Housing price, Growth and Business Cycle

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Abstract

In this paper, we investigate the link between housing price and economic growth in an overlapping generation (OLG) economy and we found that there exists a virtuous cycle between the two factors in question, in which economic growth fuels housing price appreciation which, in turn, contributes to a further savings and economic growth. Particularly, a shock that happens in housing sector is likely responsable for their short-run procycle while their long-run comovement can only be explained by a gain in non-housing technology shift. Since the effects of such shocks on the overall economy are different, knowing the nature of the shock would help the social planner in definding an appropriate policy action correcting for the shock.

Keywords: Housing price, Growth, overlapping generations

JEL Classification: R21, E32, R14

1 Introduction

Existing emperical literature has consistently addressed the co-movement between housing price and economic growth in many countries. (See for example OECD Economic Outlook(2005), Adams and Fuss(2010) or Madsen(2012))
Several issues have been widely raised trying to explain for the linkage between these two factors. In particular, Ortalo-Magn and Rady (2005), Ferrero (2012) and many other authors claimed that an economic expansion would lead to a housing boom due to the wealth effect along with the existence of borrowing constraint in which housing plays a role as a collateral in financial contract. Put differently, having higher GDP would increase individuals' wealth and make more people able to invest in housing. Therefore, housing price increases as there are more housing demand. However, Guerrieri and Iacoviello (2013) argues that the borrowing constraint no longer drives the co-movement when housing wealth is, in fact, high. They claimed that, in such case, collateral constraints become slack and wealth has little influence on housing price movement. Additionally, Tsatsaronis and Zhu (2004) using data from 17 industrialized countries and through the SVAR method concluded that in long run household income has a very small explanation power over housing price movement.
On the other hand, as saving rate is observed to be high in many countries like Korea or Taiwan, one might question if there exists a "virtuous cycle" of housing and economics growth in which a gain in housing price at first place requires individuals to save more leading to higher economic growth and then, in return, drives up housing price. Yet, the work of Deaton and Laroque (2005) developing the overlapping growth model in comparing the economy with and without land provides no support for this hypothesis. The recent work pioneering by Kahn(2009) suggest that it is the technogy shock in non-housing sector that is resposable for the two recent housing booms observed in many OECD countries. According to his work, it is the swings in labor productivity or output per hourly work in the case of the US that plays the a crucial role in pushing the housing price. In the same spirit, Borri and Reichlin (2014) investigate the OECD data from 1970 to 2011 and identify two periods of high productivity: the first one occurred through the 1970s and the second one appeared after 2000. Their empirical work also confirms the co-movement of housing price and non-housing sector productivity within these periods, i.e. real value added per hour worked, and adds that the causality is likely to go from productivity to housing sector.
To my knowledge, beside the paper of Deaton and Laroque (2005), there're no other works so far directly addressiong the impact from housing movement on overall economics. Hence, the aim of this work is to complete this task, in which we try to investigate the underlying factors that drive the synchonized evoluation between housing price and economic growth and to understand how they would affect the welfares of each generation in the overall economy.
We consider an overlapping generation economy in which individuals live for 2 periods. In our setting economy, households can allocate their disposable income between buying a house or saving in the form of capital. Housing plays a dual role in which owner occupation provides both wealth and consumption services. In the setting, we omit the rental market for housing, for renters represent only a small fraction of the population. Besides, we allow the existence of warm-glow bequest in our model. The idea of having bequest is to generate the intergenerational persistence of wealth so that any impacts

of a shock on the current wealth today can transfer to their descendants' wealth tomorrow. Thus, it can take account of the fact that even a temporary technology shock can lead to a persistent reaction of housing price via bequest mechanism.[1]. We found that a technology jump from the manufacturing sector has long-run positive effect on both housing price and capital accummulation, which suggests that the long-run procycle of housing and economic growth is likeky driven by a manufacturing technology shock. Put differently, we can say that in the existence of technology shock the housing price appreciation crowds in investment in capital sector.

Concerning the response of the steady state non-housing consumptions, we found that the young generation unambiguously benefits from the shock as their consumption rises. Indeed, raising both technology and capital stock would lead to a higher wealth which is used to finance a greater non-housing consumption. On the other hand, whether the old generation is better off after the technology shock is ambiguous. In fact, the old's consumption drops after the shock if the rate of return to capital decreases, which is the case when the elasticity between capital and productivity is great enough. Therefore, a great housing price appreciation, which is driven by a productivity shock, can lead to a redistribution of resources from the old to the young which would reduce the wealth inequality.

Taking into account two different shocks stemming from housing sector which are the housing preference shock and housing supply shock, we didn't find the long-run comovement of housing price and stock accummulation driven by these shocks. Instead, they work only in the short-run. This finding suggests that a sudden and positive shift of housing preference or a short-term shortage of housing supply is likely responsable for the short-run procycle of interest.

Overall, our findings suggest that the technology shock is likely responsible for the procycle of the two factors in long-run while the short-run comovement of them can be explained by the shock on housing sector.

The paper is organized as follows. The second section presents the overlapping generation model.The third section studies the transitional dynamics and the steady state solutions. Section four, five and six respectively investigate the the impacts of a shift on technology, housing supply and housing prefence on the economy. Section seven displays results from calibrication exercise while section eight concludes.

2 An overlapping generation model containing two assets and bequest motive

2.1 Production

In the model, there is only one homogeneous good produced in the economy, which price is assumed to be numeraire. The good can be used in non-housing consumption as well as capital formation. Nonhousing consumption and capital are perfect substitutes so that they share the same price.

