tests at the 1% significance level. It is because the average of the estimates of the degrees of freedom of the TDCC model for 125 sub-samples is 13.62 with the highest value of 9.6 for the 122th sub-sample, the lowest value of 20.20 for the 85th sub-sample and 12.88 for the first sub-sample while the other models were assumed to fit to the t-distributed innovations with generic 6 degrees of freedom. The nature of the Ku and KS tests is to emphasize on the tail behaviours of the distribution so by assuming the 6 degrees of freedom for the Student's t-distribution which is clearly higher than the estimated degrees of freedom of the TDCC, it is more likely for the other models to pass the Ku and the KS tests.

Amongst the Riskmetrics filters, the EQMA and the GEWMA models were only rejected by the two tests at the 5% significance level. For the other members in this family, the EWMA(250,0.97,0.97) and the MMA(125,0.97) are the only two models rejected by the Ku test at the 10% significance level and the MMA(125,0.95) model is the only model rejected by the KS test at the 10% significance level. Eight of the 16 EWMA models were rejected by the KS test at the 1% significant while the rest of the EWMA models was rejected by the same test at the 5% level. Also the Ku and the KS tests rejected all the MMA models at the 5% significance level except the MMA(50,0.95) model which was rejected by Ku test at the 1% level. The best model in this class is the MMA(125,0.95) which was not rejected by the Ku test with p-value being 0.112 and the second best model in this class is the MMA(125,0.97) which was only rejected by the Ku test at the 10% significance level. By this results, the Student's $t(6)$-distribution suggested by the descriptive statistic in Table 1 is appropriate for the Riskmetrics specifications to fit to the emerging data in the scope of the Ku and KS tests. The DCC-type models continue to perform better than the Riskmetrics filters except the two recent extensions which are the TDCC and the CDCC models that performed well in the previous tests were rejected by the Ku and KS tests at the 1% significance level.

Table 11: Kuiper and Kolmogorov-Smirnov Tests of the Validity of 54 Multivariate Models: Evaluation sample from 17-June-1998 to 07-May-2010

