Human Capital Accumulation And Growth

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Abstract

In this paper, we develop two simple discrete-time Ramsey models augmented with household's investment where she will divide her saving in two shares, one is physical capital and the rest be used for investing in education (human capital). The main results allow us to take progress along impacts of human capital on economic growth with three regimes. In the first and second model, there will be the steady state of economy, moreover we also prove the existence of equilibrium. With reference to the last model (with human capital accumulation), we found some conditions under which there exists growth in the long-run.

Key words: human capital, endogenous growth, economic growth, TFP: Total Factor Productivity
JEL classification:

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1 Introduction

The issues of human capital and its impacts on the total factor productivity (TFP) that have attracted a large number of studies conducted worldwide in the attempt of looking for the elements influencing the growth. The significance of human capital in today's hyper-competitive environment is tremendous and undoubted (Liepė and Sakalas, 2014). Bruno et al. (2009) assert that the supporters of endogenous growth theory pinpoint productivity growth as the key factor of East Asian success. Collins and Bosworth (1996); Lau et al. (2003) show that the total factor productivity (TFP) gains actually matter in Asian NIEs growth and that future growth can be sustained. For these authors "it is possible that the potential to adopt knowledge and technology from abroad depend on a country's stage of development. Implicitly, the TFP is one of the main factors of growth in accordance with the thesis developed by Solow (1957). In addition, Lucas (1988) postulates that human capital accumulation is pointed out as a mechanism of perpetual growth. While Bashir and Darrat (1994) indicates that countries invest more in education usually display higher per capita income. Further insight, capital is one of the major elements that support the entrepreneurs in their production and business activities. The more capital firms own, the easier they improve quantity and quality of their tools and machines, consequently the higher profitability level they could achieve. Today, in sophisticated economies, the concept "capital" does not simply stand for "physical capital", it also implies non-physical resource such as "human capital" in the form of managerial talent, education, training, skills of labors working in enterprises, etc. and the relationships between individuals, norm, trust that have certain impact on the firm productivity.(Le Van et al., 2014).

Through by many researches, the concept of human capital is still being discussed and developed. It is cleared that enables us to think of not only the years of schooling, but also of a variety of other characteristics as part of human capital investments. These include school quality, training, attitudes towards work.[1] Besides, there exists some possible classification: (1)The Becker view: human capital is the stock of knowledge or skills and this stock is directly part of the production function. (2) The Gardener view: human capital as unidimensional, since there are many dimensions or types of skills. (3) The Schultz/Nelson-Phelps view: human capital is especially useful in dealing with "disequilibrium" situations in which there is a changing environment, and workers have to adapt to this.(4) The Bowles-Gintis view: human capital is the capacity to work in organizations, obey orders, in short, adapt to life in a hierarchical/capitalist society. (5) The Spence view: human capital is the stock of skills that the labor force possesses. Returns to these skills are private in the sense that an individual's productive capacity increases with more of them.[2]

[1]Acemoglu, lecture in labor economic

[2]Handbook of Cliometrics, Claude Diebolt and Michael Haupert, editors Springer-Verlag, forthcoming, Harvard University

There were a mass of studies on human capital. The principal finding is the initial level of literacy does help predict the subsequent rate of investment, and indirectly, the rate of growth (Romer, 1989). Additionally, Romer (1986); Lucas (1988) note that the economy can achieve growth through internal factors which include human capital, population growth, and government policy.[3] Along with Galor and Moav (2004a) captures the replacement of physical capital accumulation by human capital accumulation as a prime engine of growth along the process of development and as human capital emerged as a growth engine. Moreover, Bruno et al. (2009) present that the initial value of human capital plays an essential role in the process. The higher this value, the sooner research activity and endogenous growth will take place. They also proof that research activity which produces new technologies requires more technological capital and skilled labor. Physical and technological capital are not substitutable while skilled and non-skilled labor may be substitutable, they also define and examine there exists an endogenous threshold in which is endogenously determined and related to the amount of available human capital. As well, Nelson and Phelps (1966) show the rate of return to education is greater the more technologically progressive. Likewise, Romer (1990) emphasize that the stock of human capital determines the rate of growth, that too little human capital is devoted to research in equilibrium, that integration into world markets will increase growth rates, and that having a large population is not sufficient to generate growth. Besides, Hitt and Duane (2002) illustrates on intangible human capital and social capital in terms of education, experience, cognitive skills and the firm's overall access to human capital has a positive impact on firm productivity.