The production technology is represented by a constant-return-to-scale Cobb-Douglas

[1]See for example Bellettini et al (2014), Gomes and Mendicino (2015)

production function with Hicks-neutral technology A_t in the following form:

$$F(A_t, K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}, \qquad\qquad K_0 > 0\, given \tag{1}$$

where K_t is the gross capital in the economy, A_t is the total factor productivity,and L_t is the labor demand at time t. Since capital is essential for production which means that $F(0, L_t) = 0$, capital is required to be non-negative, i.e. $K_t \geq 0$.
In intensive form, the production function expressed in term of capital per capita is:

$$f(A_t, k_t) = \frac{F(A_t, K_t, L_t)}{L_t} = A_t k_t^{\alpha} \tag{2}$$

where $k_t = K_t/L_t$ is defined as the capital per capita.

The representative firm maximizes its profit function under perfect competition. Assuming that capital stock is fully depreciated by the end of each period, in equilibrium, factors are paid at their marginal products as follows:

$$R_t = f^{'}(A_t, k_t) = A_t \alpha k_t^{\alpha-1} \tag{3}$$

$$w_t = f(A_t, k_t) - k_t f^{'}(A_t, k_t) = (1-\alpha) A_t k_t^{\alpha} \tag{4}$$

On the other hand, there's no production in the housing sector. The initial of housing supply per capita is exogenously given at time 0 as h_0. Moreover, we assume that houses, unlike capital, never deteriorate so that we have: $h_t = h_0$ at each period t.

2.2 Consumers

On the household side of the model, we consider a society with overlapping generations. Each individual lives for two periods, so that there are two generations alive at any given time. There's no population growth. The size of the population is normalized to 1.

Being young individuals supply inelastically one unit of labor, earn labor income w_t and receive a bequest b_t from their ascendants at the end of the first period. They then decide how much to consume when young c_t^y and how to divide their savings partly in capital s_{t+1} and housing goods z_{t+1}. When old, they are retired, consume c_{t+1}^o out of their earnings from capital and housing wealth and leave a bequest, b_{t+1}, to their descendants.

For now we assume that there's no uncertainty in the model and no heterogeneity.

The program of a young person born at time t is as follows:

$$U(c_t^y, c_{t+1}^o, z_{t+1}, b_{t+1}) = ln(c_t^y) + \beta ln(c_{t+1}^o) + \gamma ln(z_{t+1}) + \rho ln(b_{t+1}) \tag{5}$$

$$\text{s.t.} \quad c_t^y \geq 0, c_{t+1}^o \geq 0, z_{t+1} \geq 0, b_{t+1} \geq 0 \;\; \forall t$$

The budget constraint of a young person in time t is:

$$c_t^y + p_t z_{t+1} + s_{t+1} = w_t + b_t \tag{6}$$

where p_t is the relative price of a unit of housing sold at time t.

At the period t+1 when the individuals get old, their returns from savings and selling houses are used to finance consumption and to bequeath.

$$c^o_{t+1} + b_{t+1} = R_{t+1}s_{t+1} + p_{t+1}z_{t+1} \tag{7}$$

where R_{t+1} is the gross rate of return in capital at time t+1.

The initial old who is born in period -1, on the other hand, will decide how much they will consume and bequeath based upon their endowment of capital and housing stock. Their problem is following:

$$U(c^o_0, b_0) = \beta ln(c^o_0) + \rho ln(b_0) \tag{8}$$

$$s.t.\ \ c^o_0 + b_0 = R_0 s_0 + p_0 x_0 \tag{9}$$

where R_0 and p_0 are chosen based upon the initial capital and housing endowment k_0 and h_0 which are given at time 0.

Given k_0, h_0, $\{R_t, p_t\}_{t=0}^{\infty}$, first order conditions characterize the optimal decisions of any individuals born from time 0 onwards are the following:

$$c^y_t = \frac{(w_t + b_t)}{\Gamma} \tag{10}$$

where Γ=$1 + \beta + \rho + \gamma$.

$$c^o_{t+1} = \frac{\beta}{\Gamma} R_{t+1}(w_t + b_t) \tag{11}$$

$$z_{t+1}(p_t - \frac{p_{t+1}}{R_{t+1}}) = \frac{\gamma}{\Gamma}(w_t + b_t) \tag{12}$$

$$s_{t+1} + z_{t+1}p_t = \frac{\Gamma - 1}{\Gamma}(w_t + b_t) \tag{13}$$

$$b_{t+1} = \frac{\rho}{\Gamma} R_{t+1}(w_t + b_t) \tag{14}$$

On the other hand, solving the problem of the initial old agent in the economy gives us:

$$c^o_0 = \frac{\beta}{\beta + \rho}(R_0 s_0 + p_0 z_0) \tag{15}$$

$$b_0 = \frac{\rho}{\beta + \rho}(R_0 s_0 + p_0 z_0) \tag{16}$$

Equation (12) defines the housing demand function of agents at time t in which $p_t - \frac{p_{t+1}}{R_{t+1}}$ is used to be the user cost of housing. Since b_t and k_t are assumed to be non-negative for all t while housing supply is finite, the housing user cost must be also non-negative, i.e:

$$p_t - \frac{p_{t+1}}{R_{t+1}} \geq 0$$

Or equivalently,

$$R_t \geq \frac{p_{t+1}}{p_t} \tag{17}$$

The equation (17) suggests that returns on capital is always higher than yields from housing investment when the economy has positive capital stock.