Model type	normal		$t6$	
	Ku	KS	Ku	KS
EQMA				
(n_0)				
(250)	0.000	0.000	0.000	0.000
(125)	0.000	0.000	0.000	0.004
(75)	0.000	0.000	0.000	0.002
(50)	0.000	0.000	0.000	0.001
EWMA				
(n_0, λ_0, ν_0)				
(250,0.95,0.95)	0.000	0.000	0.000	0.000
(250,0.97,0.95)	0.000	0.000	0.000	0.002
(250,0.95,0.97)	0.000	0.000	0.000	0.001
(250,0.97,0.97)	0.000	0.000	0.000	0.002
(125,0.95,0.95)	0.000	0.000	0.000	0.000
(125,0.97,0.95)	0.000	0.000	0.000	0.001
(125,0.95,0.97)	0.000	0.000	0.000	0.001
(125,0.97,0.97)	0.000	0.000	0.000	0.002
(75,0.95,0.95)	0.000	0.000	0.000	0.000
(75,0.97,0.95)	0.000	0.000	0.000	0.000
(75,0.95,0.97)	0.000	0.000	0.000	0.000
(75,0.97,0.97)	0.000	0.000	0.000	0.001
(50,0.95,0.95)	0.000	0.000	0.000	0.000
(50,0.97,0.95)	0.000	0.000	0.000	0.000
(50,0.95,0.97)	0.000	0.000	0.000	0.000
(50,0.97,0.97)	0.000	0.000	0.000	0.000
MMA				
(n_0, ν_0)				
(250,0.95)	0.000	0.000	0.000	0.002
(250,0.97)	0.000	0.000	0.000	0.001
(125,0.95)	0.000	0.000	0.000	0.003
(125,0.97)	0.000	0.000	0.000	0.003
(75,0.95)	0.000	0.000	0.000	0.001
(75,0.97)	0.000	0.000	0.000	0.002
(50,0.95)	0.000	0.000	0.000	0.000
(50,0.97)	0.000	0.000	0.000	0.000
GEWMA				
(p, q, ν_0)				
(1,1,0.95)	0.000	0.000	0.000	0.001
(1,2,0.95)	0.000	0.000	0.000	0.001
(2,1,0.95)	0.000	0.000	0.000	0.001
(2,2,0.95)	0.000	0.000	0.000	0.001
(1,1,0.97)	0.000	0.000	0.000	0.002
(1,2,0.97)	0.000	0.000	0.000	0.002
(2,1,0.97)	0.000	0.000	0.000	0.002
(2,2,0.97)	0.000	0.000	0.000	0.002
OGARCH				
(p, q)				
(1,1)	0.000	0.000	0.000	0.000
(1,2)	0.000	0.000	0.000	0.000
(2,1)	0.000	0.000	0.000	0.000
(2,2)	0.000	0.000	0.000	0.000
CCC				
(p, q)				
(1,1)	0.000	0.000	0.000	0.000
(1,2)	0.000	0.000	0.000	0.000
(2,1)	0.000	0.000	0.000	0.000
(2,2)	0.000	0.000	0.000	0.000
DCC				
(p, q)				
(1,1)	0.000	0.000	0.000	0.000
(1,2)	0.000	0.000	0.000	0.000
(2,1)	0.000	0.000	0.000	0.000
(2,2)	0.000	0.000	0.000	0.000
ADCC				
(p, q)				
(1,1)	0.000	0.000	0.000	0.000
(1,2)	0.000	0.000	0.000	0.000
(2,1)	0.000	0.000	0.000	0.000
(2,2)	0.000	0.000	0.000	0.000
TDCC				
(p, q)				
(1,1)	0.000	0.000	0.000	0.000
CDCC				
(p, q)				
(1,1)	0.000	0.000	0.000	0.000

Table 12: Kuiper and Kolmogorov-Smirnov Tests of the Validity of 54 Multivariate Models: Evaluation sample from 17-June-1998 to 30-Aug-2004