All of the mentioned studies above demonstrated the role of human capital on production, total factor productivity, business activities and endogenous growth. However, almost all of these studies have difference aspects. In our research, we mainly aim to analyze the role of human capital and its impact on the process of endogenous economic growth. We use the classical discrete-time model (Ramsey, 1928) augmented with household's investment where she will divide her saving in two shares, one is physical capital and the rest be used for investing in education. Beside, we also base on (Lucas, 1988) where the current time allocation (non-leisure time and learning time) affects the accumulation of human capital.

Therefore, it can be seen that human capital is a key determinant, provide valuable and comprehensive insights on the production performance of domestic firms, the total factor productivity and endogenous growth of economy. At the same time, we will examine the household invests in human capital, that leads to obtain "the engine of endogenous growth". The new approach of our paper is, moreover, build the framework of theoretical model for this issue.

The remainder of this paper is organized as follows, Section 2 presents the structure of the economy. In section 3, we will show the model that is without human capital accumulation. Section 4 and 5 that present the model with human capital accumulation and section 6 is conclusion.

[3] See Rebelo (1990) and Rosenzweig (1990) and the literature cited therein for further details

2 The structure of the economy

We consider a small economy that be constituted by

**Households*

At the first period of household's life devote their entire time to the acquisition of human capital. The acquired level of human capital increases if their time investment is supplemented with capital investment in education. However, even in the absence of real expenditure individuals acquire one efficiency unit of labor-basic skills.(Galor and Moav, 2004b).

The household member is the consumer (who is also producer of the consumption good). She will divides her saving in two shares, one is physical capital and the rest of saving that uses for investing in education/human capital. And we define

$$s_t = k_{t+1} + e_t$$

**Firms*

Firms produce output using capital good and labor input. The production technology transforms labor and the capital goods into a composite good that can be either consumed or invested as the next period's capital goods input. All labor services are assumed to be identical.(Becker et al., 2015)

Considering a Cobb-Douglas production function under total depreciation of physical capital, yields

$$F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$$

with $0 < \alpha < 1$ where K_t, L_t are demand of firm for capital and labor.

Let $k_t = \frac{K_t}{N_t}$ is per capita physical capital, N_t is the total workers in the economy. We also get

$$y_t = \frac{F(K_t, L_t)}{N_t}$$
$$y_t = Ak_t^{\alpha}(\frac{L_t}{N_t})^{1-\alpha}$$

Setting $h_t = \frac{L_t}{N_t}$ is per capita effective labor.

And we also suppose that

$$h_t = \phi(e_t)$$

where $\phi(.)$ is an increasing concave function with $\phi(0) = \underline{e} > 0$, and e_t is the amount of spending in education at time t.

The technology is summarized by production function, denoted by f, let $y_t = f(k_t, h_t)$

3 Model 1. without human capital accumulation

Following the aggregate production function in section 2, and the social planner will solve the following program:

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t) \tag{1}$$

Subject to the constrain:

$$c_t + k_{t+1} + e_{t+1} \leq Ak_t^{\alpha}[\phi(e_t)]^{1-\alpha} \tag{2}$$

Where

$$\begin{aligned} f(k_t) &= Ak_t^{\alpha} \\ \phi(e_t) &= B(\underline{e} + e_t)^{\gamma} \end{aligned}$$

The Lagrangian of the problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \{\beta^t u(c_t) + \lambda_t [Ak_t^{\alpha}(\phi(e_t))^{1-\alpha} - c_t - k_{t+1} - e_{t+1}]\} \tag{3}$$

First order conditions:

$$\beta^t u'(c_t) = \lambda_t \tag{4}$$

$$\lambda_{t+1} \alpha A k_{t+1}^{\alpha-1}[\phi(e_{t+1})]^{1-\alpha} = \lambda_t \tag{5}$$

$$\lambda_{t+1}(1-\alpha)Ak_{t+1}^{\alpha}[\phi(e_{t+1})]^{-\alpha}\phi'(e_{t+1}) = \lambda_t \tag{6}$$

Moreover, the budget constraints (2) is now binding.
At steady state, these conditions become:

$$\frac{\lambda_t}{\lambda_{t+1}} = \frac{1}{\beta} \tag{7}$$

$$\frac{\lambda_t}{\lambda_{t+1}} = \alpha A k^{\alpha-1}[\phi(e)]^{1-\alpha} \tag{8}$$

$$\frac{\lambda_t}{\lambda_{t+1}} = (1-\alpha)Ak^{\alpha}[\phi(e)]^{-\alpha}\phi'(e) \tag{9}$$

From equations (7) and (8), we have the ration capital per unit of effective labor:

$$\frac{k}{\phi(e)} = (A\alpha\beta)^{\frac{1}{1-\alpha}} \tag{10}$$

Combining (9) and (10), we have:

$$\begin{aligned} \phi'(e) &= \frac{1}{A(1-\alpha)\beta}(A\alpha\beta)^{-\frac{\alpha}{1-\alpha}} \\ \phi'(e) &= \frac{\alpha}{1-\alpha}(A\alpha\beta)^{-\frac{1}{1-\alpha}} \end{aligned}$$