On the other hand, from eqn(12) and (13), one can define capital savings at period t:

$$s_{t+1} = \frac{\beta + \rho}{\gamma} z_{t+1} p_t - \frac{\beta + \rho + \gamma}{\gamma} \frac{z_{t+1} p_{t+1}}{R_{t+1}}$$

Because agents behave homogeneously and capital is the essential factor to maintain goods production, savings on capital can't go negative for any periods. Therefore,

$$R_{t+1} \geq \frac{\Gamma - 1}{\beta + \rho} \frac{p_{t+1}}{p_t} \tag{18}$$

One can realize that as long as the condition given by eqn(18) is met, the condition (17) would be satisfied accordingly.

3 Steady-state solutions and transitional dynamics

3.1 The inter-temporal equilibrium

Definition 1 *Given the capital stock per capita k_0 and housing stock per capita h_0, an inter- temporal equilibrium path is characterized by sequences of aggregate capital and housing stocks, individual consumptions,bequest, and factor prices, $\{k_t, z_t, c_t^y, c_t^o, b_t, R_t, w_t, p_t\}_{t=0}^{\infty}$, such that:*
(i) for all $t > 0$, $(c_t^y, c_{t+1}^o, z_{t+1}, b_{t+1})$ solves the problem of an agent born in period t given by eqn(1)-(3)
(ii) (c_0^o, b_0) solves the problem of the initial old given by eqn(4) and (5)
(iii) (k_t, R_t, w_t) satisfies the optimality conditions of a firm given by eqn(7)-(9)
(iv) housing, capital, labor and nonhousing goods markets clear.

$$z_t = h_0 \tag{19}$$

$$k_{t+1} = s_{t+1} \tag{20}$$

$$L_t = 1 \tag{21}$$

$$f(A_t, k_t) = c_t^y + c_t^o + s_{t+1} \tag{22}$$

The eqn(22) is also known as the feasibility constraint for the whole economy at any time t.

By rewriting bequest as a function of current and future capital so that:

$$b_t = \frac{\Gamma\rho}{\Gamma\beta + \rho}\left(\frac{\Gamma - 1}{\Gamma} w(k_t) + k_t R(k_t) - k_{t+1}\right) \tag{23}$$

We can easily see that the dynamic system can be characterized by the dynamics of capital and housing price (k_t, p_t) while the other equilibrium quantities can be obtained from these three variables. Replacing eqn (19), (20) and (23) into eqn(12) and eqn(13) we can derive the dynamics of housing price:[2]

$$p_{t+1} = \alpha A_{t+1} k_{t+1}^{\alpha-1} p_t - \frac{\gamma\alpha}{(\Gamma - 1)h_0}(E A_t A_{t+1} k_t^{\alpha} k_{t+1}^{\alpha-1} - M A_{t+1} k_{t+1}^{\alpha})) \tag{24}$$

[2]Detailed proof is given in Appendix

and also the dynamics of capital accummulation:

$$k_{t+1} = \frac{E}{M+1} A_t k_t^\alpha - \frac{h_0}{M+1} p_t \tag{25}$$

where $E \equiv \frac{(\Gamma-1)(\rho+\beta(1-\alpha))}{\Gamma\beta+\rho}$ and $M \equiv \frac{(\Gamma-1)\rho}{\Gamma\beta+\rho}$

Defining a new variable called x_t where $x_t = \frac{p_t}{k_t^\alpha}$, the dynamic system of (k_t, p_t) instead can be reduced to the system of soly x_t.
In this setting, the law of motion of x_t is following:[3]

$$x_{t+1} = (M+1)\alpha A_{t+1} \frac{x_t - \frac{\gamma E A_t}{h_0(\Gamma-1)}}{EA_t - h_0 x_t} + \frac{\gamma\alpha M A_{t+1}}{h_0(\Gamma-1)} \tag{26}$$

Notice thatx_t has to satisfy the following condition $x_t > 0$ and $x_t \neq \frac{E}{h_0}$ as k_t and p_t are required to be strictly greater than 0. Moreover, x_t is not a predetermined variable since its initial value x_0 depending on the given inital value k_0 and the choice of initial housing price p_0, which is a forward-looking value. Since the setting economy is assumed to be deterministic, agents would choose the starting housing price rationally so that the economy would converge after a period of time

3.2 Steady State solutions

In what follows, we are going to study the economy in the long run. A non-trivial steady-state equilibrium is defined by the following system of equations:

$$c^y = \frac{1}{\Gamma - R(k)\rho} w(k) \tag{27}$$

$$c^o = \frac{\beta R}{\Gamma - R(k)\rho} w(k) \tag{28}$$

$$h_0 p(1 - \frac{1}{R(k)}) = \frac{\gamma}{\Gamma - \rho R(k)} w(k) \tag{29}$$

$$b = \frac{\rho R(k)}{\Gamma - R(k)\rho} w(k) \tag{30}$$

$$k - (\frac{\Gamma-1}{\Gamma} - \frac{\gamma}{\Gamma}\frac{\alpha A k^{\alpha-1}}{\alpha A k^{\alpha-1} - 1})(1-\alpha)Ak^\alpha - \frac{\rho}{\Gamma}\alpha A k^\alpha = 0 \tag{31}$$

$$h_0 x^2 + (\alpha(M+1) - \frac{\alpha\gamma M}{\Gamma-1})Ax - \frac{\alpha\gamma E A^2}{(\Gamma-1)h_0} = 0 \tag{32}$$

Notice that the system at steady state can be characterized as a function of x. Hence, the number of steady state equilibria in the model can be obtained by finding the number of roots of x given by eqn(32).

