Limit cycles under a negative effect of pollution on consumption demand: the role of an environmental Kuznets curve

Stefano BOSI
EPEE, University of Evry

David DESMARCHELIER
LEM, University of Lille

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Abstract

Since Heal (1982), there is a theoretical consensus about the occurrence of limit cycles (through a Hopf bifurcation) under a positive effect of pollution on consumption demand (*compensation effect*) and about the impossibility under a negative effect (*distaste effect*). However, recent empirical evidence advocates for the relevance of *distaste effects*.

Our paper challenges the conventional view on the theoretical ground and reconciles theory and evidence. The Environmental Kuznets Curve (pollution first increases in the capital level then decreases) plays the main role. Indeed, the standard case à la Heal (limit cycles only under a *compensation effect*) only works along the upward-sloping branch of the curve while the opposite (limit cycles only under a *distaste effect*) holds along the downward-sloping branch.

Welfare effects of taxation also change according to the slope of the EKC.

JEL Classification: E32, O44.

1 Introduction

Human activities pollute and pollution affects them in turn. Theorists have considered the pollution effects on production and, to a lesser extent, on preferences. In particular, they have focused on consequences on consumption demand. However, as shown by Michel and Rotillon (1995), the pollution effect on this demand remains ambiguous from a theoretical point of view. More precisely, on the one hand, pollution promotes consumption demand through what they call a *compensation effect* (households consume more to compensate the utility loss due to a higher pollution). On the other hand, if household likes to consume in a pleasant environment, a rise in the pollution level reduces the consumption demand (Michel and Rotillon call this phenomenon *distaste effect*). In a seminal paper, Heal (1982) highlighted the saddle-path stability of an environmental Ramsey economy under a *distaste effect* while the occurrence of a limit cycle through a Hopf bifurcation under a sufficiently large *compensation effect*. Since, different papers have pointed out these results as a robust feature of Ramsey models with pollution effects on preferences.

To the best of our knowledge, there are no papers with empirical evidence about the direct effects of pollution on consumption demand. Nevertheless, in a recent contribution, Finkelstein *et al.* (2013) have shown that a poorer health leads to a lower marginal utility of consumption. Indirectly, such a complementarity between health and consumption says something about the pollution effects on consumption demand. Indeed, medical research has largely documented the negative pollution effect on human health.[1] Thus, a higher pollution level lowers health which lowers consumption demand in turn. Therefore, the empirical evidence provided by Finkelstein *et al.* (2013) seems to confirm the existence of a *distaste effect* which casts some doubts on the plausibility of endogenous cycles pointed out by Heal (1982).

Intuitively, endogenous limit cycles appear in Heal (1982) because a higher pollution increases the consumption demand (*compensation effect*) and decreases saving and capital accumulation. Under a sufficiently large *compensation effect*, the capital stock lowers entailing a drop in future pollution at the end. Hence, a rise in pollution today implies a drop in pollution tomorrow and an endogenous fluctuation. Thus, a *compensation effect* promotes (limit) cycles because of a positive relation between capital and pollution. Heal's results rest on this monotonicity. In order to reconcile the empirically evidence (*distaste effect*) with the possibility of limit cycles, we have to go beyond Heal's argument and, clearly, to renounce to monotonicity.

Fortunately, monotonicity does not seem to fit the evidence. The Environmental Kuznets Curve (EKC) literature points out that, beyond a critical capital level, the relation between capital and pollution is no longer positive but negative.[2] A negative relation seems to rule out the occurrence of endogenous cycles under a *compensation effect*, but to promote them under a *distaste effect*. Our paper aims at providing a theoretical framework based on the ECK to overturn Heal's conclusion while fitting the evidence.

According to Kijima *et al.* (2010), four arguments provide a rationale for the EKC. (1) EKC accounts for an historical economic process: first, the industrialization (from a clean agriculture to a dirty industry); then, the tertiarization (from a dirty industry to clean services). (2) Internalizing pollution externalities requires advanced institutions that exist only in mature economies. (3) There exists a threshold beyond which abatement becomes profitable. (4) Abatement activities exhibit increasing returns.[3]

According to Fernandez *et al.* (2012), EKC makes local indeterminacy and endogenous fluctuations less demanding. These authors develop a continuous-time Ramsey economy with endogenous labor supply and show that conditions

[1] The reader is referred to a brief survey on the pollution effects on human health by Kampa and Castanas (2008).