Model type	normal		$t6$	
	Ku	KS	Ku	KS
EQMA				
(n_0)				
(250)	0.000	0.000	0.037	0.020
(125)	0.000	0.000	0.050	0.036
(75)	0.000	0.000	0.043	0.023
(50)	0.001	0.001	0.017	0.013
EWMA				
(n_0, λ_0, ν_0)				
(250,0.95,0.95)	0.002	0.003	0.007	0.006
(250,0.97,0.95)	0.000	0.001	0.028	0.013
(250,0.95,0.97)	0.001	0.002	0.032	0.015
(250,0.97,0.97)	0.000	0.001	0.066	0.031
(125,0.95,0.95)	0.002	0.003	0.009	0.007
(125,0.97,0.95)	0.001	0.002	0.037	0.015
(125,0.95,0.97)	0.001	0.002	0.032	0.017
(125,0.97,0.97)	0.000	0.001	0.066	0.031
(75,0.95,0.95)	0.003	0.003	0.003	0.005
(75,0.97,0.95)	0.002	0.002	0.020	0.009
(75,0.95,0.97)	0.002	0.002	0.020	0.013
(75,0.97,0.97)	0.001	0.001	0.037	0.015
(50,0.95,0.95)	0.003	0.003	0.001	0.005
(50,0.97,0.95)	0.002	0.004	0.002	0.008
(50,0.95,0.97)	0.003	0.005	0.002	0.005
(50,0.97,0.97)	0.003	0.004	0.004	0.007
MMA				
(n_0, ν_0)				
(250,0.95)	0.000	0.000	0.024	0.027
(250,0.97)	0.000	0.000	0.012	0.017
(125,0.95)	0.000	0.000	0.112	0.055
(125,0.97)	0.000	0.000	0.076	0.048
(75,0.95)	0.000	0.000	0.024	0.011
(75,0.97)	0.000	0.000	0.032	0.015
(50,0.95)	0.002	0.003	0.009	0.011
(50,0.97)	0.001	0.002	0.015	0.011
GEWMA				
(p, q, ν_0)				
(1,1,0.95)	0.000	0.000	0.028	0.042
(1,2,0.95)	0.000	0.000	0.028	0.036
(2,1,0.95)	0.000	0.000	0.032	0.042
(2,2,0.95)	0.000	0.000	0.032	0.042
(1,1,0.97)	0.000	0.000	0.028	0.042
(1,2,0.97)	0.000	0.000	0.024	0.042
(2,1,0.97)	0.000	0.000	0.024	0.036
(2,2,0.97)	0.000	0.000	0.028	0.042
OGARCH				
(p, q)				
(1,1)	0.000	0.000	0.050	0.036
(1,2)	0.000	0.000	0.058	0.042
(2,1)	0.000	0.000	0.050	0.036
(2,2)	0.000	0.000	0.058	0.042
CCC				
(p, q)				
(1,1)	0.000	0.000	0.066	0.048
(1,2)	0.000	0.000	0.050	0.042
(2,1)	0.000	0.000	0.058	0.048
(2,2)	0.000	0.000	0.058	0.048
DCC				
(p, q)				
(1,1)	0.000	0.000	0.043	0.042
(1,2)	0.000	0.000	0.037	0.042
(2,1)	0.000	0.000	0.043	0.042
(2,2)	0.000	0.000	0.032	0.036
ADCC				
(p, q)				
(1,1)	0.000	0.000	0.066	0.036
(1,2)	0.000	0.000	0.066	0.036
(2,1)	0.000	0.000	0.076	0.036
(2,2)	0.000	0.000	0.087	0.042
TDCC				
(p, q)				
(1,1)	0.000	0.000	0.000	0.001
CDCC				
(p, q)				
(1,1)	0.001	0.003	0.000	0.002

4 Concluding remarks

The introduction of the DCC-GARCH model of Engle (2002) initiated a large number of studies that extend the DCC model or apply the DCC specifications in risk management, portfolio selection or in the analysis of market interdependence. However, the increasing extensions of the DCC model, as well as the convenience of the Riskmetrics filters, raise the question of the uncertainty of the multivariate volatility models, specifically when they are fitted to the data of emerging financial markets. This paper focuses on the evaluations of the 54 volatility models categorized into 10 classes to provide the detail analysis of model selection in the context of the emerging markets. The methods of evaluation comprise in-sample evaluations, which used the maximized log-likelihood values, AICs and SBICs and out-of-sample evaluations, and used the VaR-based diagnostic test with both the pre-determined weights and the optimal weights in portfolio selection. Moreover, the Kolmogorov-Smirnov and the Kuiper tests are also suggested to test the out-of-sample fit of the 54 models.

In summary, the in-sample and out-of-sample evaluations using the different statistics provide an in-depth view of the applications of the volatility models to the data including 19 emerging financial markets and the US market. The DCC-type models generally performed better than the Riskmetrics specifications and O-GARCH models in both types of the evaluations. Specifically, the TDCC model was selected to be the best model by both of the AIC and SBIC in the in-sample evaluations.

In active risk management, the optimally-computed portfolio return allows for computing the Information Ratio which evaluates how volatility models perform in trading. The IR values were all positive, which indicates that all volatility models perform well in trading. However, the DCC-typed models continued to outperform the Riskmetrics filters in this evaluation by producing significantly higher values of IR. Models under the Student's $t(6)$-distribution had higher IR values than the values of those under Gaussian distribution.