Proposition 2 *There exists one equilibrium of the economy with positive capital stock and housing stock.*

[3] See Appendix for details

Proof. From eqn(32), define

$$g(x) = h_0 x^2 + (\alpha(M+1) - \frac{\alpha\gamma M}{\Gamma - 1})Ax - \frac{\alpha\gamma E A^2}{(\Gamma - 1)h_0}$$

It's easy to see that $g(0) = -\frac{\gamma\alpha E}{(\Gamma-1)h_0} < 0$ when $x = 0$
Hence, if the function $g(x)$ has two roots which are x_1 and x_2 and if we assume that $x_1 < x_2$ then:

$$x_1 < 0 < x_2$$

As the solutions of $g(x) = 0$ has to be strictly greater than 0, there's one root x_2 that satisfies such condition. Hence, the model has a unique non-trivial equilibrium. ■

3.3 The dynamic path of the economy

The economy in dynamic path can be featured by the dynamics of x_t given by eqn(26). Let us now provide the local analysis of the dynamic system.

Observe firstly that x_{t+1} receives negative values when x_t either tends to 0 or to $+\infty$. Indeed,
When $x_t \to 0$, i.e. $\frac{p_t}{k_t^\alpha} \to 0$ due to either $p_t \to 0$ or $k_t \to \infty$, then $x_{t+1} \to -\frac{\gamma\alpha}{(\Gamma-1)h_0}$ (< 0).
When $x_t \to \infty$, i.e. $\frac{p_t}{k_t^\alpha} \to \infty$, then $x_{t+1} \to -\frac{\alpha}{h_0}(1 + M(\frac{\beta+\rho}{\Gamma-1}))$ (< 0).
Besides, when $x_t \to (\frac{E}{h_0})^-$, from eqn(26) we get $x_{t+1} \to +\infty$. When $x_t \to (\frac{E}{h_0})^+$, from eqn(26) we get $x_{t+1} \to -\infty$.

Secondly, it can be proved that x_t is monotonic over time. Indeed,

$$\frac{\partial x_{t+1}}{\partial x_t} = \frac{\beta + \rho}{(E - h_0 x_t)^2} E > 0$$

Figure 1 below displays the global dynamics of x_t. It can be seen that the system has a unique steady state x_2 in which the long-run capital stock and housing price are strictly positive.

Proposition 3 *The steady state is locally unstable.*

Proof. As the function of x_t cuts the 45 line from below, the tangent at the point x_2 is greater than 1. Hence the equilibria x_2 is locally unstable. ■

4 Impact of a non-housing productivity shift

4.1 Long term effects

4.1.1 Impact on factor prices

From eqn(31), one can easily prove that[4]

$$\frac{\partial k}{\partial A} > 0$$

[4] Proof given in the Appendix

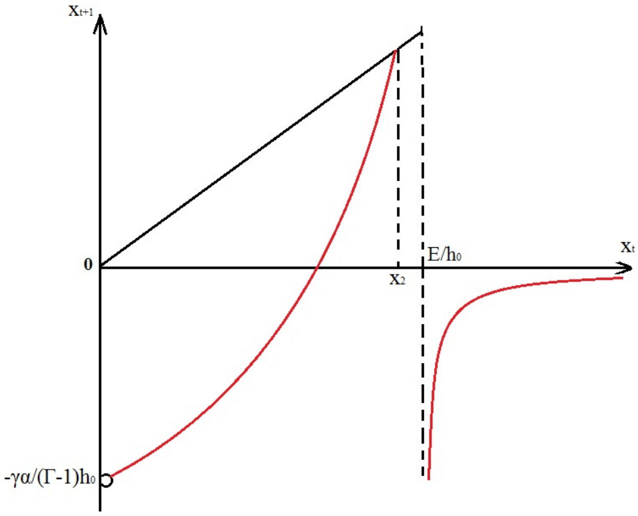

Figure 1: Phase diagram:The global dynamics of x_t .

which means that a permanent shift in technology increases capital stock in the long-run.

On the other hand, one can find the long-run relationship between x and R, the gross return on capital, is as follows: [5]

$$\frac{x}{A} = \frac{p}{w} = \frac{\gamma(ER - \gamma M)}{(R-1)(1-\alpha)(\Gamma-1)h_0} \tag{33}$$

Therefore, how x reacts to a productivity shift depends on the reaction of R. We are now in the position to examine the effect of a rise in the technology at time 1, A_1, on the economic factors in long-run.

Eqn(31) gives us a function to define the return on capital R in long-run as following:

$$\Delta(R(k)) \equiv \alpha[\beta(1-\alpha)+\rho]R^2 - [\Gamma - 1 + \alpha + \alpha\rho]R + \Gamma = 0 \tag{34}$$

It can be seen that the steady state R doesn't depend on A as $\frac{\partial R}{\partial A} = 0$. A return on capital receives two opposite effects when a technology shock happens: an increase in A directly that drives up the rate of return and an increase in the capital stock that, instead, decreases the return rate. With this particular setting, these two impacts fully offset each other so that the rate R doesn't change when A changes.

$$\frac{\partial(x/A)}{\partial A} = \frac{\partial(p/w)}{\partial A} = 0$$

Put differently,

$$\frac{\partial(p/w)}{\partial A} = \frac{1}{w}\frac{\partial p}{\partial A} - \frac{p}{w^2}\frac{\partial w}{\partial A} = 0$$

Or equivalently,

$$\frac{1}{w}\frac{\partial p}{\partial A} = \frac{p}{w^2}\frac{\partial w}{\partial A} \tag{35}$$

[5]Obtaining from defining at steady state eqn(47) from Appendix

which means that the response of housing price to a shock in technology is similar with the one of wage. As

$$\frac{\partial w}{\partial A} = k^{\alpha} + A\frac{\partial k}{\partial A} > 0$$

It leads to

$$\frac{\partial p}{\partial A} > 0$$

Proposition 4 *A permanent technology shift will increase both housing price and capital stock in the long-run.*

4.1.2 Impact on non- factor- prices

An positive shift in technology leads to a higher the steady state production. Indeed[6],

$$\frac{\partial f(k)}{\partial A} = k^{\alpha}(1 + \alpha\epsilon_{k,A}) \tag{36}$$

where $\epsilon_{k,A} = \frac{\partial k/k}{\partial A/A}$ is the elasticity of capital with respect to technology. According the results in section 4, we know that $\epsilon_{k,A} > 0$.