Notice that $\phi(e_t) = B(\underline{e} + e_t)^\gamma$, hence $\phi'(e_t) = B\gamma(\underline{e} + e_t)^{\gamma-1}$ Thus

$$\begin{aligned}
\gamma B(\underline{e} + e)^{\gamma-1} &= \frac{\alpha}{1-\alpha}(A\alpha\beta)^{-\frac{1}{1-\alpha}} \\
(\underline{e} + e)^{\gamma-1} &= \frac{\alpha}{\gamma(1-\alpha)}(A\alpha\beta)^{-\frac{1}{1-\alpha}}\frac{1}{B} \\
(\underline{e} + e)^{1-\gamma} &= \frac{\gamma(1-\alpha)}{\alpha}(A\alpha\beta)^{\frac{1}{1-\alpha}}B
\end{aligned}$$

So, we have:

$$e = [B\gamma\frac{1-\alpha}{\alpha}]^{\frac{1}{1-\gamma}}[A\alpha\beta]^{\frac{1}{(1-\alpha)(1-\gamma)}} - \underline{e} \tag{11}$$

Since $e \geq 0$ so that $[B\gamma(1-\alpha)]^{\frac{1}{1-\gamma}}[A\beta]^{\frac{1}{(1-\alpha)(1-\gamma)}}\alpha^{\frac{1}{1-\gamma}(\frac{1}{1-\alpha}-1) \geq \underline{e}}$
From equation (10), we get

$$\begin{aligned}
k &= (A\alpha\beta)^{\frac{1}{1-\alpha}}\phi(e) \\
k &= (A\alpha\beta)^{\frac{1}{1-\alpha}}B(\underline{e} + e)^\gamma
\end{aligned}$$

$$k = [B(A\alpha\beta)^{1/1-\alpha}]^{\frac{1}{1-\gamma}}[\frac{1-\alpha}{\alpha}\gamma]^{\frac{\gamma}{1-\gamma}} \tag{12}$$

where $e + \underline{e} = [B\gamma\frac{1-\alpha}{\alpha}(A\alpha\beta)^{\frac{1}{1-\alpha}}]^{\frac{1}{1-\gamma}}$
Finally, from the budget constraint, equations (11) and (12) and notice that $f(k) = Ak^\alpha$, generates the following result, as follows

$$\begin{aligned}
c + k + e &= f(k)[\phi(e)]^{1-\alpha} \\
c &= Ak[\frac{\phi(e)}{k}]^{1-\alpha} - k - e \\
c &= \frac{k}{\alpha\beta} - k - e \\
c &= k[\frac{1}{\alpha\beta} - 1] - e
\end{aligned}$$

$$c = \underline{e} + [B(A\alpha\beta)^{1/1-\alpha}]^{\frac{1}{1-\gamma}}[(\frac{1}{\alpha\beta} - 1)(\frac{1-\alpha}{\alpha}\gamma)^{\frac{\gamma}{1-\gamma}} - (\frac{1-\alpha}{\alpha}\gamma)^{\frac{1}{1-\gamma}}] \tag{13}$$

At the steady state, we have the solutions, as follows

$$\begin{aligned}
e_s &= [B\gamma\frac{1-\alpha}{\alpha}]^{\frac{1}{1-\gamma}}[A\alpha\beta]^{\frac{1}{(1-\alpha)(1-\gamma)}} - \underline{e} \\
k_s &= [B(A\alpha\beta)^{1/1-\alpha}]^{\frac{1}{1-\gamma}}[\frac{1-\alpha}{\alpha}\gamma]^{\frac{\gamma}{1-\gamma}} \\
c_s &= \underline{e} + [B(A\alpha\beta)^{1/1-\alpha}]^{\frac{1}{1-\gamma}}[(\frac{1}{\alpha\beta} - 1)(\frac{1-\alpha}{\alpha}\gamma)^{\frac{\gamma}{1-\gamma}} - (\frac{1-\alpha}{\alpha}\gamma)^{\frac{1}{1-\gamma}}]
\end{aligned}$$

Proposition 1 *There exists the steady state solutions. Obviously, $c_s > 0$, that lead to:*
i. c_s and k_s are increase with either e_s or(/and) A, B;
ii. k_s is increase with either γ or(/and) β

Proposition 2 *Global convergence. There exists the optimal sequence of problem (P), $\{c_t^*, k_{t+1}^*, e_{t+1}^*\}$ is monotonic and converges to steady state (c_s, k_s, e_s)*

Proof.
Consider the problem (P).