Based on VaR analysis using optimally-weighted portfolio returns, the DCC(1,1) model was ranked as the best model while the TDCC was ranked as the second best model in the VaR-based test. The ADCC(1,1) and the CCC(1,1) models also passed the VaR-based diagnostic test at the 1% and the 5% significance levels, respectively, if assumed to be under the Student's $t(6)$-distribution. However, all Riskmetrics filters were rejected at the 1% significance level by the test under either of the assumptions. The limit of this test is that it is hard to realise in practice when it needs to compute the optimal portfolio weights which are conditionally suggested by a volatility model. However, transaction costs are more likely to prevent risk managers from obtaining the expected optimal portfolio weights.

An alternative approach suggested to overcome the difficulty is the passive risk management by which the weights of portfolio are set equally. This test was performed using two values of risk tolerance probability, $\alpha = 1\%$ and $\alpha = 5\%$. With $\alpha = 1\%$, models using $t(6)$-distributed innovations outperformed those using Gaussian innovations. However, with larger risk tolerance of 5%, models under Student's t-distribution assumption were rejected in favour of those under Gaussian assumption. This contradictory result indicated that the VaR-based diagnostic test in passive risk management is not consistent when the risk tolerance changes. This is due to the fact that this test only used a volatility model to compute VaR constraint rather than to directly compute the portfolio return. The test is more likely to be inconsistent and biased when financial returns experience unexpected shortfalls. Following this test, the choice of the risk tolerance probability is decisive in selecting the best model. However, it is interesting that the DCC(1,1) and the TDCC models consistently managed to be in the top 2 models regardless to the choice of risk tolerance probability.

Using the Kuiper and the Kolmogorov-Smirnov tests with the exclusion the period of the Global financial crisis from evaluation sample, the DCC models were chosen to be the best models. The Riskmetrics filters were also reasonably suggested by the tests. However, the TDCC and the CDCC models were rejected at 1% levels. This is explained by the fact that the TDCC model used the endogenous degrees of freedom for Student's t-distribution which is lower than the generic 6 degrees of freedom for the other models and the CDCC model was designed to fit to the large-scaled portfolios rather than such the medium-scaled portfolio as in this study. The different approach of the Ku and the KS tests in evaluating the models by mainly focusing on the tail behaviour of the innovations to deliver the different test results by which the attractive TDCC and CDCC models were rejected contribute to highlight the fact that there is no best model at all times. This is importantly helpful for investors and risk managers in choosing the most appropriate volatility model.

The Student's $t(6)$-distribution assumption is more relevant than the Gaussian distribution assumption for all of the volatility models to fit to the data from the emerging financial markets. The TDCC model outperformed the other models in in-sample evaluation and performed well in the VaR-based analysis but failed to pass the Ku and KS tests. This is because of the difference between the estimates of the endogenous degrees of freedom of the multivariate Student's t-distribution (centering around 13.62) of the TDCC model and the estimates of the same parameter of the univariate Student's t-distribution given by the univariate t-GARCH model (centering around 5.744). This could be a suggestion for the future research.

The VaR-based diagnostic tests as well as the Ku and KS tests provide more practical ways to evaluate volatility models than the usual in-sample evaluation. It helps both financial authorities, who use passive risk management, and investors, who use active risk management, in choosing the best model for their own purposes. That there is no best model at all times highlights the fact that it is unlikely to find a model that works well in both calm and noisy periods of financial markets. A model with t-distribution assumption is more relevant for crisis period but too conservative in normal period. Hence, designing a model that can deal with both calm and turmoil period is still a real challenge. Therefore, the results in this paper could not be generalised. However, it helps us to find the best practical model that work relatively well during the time of the Global financial crisis. It is the TDCC model, which is suggested to be applied in the next research for the test of financial contagion.