One can derive the consumption of an young agent as a function of current and future capital:

$$c_t^y = \frac{1}{\Gamma - 1}EA_t k_t^{\alpha} - \frac{1}{\Gamma - 1}Mk_{t+1} \tag{37}$$

The impact of a shift in technology on the young consumption at steady state can be defined as:

$$\frac{\partial c^y}{\partial A} = \frac{1}{\Gamma - 1}(Ek^{\alpha} + (ER - M)\frac{\partial k}{\partial A}) > 0\ ^7 \tag{38}$$

Therefore, the new steady state consumption of the young is also higher in response to the shock.

On the other hand, the effect of productivity shock on the yield of capital is ambiguous. A gain in technology has a direct and positive effect on the interest rate. Yet, it can negatively and indirectly impact the rate via the reduction of marginal product of capital due to a gain in capital stock. Hence, whether the rate of return to capital increases or falls after the shock depends on which effect dominates.

From eqn(6), one can compute:

$$\frac{\partial R}{\partial A} = \alpha k^{\alpha-1}(1 - (1 - \alpha)\epsilon_{k,A}) \tag{39}$$

Defining $\bar{\epsilon} = 1/(1 - \alpha)$, then:

(i) $\frac{\partial R}{\partial A} > 0$ if $0 < \epsilon_{k,A} < \bar{\epsilon}$, i.e. the elasticity between capital stock and productivity is small enough.

(ii) Otherwise, $\frac{\partial R}{\partial A} < 0$ if $\epsilon_{k,A} > \bar{\epsilon}$, i.e. capital responds substantially to a change in productivity.

Due to our setting, the decisions of the old on consumption and bequest are also constant shares of their wealth, which is made up by housing and financial wealth. As

[6]Proof for this subsection can be found on Appendix

[7]This result was proved in section 4

the return to capital rises in response to the shock due to a small shift of capital, the old's wealth clearly increases in response to the shock. As a result, the steady state bequest and consumption of the old generation will obviously increase. This result is consistent with the work of Bruecker and Pereira (1994).

However, when the elasticity between capital rate of return and the shock is great enough, one can prove that bequest and consumption of the old decrease accordingly.
Proof. A change of bequest at steady state due to the shock can be obtained by:

$$\frac{\partial b}{\partial A} = \frac{\Gamma - 1 + \alpha}{\Gamma} k^{\alpha} + \frac{k}{A}\frac{\partial b}{\partial k}\epsilon_{k,A} \tag{40}$$

where

$$\tfrac{\partial b}{\partial k} = \tfrac{\Gamma - 1 + \alpha}{\Gamma} R(k) - 1$$

It defines how bequest changes in response to an economic growth. One can prove that $\frac{\partial b}{\partial k} < 0$. Indeed, this result is supported by the fact that the intergenerational giving, i.e. bequest, is observed to be lower in developed countries than in developing countries.

Consider the exceptional case where $\epsilon_{k,A} = \bar{\epsilon} = 1/(1-\alpha)$.By replacing it to the eqn(50) one has:

$$\frac{\partial b}{\partial A}(\bar{\epsilon}) = \frac{\Gamma - 1 + \alpha}{\Gamma}\frac{k^{\alpha}}{1-\alpha} - \frac{k}{A(1-\alpha)}$$

Using the result that $\frac{\partial b}{\partial A}(\bar{\epsilon}) < 0$[8], one can prove that for any $\epsilon_{k,s} > \bar{\epsilon}$,

$$\frac{\partial b}{\partial A}(\epsilon) < \frac{\partial b}{\partial A}(\bar{\epsilon}) < 0$$

It means that bequest in long run will decrease if interest rate responds negatively to a gain in technology. ■

Similarly, in this scenario, the new steady state consumption of the old generation will be lower in response to the positive shock.

Proposition 5 *A technology shock has unambiguously positive effect on consumption of the young generation via income mechanism.*
However, it has an opposite effect on the old generation's consumption and the amount bequeathed depending on how much capital reacts to the productivity shock. In fact, these new steady states will be lower if the response of capital is above a certain threshold so that the return to capital eventually reduces after a shock.
Otherwise, they will respond positively.

5 Impact of a housing supply shift

5.1 Long term effects on factor prices

Proposition 6 *A unanticipated permanent drop in housing supply will be fully absorbed in housing market so that housing price immediately increases while capital accumulation isn't affected.*

[8] The proof is given in the Appendix

Proof. It can be seen from the equation (31), which is used to define the long-run capital stock, that the steady state capital doesn't depend on the stock of housing. As a result, any changes from housing supply has no impact on the way capital is accumulated. On the other hand, from eqn(29) we have the product:

$$ph_0 = \frac{\gamma R(k)w(k)}{(\Gamma - \rho R(k))(R(k) - 1)}$$

which is a constant in the long-run. As a result, any changes in housing stock would be fully offset by the adjustment of housing price. ■

5.2 Short term effects on factor prices

We are now in position to investigate the impact of a negative and temporary change in housing stock. In reality, it would be the case when there is a natural disaster that causes great damage of houses or there is the entry of baby boom generation into its house-buying years causing the housing stock per capita decreases during such period. Figure 3 below illustrates the evolution of x in which x^* and x'^* correspond to the steady state values of x before and after the negative shock, respectively. Since, in the long run, only housing price is influenced by the shock, we obviously obtain:

$$x'^* > x^* \ as \ h'_t < h_t$$

When the shock occurs at time t, x_t jumps to the right as p_t increases while k_t, that is predetermined, doesn't change. Since the shock is temporary, $x_t < x'^*$ and x_t will follow the diverging path of the new system AB until it reaches the old steady state x^*. It suggests that the two factors co-move in the short run in which a gain is capital accumulation is followed by a gain in housing price.