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t),$$
$$\text{s.c } c_t + k_{t+1} + e_{t+1} \leq f(k_t, e_t) + (1-\delta)k_t \text{ for all } t.$$

For $s > 0$, define

$$\varphi(s) = \max_{k+e=s} \psi(k, e)$$

where $\psi(k, e) = f(k, e) + (1-\delta)k$. Denote by $k(s), e(s)$ solution of $\max_{k+e=s}(f(k, e) + (1-\delta)k)$. By Inada condition, we have $\varphi'(0) = +\infty$. The problem (P) becomes (Q):

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t),$$
$$\text{s.c } c_t + s_{t+1} \leq \varphi(s_t) \text{ for all } t.$$

This is a standard Ramsey model, and it is well known that the optimal sequence $\{c_t^*, s_{t+1}^*\}$ is monotonic and converges to (c_s, s_s). To prove the the monotonicity of $\{k_{t+1}^*, e_{t+1}^*\}$, we will prove that the functions k_s, e_s are increasing in s. Indeed, $k(s)$ is solution to

$$\psi_1(k, s-k) - \psi_2(k, s-k) = 0.$$

Define $z(k, s) = \psi_1(k, s-k) - \psi(k, s_k) = 0$. By using the implicit theorem, we get

$$\begin{aligned}
k'(s) &= -\frac{\frac{\partial z}{\partial s}}{\frac{\partial z}{\partial k}} \\
&= -\frac{\psi_{12}(k, s-k) - \psi_{22}(k, s-k)}{\psi_{11}(k, s-k) - \psi_{12}(k, s-k) - \psi_{12}(k, s-k) + \psi_{22}(k, s-k)} \\
&= \frac{\psi_{12}(k, s-k) - \psi_{22}(k, s-k)}{-\psi_{11}(k, s-k) + 2\psi_{12}(k, s-k) - \psi_{22}(k, s-k)}.
\end{aligned}$$

From the concavity and the separable form of production function, we get $\psi_{12} > 0$, $\psi_{11}, \psi_{22} < 0$, and we can verify easily that $0 < k'(s) < 1$, which implies $k(s)$ and $e(s)$ are increasing function of s. Hence we have the optimal sequence of problem (P), $\{c_t^*, k_{t+1}^*, e_{t+1}^*\}$ is monotonic and converges to steady state (c_s, k_s, e_s).

4 Model 2. with human capital accumulation

In this section, we will present the main results of the model when there exists the human capital accumulation. With reference to all of the set up previously, we also have the following program The social planner will also maximize

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t)$$

Subject to the constrains:

$$c_t + k_{t+1} + e_t(1-n_t) \leq Ak_t^\alpha [n_t h_t]^{1-\alpha} \tag{14}$$

$$h_{t+1} \leq (1-\delta)h_t + \phi[e_t, (1-n_t)] \tag{15}$$

Where
- $\phi[e_t, (1-n_t)] = Be_t^\gamma (1-n_t)^{1-\gamma}$
- n_t be working time and
- $1 - n_t$ denoted by learning time.

Taking the Lagrange equation, yields

$$\sum_{t=0}^{\infty} \{\beta^t u(c_t) - \lambda_t [c_t + k_{t+1} + e_t(1-n_t) - Ak_t^\alpha (n_t h_t)^{1-\alpha}]$$
$$- \mu_t [h_{t+1} - (1-\delta)h_t + Be_t^\gamma (1-n_t)^{1-\gamma}]\}$$

First oder conditions:

$$\beta^t u'(c_t) = \lambda_t \tag{16}$$

$$\lambda_{t+1} R_{t+1} = \lambda_t \tag{17}$$

$$\lambda_{t+1} w_{t+1} n_{t+1} + \mu_{t+1}(1-\delta) = \mu_t \tag{18}$$

$$\mu_t B\gamma e_t^{\gamma-1} (1-n_t)^{1-\gamma} = \lambda_t (1-n_t) \tag{19}$$

$$\mu_t B e_t^\gamma (1-\gamma)(1-n_t)^{-\gamma} = \lambda_t (w_t h_t + e_t) \tag{20}$$

Where $R_{t+1} = \alpha A(\frac{k_{t+1}}{n_{t+1}h_{t+1}})^{\alpha-1}$ and $w_{t+1} = (1-\alpha)A(\frac{k_{t+1}}{n_{t+1}h_{t+1}})^\alpha$. Moreover, the budget constraints (14) and (15) are now binding.
At the steady state and From equations (16) and (17)

$$R_{t+1} = \frac{\lambda_t}{\lambda_{t+1}} = \frac{\beta^t u'(c_t)}{\beta^{t+1} u'(c_{t+1})}$$
$$R_{t+1} = \frac{1}{\beta} \equiv R$$