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Figure 1: Daily Returns of Eight Asian Countries

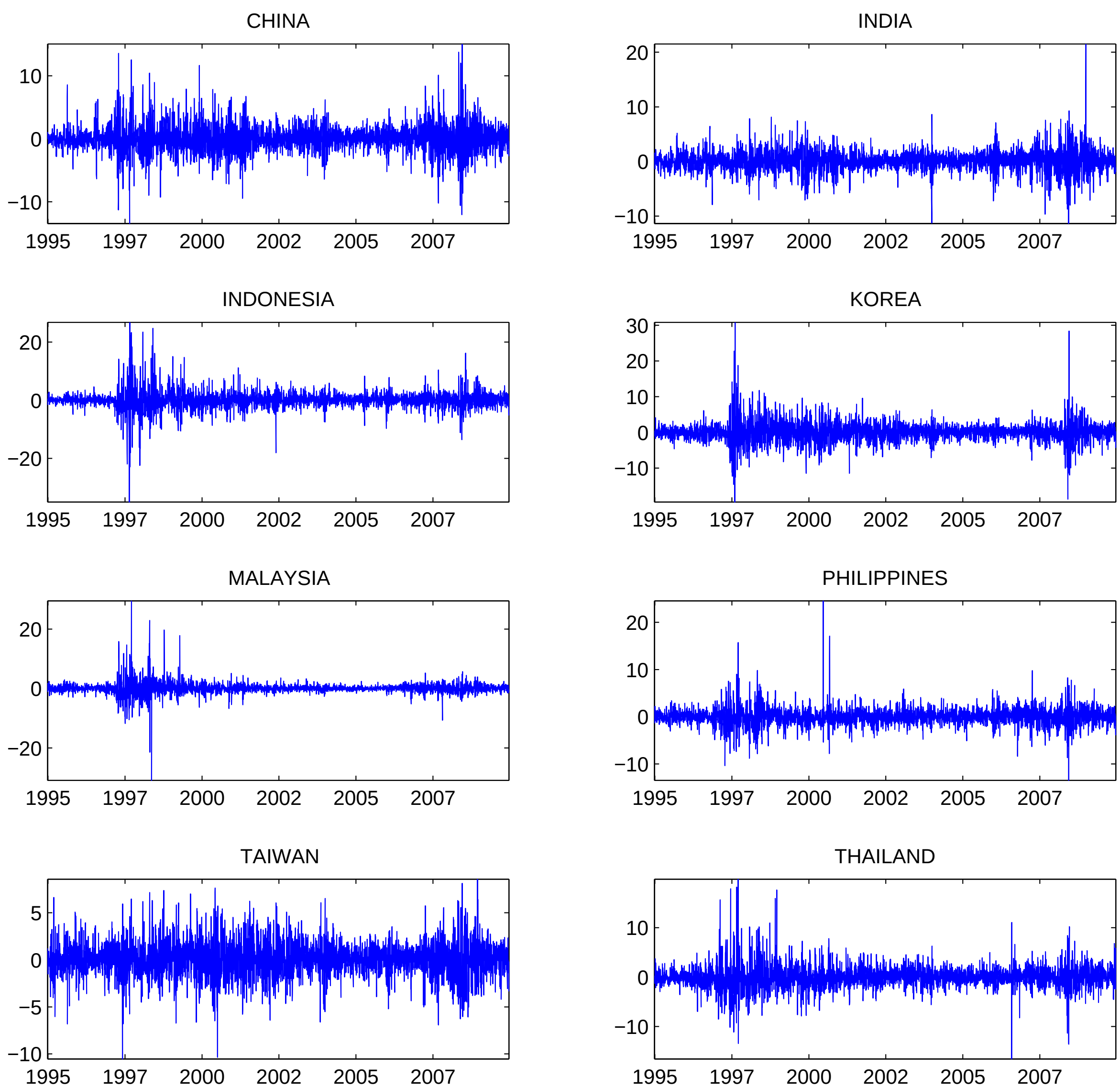