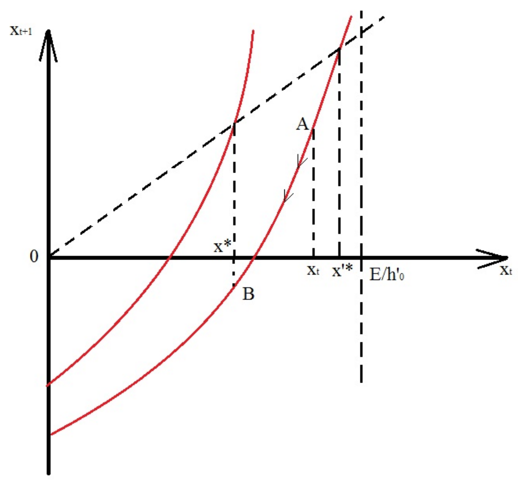

Figure 2: Transitory housing supply shock: The economy moves from A to B

Proposition 7 *The capital accumulation behaves similarly with housing price in response to negative temporary shock on housing supply in which they all increase in the first place before falling back to the initial state.*

Proof. From the dynamics of housing price given by eqn(24), one can define the dynamic relationship between x_t and k_t as follows [9]:

$$x_{t+1} = \alpha \frac{k_t^\alpha}{k_{t+1}}(x_t - \frac{\gamma E}{(\Gamma - 1)h_{t+1}}) + \frac{\alpha\gamma M}{(\Gamma - 1)h_{t+1}} \tag{41}$$

Therefore

$$\frac{\partial x_{t+1}}{\partial h_{t+1}} = \frac{\partial x_t}{\partial h_{t+1}}\frac{\alpha k_t^\alpha}{k_{t+1}} + \frac{\alpha\gamma}{(\Gamma - 1)h_t^2}(\frac{Ek_t^\alpha}{k_{t+1}} - M) - (x_t - \frac{\gamma E}{(\Gamma - 1)h_{t+1}})(\frac{\alpha k_t^\alpha}{k_{t+1}})\frac{\partial k_{t+1}}{\partial h_{t+1}}$$

One can easily prove that $\frac{Ek_t^\alpha}{k_{t+1}} - M > 0$ [10] for all t so that the sum of the two terms on the right hand side is always positive. To have $\frac{\partial x_{t+1}}{\partial h_{t+1}} < 0$ it requires that $\frac{\partial k_{t+1}}{\partial h_{t+1}} > 0$. As a result, k_{t+1} increases in response to the shock. ■

6 Impact of a housing preference shift

6.1 Long term effects on factor prices

Proposition 8 *A positive and permanent shock on housing preference induces an increase in housing demand and crowds out the steady state capital stock via the budget constraint.*

Proof. One can prove easily that $\frac{\partial p}{\partial \gamma} > 0$ which explains the fact that, when housing stock is fixed, a stronger desire for homeownership would obviously lead to an increase in housing price.
Consider the $\delta(R(k))$ function that we defined in eqn(34). One can derive:

$$\frac{\partial k}{\partial \gamma} = -\frac{\partial \delta/\partial \gamma}{\partial \delta/\partial k} \tag{42}$$

Indeed,

$$\frac{\partial \delta}{\partial \gamma} = 1 - R/\alpha < 0$$

And one can also prove that $\frac{\partial \delta}{\partial k} < 0$.
Hence, from eqn(42) we can prove that $\frac{\partial k}{\partial \gamma} < 0$ ■

This result is consistent with the work of Deaton and Laroque (2001) about the behavior of capital when land is introduced into the model.

6.2 Short term effects on factor prices

We now consider a positive and transitory shock on housing preference at time t. Since capital stock is a predetermined variable, it doesn't respond simultaneously to the shock, i.e., $\frac{\partial k_t}{\partial \gamma} = 0$. On the other hand, a gain in γ means a higher housing desirability which leads to an increase in housing price as the housing stock is assumed to be constant.

[9] For simplicity, here we assume A_t is always equal to 1

[10] From the eqn(25), it's obvious that $k_{t+1} < \frac{E}{M+1}k_t^\alpha$. Hence, it's clear that $\frac{Ek_t^\alpha}{k_{t+1}} - M > 0$ for all t

Therefore, housing price immediately shifts at the time the shock happens, i.e: $\frac{\partial p_0}{\partial \gamma} > 0$.

Let us now define the behavior of capital stock at time $t+1$. From eqn(25), one can easily derive:

$$\frac{\partial k_{t+1}}{\partial \gamma_t} = \frac{\rho + \beta(1-\alpha)}{\Gamma_t^2(\beta+\rho)} k_t^{\alpha} - \frac{(\Gamma_t - 1)\beta\rho}{\Gamma_t^2(\beta+\rho)^2} h_0 p_t - \frac{h_0}{M_t + 1}\frac{\partial p_t}{\partial \gamma_t}$$

where $\Gamma_t = 1 + \beta + \rho + \gamma_t$.
Proof given in the Appendix can show that the deduction made by the first two terms in the right hand side is always positive. Hence, as long as $\frac{\partial p_t}{\partial \gamma_t}$ is small enough, i.e. housing price doesn't rise that much, capital at time $t+1$ would increase. We can write:
$\frac{\partial k_{t+1}}{\partial \gamma_t} > 0$ if $\frac{\partial p_t}{\partial \gamma_t}$ is small enough.