And then we have the ratio capital per unit of effective labor:

$$\frac{k}{nh} = (A\alpha\beta)^{1/(1-\alpha)} \tag{21}$$

Besides, we also have

$$R = \alpha A(\frac{k}{nth})^{\alpha-1} = \frac{1}{\beta}$$
$$w_t = (1-\alpha)A(A\alpha\beta)^{\alpha/(1-\alpha)} \equiv w$$

Reformulating (15), we get

$$\frac{h}{e} = \frac{1}{\delta}B(\frac{e}{1-n})^{\gamma-1} \tag{22}$$

At the same time, divide (20) by (19), generates

$$w\frac{h}{e} = \frac{1-2\gamma}{\gamma} \tag{23}$$

Combining equations (22) and (23), yields

$$\frac{e}{1-n} = (B\frac{w}{\delta}\frac{\gamma}{1-2\gamma})^{\frac{1}{1-\gamma}} \tag{24}$$
$$\frac{h}{e} = \frac{1}{w}\frac{1-2\gamma}{\gamma} \tag{25}$$

Return to equation (19), this following equation below equivalently

$$\lambda_{t+1} = \mu_{t+1}(\frac{e}{1-n})^{\gamma-1}\frac{B\gamma}{1-n} \tag{26}$$

Replacing equations (26)into (18), get

$$\mu_{t+1}\frac{n}{1-n}wB\gamma(\frac{e}{1-n})^{\gamma-1}\mu_{t+1}(1-\delta) = \mu_t \tag{27}$$

From equations (26), yields

$$\frac{\mu_t}{\lambda_t} = \frac{\mu_{t+1}}{\lambda_{t+1}} = (\frac{e}{1-n})^{1-\gamma}\frac{1-n}{B\gamma} \tag{28}$$

In addition, according to equations (24), (27) and (28) with $R = \frac{\mu_t}{\mu t+1} = \frac{\lambda_t}{\lambda t+1} = \frac{1}{\beta}$, we will have

$$\frac{n}{1-n} = \frac{1-\beta(1-\delta)}{B\beta w\gamma}\frac{w}{\delta}B\frac{\gamma}{1-2\gamma}$$
$$\frac{n}{1-n} = \frac{1-\beta(1-\delta)}{(1-2\gamma)\beta\delta}$$

$$\frac{n}{1-n} = \frac{1}{1-2\gamma}(1+\frac{1-\beta}{\beta}\frac{1}{\delta}) \tag{29}$$

Let $\theta = \frac{1}{1-2\gamma}(1+\frac{1-\beta}{\beta}\frac{1}{\delta})$, we archive

$$n = \frac{\theta}{1+\theta} \tag{30}$$
$$1-n = \frac{1}{1+\theta} \tag{31}$$

Continuously, subtituting equation (31) into (24), we get

$$e = \frac{1}{1+\theta}(B\frac{w}{\delta}\frac{\gamma}{1-2\gamma})^{\frac{1}{1-\gamma}} \tag{32}$$

Insert (32) into (25), we also get

$$h = \frac{1}{w}\frac{1-2\gamma}{\gamma}\frac{1}{1+\theta}(B\frac{w}{\delta}\frac{\gamma}{1-2\gamma})^{\frac{1}{1-\gamma}} \tag{33}$$

From equation (21), we have

$$k = \frac{\theta}{1+\theta}\frac{1}{w(1+\theta)}\frac{1-2\gamma}{\gamma}[B\frac{w}{\delta}\frac{\gamma}{1-2\gamma}]^{\frac{1}{1-\gamma}}[A\alpha\beta]^{\frac{1}{1-\alpha}} \tag{34}$$

Finnaly, from the budget constraint (14) is binded, and from three equations (32) (33) and (34) give the following solution, as follows

$$c = Ak(\frac{k}{nh})^{\alpha-1} - k - e(1-n)$$
$$c = Ak(\frac{k}{\alpha\beta}) - k - e(1-n)$$

$$c = k[\frac{1}{\alpha\beta} - \frac{w}{\theta}\frac{\gamma}{1-2\gamma}\frac{1}{(A\alpha\beta)^{\frac{1}{1-\alpha}}} - 1] \tag{35}$$

where

$$e(1-n) = \frac{1}{\theta}w\frac{\gamma}{1-2\gamma}\frac{1}{(A\alpha\beta)^{\frac{1}{1-\alpha}}}k$$