Figure 2: Daily Returns of Five Latin American Countries

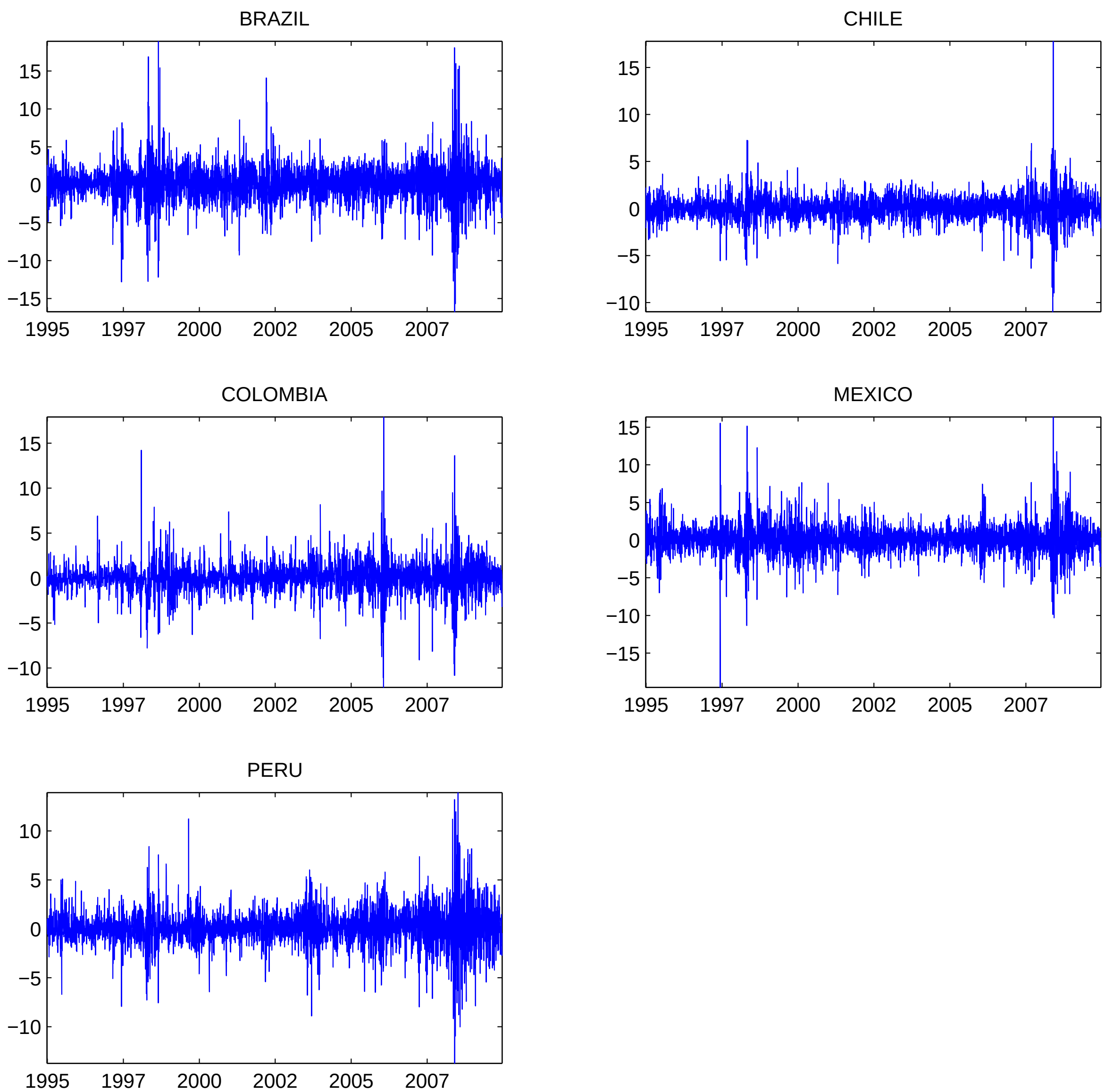

Figure 3: Daily Returns of Six European Countries and the US