7 Numerical analysis

In this section, we first focus on the dynamics of housing price and capital accumulation in response to a positive and temporary shock on housing preference and housing supply. Table 1 below summarizes our calibrated parameters.

Table 1: Calibrated Parameter

Parameter	Value
α	0.3
β	0.3
γ	0.2
ρ	0.2
h_0	1
A	20

We present hereafter the simulation results concerning a negative and temporary shock to housing supply lasting for 2 periods. As we can see from the figure 3 below, housing price and capital stock clearly co-move in response to such shock as confirming the findings that the procycle of them in the short-run is driven from the housing sector. Besides, it's interesting to note that in this case young generations will be better off as their consumptions increase at the cost of the old.

The evolutions of the variables of our interest in response to a positive 1 period housing preference shock are displayed in the figure 4. We can see that, at the beginning of the shock, capital stock and housing price moves cyclically in which a positive shift in capital stock takes place after a gain in housing price. By shifting preference toward housing, it increases, in the short-run, savings for both housing and capital through a reduction in consumption and results in a higher wages and a lower yield on capital as the its supply increases. Moreover, it can be seen that both young and old generations are worse off due to the positive shock.

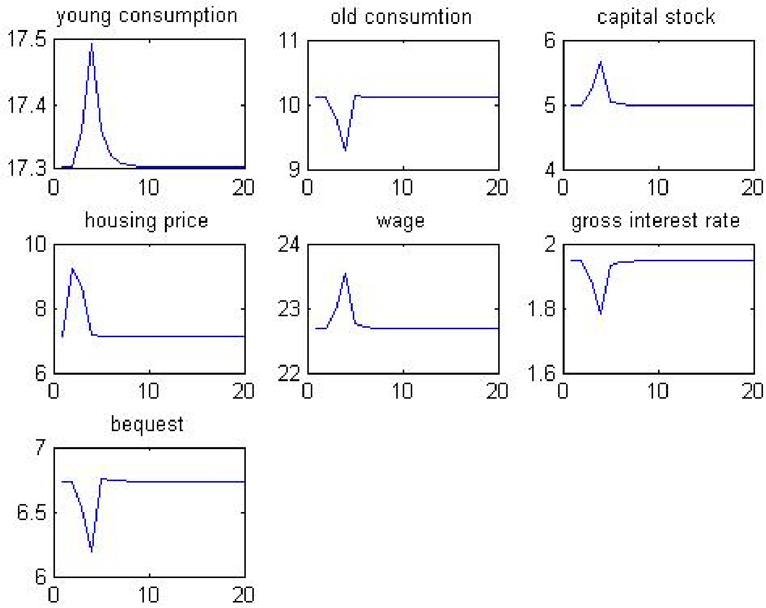

Figure 3: Transitional dynamics after a negative temporary housing supply shock : h decreases from 1 to 0.5 for 2 periods.
Note: The y-axis measures actual values of each variable

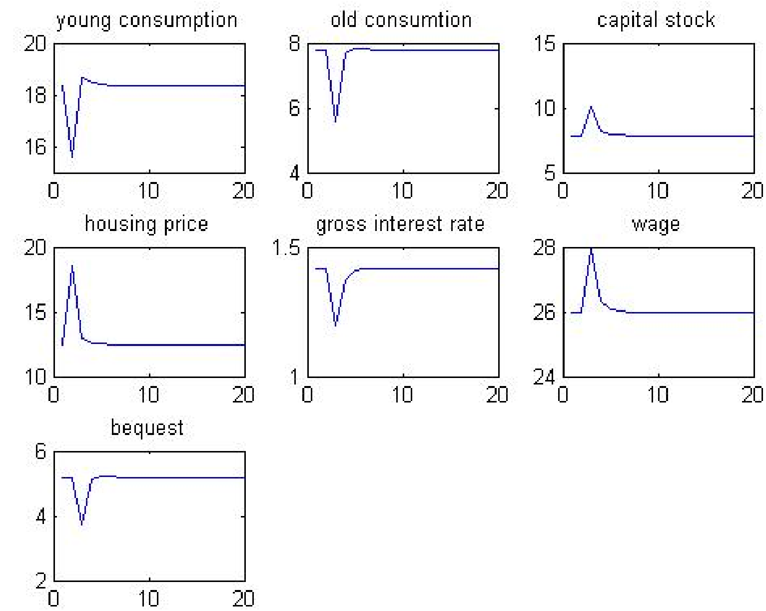

Figure 4: Transitional dynamics after housing preference shock over time: γ increases from 0.2 to 0.5 for 1 period.
Note: The y-axis measures actual values of each variable

8 Conclusion

In this paper, we investigate the co-movement between housing price and capital accumulation and we find that there exists a short-run virtuous cycle in which economic growth fuels housing price appreciation which, in turn, contributes to a further savings and economic growth via either a shock on housing supply or on housing preference. Besides, as employing a very simple set-up, we also find that a positive technology shift leads to a long-run housing price appreciation and higher capital stock, which confirms the hypothesis that technology shift could have been one of the main factor that drove the recent housing price appreciation. The effect of the shift on steady-state output is also positive.

It is believed that a housing boom will increase the inequality between the rich (the old) and the poor (the young). Since only the rich invests in housing, a gain in housing price leads to an increase in their housing wealth whereas the poor becomes less capacity to afford it due to their borrowing constraint. However, we have showed that it's not necessary the case. In fact, a housing boom, triggered by the productivity shock, can rg) under the condition that capital stock responds strong enough to the technology shift. If it's so, the rental rate of capital will fall in response to the productivity shock which leads to a higher the young's wealth and a lower the old's wealth. As a result, consumption of the young at the new steady state is greater while the old reduces their consumption after the shock.