We have the steady state solutions

$$\begin{aligned}
e^s &= \frac{1}{1+\theta}[B\frac{w}{\delta}\frac{\gamma}{1-2\gamma}]^{\frac{1}{1-\gamma}}\\
k^s &= \frac{\theta}{1+\theta}\frac{1}{w(1+\theta)}\frac{1-2\gamma}{\gamma}[B\frac{w}{\delta}\frac{\gamma}{1-2\gamma}]^{\frac{1}{1-\gamma}}[A\alpha\beta]^{\frac{1}{1-\alpha}}\\
h^s &= \frac{1}{w}\frac{1-2\gamma}{\gamma}\frac{1}{1+\theta}[B\frac{w}{\delta}\frac{\gamma}{1-2\gamma}]^{\frac{1}{1-\gamma}}\\
c^s &= k[\frac{1}{\alpha\beta} - \frac{w}{\theta}\frac{\gamma}{1-2\gamma}\frac{1}{(A\alpha\beta)^{\frac{1}{1-\alpha}}} - 1]\\
n &= \frac{\theta}{1+\theta}\\
1-n &= \frac{1}{1+\theta}
\end{aligned}$$

Where

$$w = \frac{1-\alpha}{\alpha}\frac{1}{\beta}(A\alpha\beta)^{\frac{1}{1-\alpha}}$$
$$\theta = \frac{1}{1-2\gamma}[1 + \frac{1-\beta}{\beta}\frac{1}{1-\delta}]$$

Proposition 3 *A necessary condition for the existence of a steady state solution is:*
i. $\gamma < \frac{1}{2}$ *or/(and)*
ii. δ *(depreciation of human capital) closed to 0*

5 Model 3. with human capital accumulation and endogenous growth

The social planner also maximizes

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t)$$

Subject to the constraint:

$$c_t + k_{t+1} + e_t \leq A k_t^{\alpha} (n_t h_t)^{1-\alpha} \tag{36}$$
$$h_{t+1} \leq (1-\delta) h_t + \phi[e_t, (1-n_t)] \tag{37}$$

Where
$\phi[e_t, (1-n_t)] = B e_t^{\gamma} h_t^{1-\gamma} (1-n_t)^{\sigma}$
Taking the Lagrange equation, yields

$$\sum_{t=0}^{\infty} \{\beta^t u(c_t) - \lambda_t [c_t + k_{t+1} + e_t - A k_t^{\alpha} (n_t h_t)^{1-\alpha}]$$
$$-\mu_t [h_{t+1} - (1-\delta) h_t + B e_t^{\gamma} h_t^{1-\gamma} (1-n_t)^{\sigma}]\}$$

First oder conditions:

$$\beta^t u' c_t = \lambda_t \tag{38}$$
$$\lambda_{t+1} R_{t+1} = \lambda_t \tag{39}$$
$$\lambda_{t+1} w_{t+1} n_{t+1} + \mu_{t+1} [1 - \delta + (1-\gamma) B e_{t+1}^{\gamma} h_{t+1}^{-\gamma} (1-n_{t+1})^{\sigma}] = \lambda_t \tag{40}$$
$$\mu_t B \gamma e_t^{\gamma-1} h_t^{1-\gamma} (1-n_t)^{\sigma} = \lambda_t \tag{41}$$
$$\mu_t B e_t^{\gamma} h_t^{1-\gamma} \sigma (1-n_t)^{\sigma-1} = \lambda_t w_t h_t \tag{42}$$

Where $R_{t+1} = \alpha A (\frac{k_t}{n_t h_t})^{\alpha-1}$ and $w_t = (1-\alpha) A (\frac{k_t}{n_t h_t})^{\alpha}$. Moreover, the budget constraints (36) and (37) are now binding.
If there are solutions of the form: $\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \rho$, $\forall t$
Combining with equations (38) and (39), we have:

$$R_{t+1} = \frac{\lambda_t}{\lambda_{t+1}} = \frac{\beta^t u'(c_t)}{\beta^{t+1} u'(c_{t+1})}$$

And then

$$R_{t+1} = \frac{\rho}{\beta} \equiv R$$
$$\frac{k_t}{n_t h_t} = (\frac{A\alpha\beta}{\rho})^{1/(1-\alpha)}$$

Besides,

$$R = \alpha A(\frac{k_t}{n_t h_t})^{\alpha-1} = \frac{\rho}{\beta}$$
$$w_t = (1-\alpha)A(\frac{A\alpha\beta}{\rho})^{\alpha/(1-\alpha)} \equiv w$$

Reformulating (37) we get:

$$\rho = (1-\delta) + B(\frac{e_t}{h_t})^{\gamma}(1-n)^{\sigma} \tag{43}$$

Divide (42) by (41) we have:

$$\frac{e_t}{h_t} = \frac{\gamma}{\sigma} w(1-n) \tag{44}$$

In addition, according to (42) we have:

$$\frac{\mu_t}{\lambda_t} = \frac{w}{\sigma}(1-n)^{1-\sigma} B^{-1}(\frac{e_t}{h_t})^{-\gamma} \tag{45}$$

Combining (45) and (40) (and the fact that $\frac{e_t}{h_t}$ is constant) we have:

$$\lambda_{t+1} wn + [\frac{w}{\sigma}(1-n)^{1-\sigma} B^{-1}(\frac{e_{t+1}}{h_{t+1}})^{-\gamma}\lambda_{t+1}][(1-\delta) + (1-\gamma)Be_{t+1}^{\gamma} h_{t+1}^{-\gamma}(1-n)^{\sigma}] =$$
$$= \lambda_t \frac{w}{\sigma}(1-n)^{1-\sigma} B^{-1}(\frac{e_t}{h_t})^{-\gamma}$$

Generating this equation above, yield

$$\lambda_{t+1}\{wn + [(1-\delta) + (1-\gamma)Be_{t+1}^{\gamma} h_{t+1}^{-\gamma}(1-n)^{\sigma}]\frac{w}{\sigma}(1-n)^{1-\sigma} B^{-1}(\frac{e_{t+1}}{h_{t+1}})^{-\gamma}\} =$$
$$= \lambda_t \frac{w}{\sigma}(1-n)^{1-\sigma} B^{-1}(\frac{e_t}{h_t})^{-\gamma}$$
$$wn + \frac{1-\gamma}{\sigma} w(1-n) = (\frac{\rho}{\beta} - 1 + \delta)[\frac{w(1-n)}{\sigma} \frac{1}{(1-n)^{\sigma} B(\frac{e_t}{h_t})^{\gamma}}]$$

Remark $\frac{\lambda_t}{\lambda_{t+1}} = \frac{\rho}{\beta}$
But according to (43), $(1-n)^{\sigma} B(\frac{e_t}{h_t})^{\gamma} = \rho - 1 + \delta$, thus we obtain:

$$wn + \frac{1-\gamma}{\sigma} w(1-n) = (\frac{\rho}{\beta} - 1 + \delta)[\frac{w(1-n)}{\sigma} \frac{1}{\rho - 1 + \delta}]$$

$$n+\frac{1-\gamma}{\sigma}(1-n)=(\frac{\rho}{\beta}-1+\delta)[\frac{(1-n)}{\sigma}\frac{1}{\rho-1+\delta}] \tag{46}$$

Let v denotes $1-n$, then (46) gives:

$$\frac{1-v}{v}+\frac{1-\gamma}{\sigma}=\frac{1}{\sigma}\frac{(\rho/\beta)-1+\delta}{\rho-1+\delta}$$

$$\frac{1}{v}=\frac{\sigma+\gamma-1}{\sigma}+\frac{1}{\sigma}\frac{(\rho/\beta)-1+\delta}{\rho-1+\delta} \tag{47}$$

Combining (43), (44), (47) and the fact that $w=(1-\alpha)A(\frac{A\alpha\beta}{\rho})^{\alpha/(1-\alpha)}$, we have

$$\rho-1+\delta=B(\frac{\gamma}{\sigma}w)^{\gamma}(1-n)^{\gamma+\sigma}$$

$$\frac{\rho-1+\delta}{(1-n)^{\gamma+\sigma}}=B(\frac{\gamma}{\sigma}w)^{\gamma}$$

$$\frac{\rho-1+\delta}{\sigma}[\sigma+\gamma-1+\frac{(\rho/\beta)-1+\delta}{\rho-1+\delta}]^{\sigma+\gamma}=B[\frac{\gamma}{\sigma}(1-\alpha)A(A\alpha\beta)^{\frac{\alpha}{1-\alpha}}]^{\gamma}\rho^{-\gamma\frac{\alpha}{1-\alpha}}$$

$$\frac{\rho-1+\delta}{\sigma}[\sigma+\gamma-1+\frac{(\rho/\beta)-1+\delta}{\rho-1+\delta}]^{\sigma+\gamma}\rho^{\gamma\frac{\alpha}{1-\alpha}}=B[\frac{\gamma}{\sigma}(1-\alpha)A(A\alpha\beta)^{\frac{\alpha}{1-\alpha}}]^{\gamma} \tag{48}$$

The steady state of growth (ρ) is given by:

$$\varphi(\rho)=B[\frac{\gamma}{\sigma}(1-\alpha)A(A\alpha\beta)^{\frac{\alpha}{1-\alpha}}]^{\gamma}$$

Where

$$\varphi(\rho)=\frac{\rho-1+\delta}{\sigma}[\sigma+\gamma-1+\frac{(\rho/\beta)-1+\delta}{\rho-1+\delta}]^{\sigma+\gamma}\rho^{\gamma\frac{\alpha}{1-\alpha}}$$

Proposition 4 *There exists growth in the economy ($\rho>1$), if*
i. δ (depreciation of human capital) is small (is close to 0) or(/and)
ii. A, B, β and α are big enough.