On the other hand, our findings suggest that the shocks that takes place in housing sector like housing supply or housing preference are likely responsible for the procycle of the two factors in short-run. However, the influences of such shifts on consumption level of two generations are clearly different. Finally, for future work, the model needs further investigation about the reaction of homeowners' consumption and welfare during the housing boom and bust in order to see if one has to construct different fiscal policies on housing and capital that aims at redistributing the wealth.

Appendices

1. The dynamics of capital and housing stock

The feasibility constraint of the economy at any time t is derived as follows:

$$f(k_t) = c_t^y + c_t^0 + s_{t+1}$$

Then replacing the eqn(14), (15) and (18) into the feasibility constraint, we have:

$$\frac{w_t + b_t}{\Gamma} + \frac{\beta}{\rho} b_t + k_{t+1} = f(k_t)$$

Hence, we can rewrite bequest as a function of the current and future capital stock:

$$b_t = \frac{\Gamma\rho}{\Gamma\beta + \rho}\left(\frac{\Gamma - 1}{\Gamma} w(k_t) + k_t R(k_t) - k_{t+1}\right) \tag{43}$$

where $R(k_t)$ and $w(k_t)$ are defined by eqn(8) and (9), respectively. Since

$$k_{t+1} = \frac{\Gamma - 1}{\Gamma}(w_t + b_t) - h_0 p_t$$

we can rewrite it using the eqn(42) and obtain the dynamics of capital:

$$k_{t+1} = \frac{E}{1+M} A_t k_t^{\alpha} - \frac{h_0}{1+M} p_t \tag{44}$$

where $E \equiv \frac{(\Gamma-1)(\rho+\beta(1-\alpha))}{\Gamma\beta+\rho}$ and $M \equiv \frac{(\Gamma-1)\rho}{\Gamma\beta+\rho}$ And also from eqn(16) we can define the evolutionary law of housing price:

$$p_{t+1} = \alpha A_{t+1} k_{t+1}^{\alpha-1} p_t - \frac{\gamma\alpha}{(\Gamma-1)h_0}(E A_t k_t^{\alpha} - M k_{t+1})) A_{t+1} k_{t+1}^{\alpha-1} \tag{45}$$

Hence, we can reduce the dynamic system of three equations to the system of two which includes capital stock and housing price that can be characterized by eqn(43) and (44) above.

2. Proof the dynamics of x_t given in eqn(26)

From eqn(25), one can rewrite:

$$\frac{k_{t+1}}{k_t^{\alpha}} = \frac{E A_t}{M+1} - \frac{h_0}{M+1} \frac{p_t}{k_t^{\alpha}} \tag{46}$$

From eqn(24), one can derive:

$$\frac{p_{t+1}}{k_{t+1}^{\alpha}} = \alpha A_{t+1} \frac{k_t^{\alpha}}{k_{t+1}} \left(\frac{p_t}{k_t^{\alpha}} - \frac{\gamma E A_t}{(\Gamma-1)h_0}\right) + \frac{\alpha\gamma M A_{t+1}}{(\Gamma-1)h_0} \tag{47}$$

Defining $x_t = \frac{p_t}{k_t^{\alpha}}$ and replacing eqn(46) into (47) we obtain:

$$x_{t+1} = (M+1)\alpha A_{t+1} \frac{x_t - \frac{\gamma E A_t}{h_0(\Gamma-1)}}{E A_t - h_0 x_t} + \frac{\gamma\alpha M A_{t+1}}{h_0(\Gamma-1)} \tag{48}$$

3. Prove the results in subsection 4.1.2

Consider $\overline{R} = \frac{\alpha\Gamma}{\Gamma-1+\alpha}$. According to our setting, $\overline{z} = \frac{\Gamma}{\Gamma-1+\alpha}$
Replacing z into the function $\Delta(z(k))$, one obtains:

$$\Delta(\overline{z} = \frac{\Gamma}{\Gamma-1+\alpha}) = \alpha(\beta(1-\alpha)+\rho)\frac{\Gamma^2}{(\Gamma-1+\alpha)^2} - \frac{(\Gamma-1+\alpha+\alpha\rho)\Gamma}{\Gamma-1+\alpha} + \Gamma$$

Then by simple operation, one can prove that $\Delta(\overline{z}) > 0$ which means that

$$R < \overline{R} = \frac{\alpha\Gamma}{\Gamma-1+\alpha}$$

As a result, we also have:

$$R < \frac{\Gamma}{\Gamma-1+\alpha} \tag{49}$$

because $0 < \alpha < 1$.
Now, considering the special case where $\bar{\epsilon}_{k,A} = \frac{1}{1-\alpha}$, using the result given by eqn (45) we have:

$$\frac{\partial b}{\partial A}(\bar{\epsilon}_{k,A}) = \frac{\Gamma - 1 + \alpha}{\Gamma} k^{\alpha} + (\frac{(\Gamma - 1 + \alpha)R}{\Gamma} - 1)\frac{k}{A(1-\alpha)} < 0 \tag{50}$$

So for any $\epsilon_{k,A} > \bar{\epsilon}$ using the results given by eqn (45) and (49) we know that:

$$\frac{\partial b}{\partial A}(\epsilon_{k,A}) < \frac{\partial b}{\partial A}(\bar{\epsilon}) < 0$$

And it is the case when capital responds aggressively to a change in technology, i.e. $\frac{\partial R}{\partial A} < 0$.

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