Proof.
Let $\varphi(\rho)\equiv\frac{\rho-1+\delta}{\sigma}[\sigma+\gamma-1+\frac{(\rho/\beta)-1+\delta}{\rho-1+\delta}]^{\sigma+\gamma}\rho^{\gamma\frac{\alpha}{1-\alpha}}$, then we can verify that: If $\sigma+\gamma<1$, then $\varphi(\rho)$ is strictly increasing. As $\varphi(1-\delta)=0$, there must be an $\bar{\rho}>1-\delta$ such that $\varphi(\bar{\rho})=B[\frac{\gamma}{\sigma}(1-\alpha)A(A\alpha\beta)^{\frac{\alpha}{1-\alpha}}]^{\gamma}$. Furthermore, $\bar{\rho}>1$ if and only if $\varphi(1)<B[\frac{\gamma}{\sigma}(1-\alpha)A(A\alpha\beta)^{\frac{\alpha}{1-\alpha}}]^{\gamma}$, which is equivalent to:

$$\frac{\delta}{\sigma}(\sigma+\gamma+\frac{1-\beta}{\beta\frac{1}{\delta}})^{\sigma+\gamma}<B[\frac{\gamma}{\sigma}(1-\alpha)A(A\alpha\beta)^{\frac{\alpha}{1-\alpha}}]^{\gamma}$$

which is true if δ is close to 0, or (/and) A, B, β and α are big enough.

6 Conclusions

This paper is basically modified from the classical Ramsey model in discrete time that has explored the potential impact of human capital on economic growth. The crucial results allow us to make progress along three lines. First, we demonstrate the steaty state of the economy and the existence of equilibrium in the first and second scenario where the model with and without human capital accumulation. In the last regime, the model with human capital accumulation, we found some conditions under which there exists growth in the long-run.

References

Bashir, A.-H. M. and A. F. Darrat (1994). Human capital, investment and growth: Some results from an endogenous growth model. *Journal of Economics and Finance 18*(1), 67–80.

Becker, R. A., K. Borissov, and R. S. Dubey (2015, September). Ramsey equilibrium with liberal borrowing. Caepr Working Papers 2015-003, Center for Applied Economics and Policy Research, Economics Department, Indiana University Bloomington.

Bruno, O., C. Le Van, and B. Masquin (2009, August). When does a developing country use new technologies? *Economic Theory 40*(2), 275–300.

Collins, S. M. and B. Bosworth (1996). Economic growth in east asia: Accumulation versus assimilation. *Brookings Papers on Economic Activity 27*(2), 135–204.

Galor, O. and O. Moav (2004a). From physical to human capital accumulation: Inequality and the process of development. Ge, growth, math methods, EconWPA.

Galor, O. and O. Moav (2004b). From physical to human capital accumulation: Inequality and the process of development. *The Review of Economic Studies 71*(4), 1001–1026.

Hitt, M. A. and R. Duane (2002). The essence of strategic leadership: Managing human and social capital. *Journal of Leadership & Organizational Studies 9*(1), 3–14.

Lau, L. J., J. Park, L. J. Lau, and J. Park (2003). The sources of east asian economic growth revisited. In *Cornell University*, pp. 6–7. Kluwer Academic Publishers.

Le Van, C., A. N. Nguyen, and N.-M. Nguyen (2014). Growth strategy with social capital and physical capital-theory and evidence: the case of vietnam.

Liepė, Ž. and A. Sakalas (2014). Evaluation of human capital role in the value creation process. *Procedia-Social and Behavioral Sciences 156*, 78–82.

Lucas, R. (1988). On the mechanics of economic development. *Journal of Monetary Economics 22*(1), 3–42.

Nelson, R. R. and E. S. Phelps (1966). Investment in humans, technological diffusion, and economic growth. *The American Economic Review*, 69–75.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy 94*(5), 1002–1037.

Romer, P. M. (1989, November). Human capital and growth: Theory and evidence. NBER Working Papers 3173, National Bureau of Economic Research, Inc.

Romer, P. M. (1990, October). Endogenous technological change. *Journal of Political Economy 98*(5), S71–102.

Solow, R. M. (1957). Technical Change and the Aggregate Production Function. *The Review of Economics and Statistics 39*(3), 312–320.