[2] The reader is referred to a survey on the EKC compiled by Kijima *et al.* (2010).

[3] Andreoni and Levinson (2001), and, more recently, Managi (2006) and Managi and Kaneko (2009) have found an empirical evidence of increasing returns to scale for abatment activities.

for equilibrium indeterminacy are compatible with the existence of an EKC.

Taking in account facts and criticisms, we build a simple continuous-time Ramsey economy where a pollution externality, coming from capital accumulation, affects the marginal utility of consumption. We introduce a government which levies a proportional tax on capital utilization at the firm level to finance depollution expenditures. As seen above, to fit evidence, we assume that depollution efficiency increases in the abatement effort (Andreoni and Levinson (2001) among the others).

Our standard assumptions on preferences and technology ensure the existence of a unique positive steady state. The introduction of increasing depollution efficiency leads to the existence of an EKC at the steady state.

We show that, in the long run, a pollution tax is never welfare-improving when the economy is located on the downward-sloping branch of the EKC.

We prove that, in the short run, along the same branch of the EKC, limit cycles (through a Hopf bifurcation) arise if and only if preferences exhibit a *distaste effect*. To the best of our knowledge, our model is the first attempt to present the *distaste effect* as a source of macroeconomic volatility. The existence of empirically grounded *distaste effects* is no longer a sufficient argument to rule out the occurrence of limit cycles.

The rest of the paper is organized as follows: we present the model (Sections 2 to 5), we study the equilibrium dynamics in the short and long run (Sections 6 to 11), we solve and simulate the isoelastic case (Section 12 and 13), we conclude (Section 14). All the technical proofs are gathered in the Appendix (Section 15).

2 Firms

We consider a simple Ramsey economy with a pollution accumulation coming from capital utilization. A government levies a carbon tax in order to finance depollution expenditures according to a balanced budget rule. Depollution efficiency is endogenous and is characterized by increasing returns for abatement activities.

A representative firm produces a single output. A constant returns to scale technology is represented by an aggregate production function: $Y(t) = F(K(t), L(t))$, where $K(t)$ and $L(t)$ are the aggregate demands for capital and labor at time t. For notational parsimony, the time argument t will be omitted in the following for any variable. Following Itaya (2008) and Fernandez *et al.* (2012), any firm must pay a pollution tax τ levied by a public authority on physical capital.

Assumption 1 *The production function $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is C^1, homogeneous of degree one, strictly increasing and concave. Standard Inada conditions hold.*

Any firm chooses the amount of capital and labor to maximize the profit taking as given the real interest rate r and the real wage w. The program $\max_{K,L} [F(K, L) - rK - \tau K - wL]$ is correctly defined under Assumption 1 and the first-order conditions write:

$$r + \tau = f'(k) \equiv R(k) \text{ and } w = f(k) - kf'(k) \equiv w(k) \tag{1}$$

where $f(k) \equiv F(k,1)$ is the average productivity and $k \equiv K/L$ denotes the capital intensity at time t. We introduce the capital share in total income and the elasticity of capital-labor substitution:

$$\alpha(k) \equiv \frac{kf'(k)}{f(k)} \text{ and } \sigma(k) = \alpha(k)\frac{w(k)}{kw'(k)}$$

In addition, we determine the elasticities of factor prices:

$$\frac{kR'(k)}{R(k)} = -\frac{1-\alpha(k)}{\sigma(k)} \text{ and } \frac{kw'(k)}{w(k)} = \frac{\alpha(k)}{\sigma(k)} \tag{2}$$

3 Households

Any household earns a capital income rh and a labor income wl where h and l denote the individual wealth and labor supply at time t. For simplicity, we assume also that the household supplies inelastically one unit: $l = 1$. Thus, households consume and save their income according to the budget constraint

$$c + \dot{h} \leq (r - \delta) h + w \tag{3}$$

where $\dot{h}$ denotes the time-derivative of wealth. The gross investment includes the capital depreciation at the rate δ.

For the sake of simplicity, the population of consumers-workers is constant over time and normalized to one: $N = 1$. Such a normalization implies $L = Nl = l = 1$, $K = Nh = h$ and $h = K/N = kl = k$.

In the following, P will denote the stock of pollution (aggregate externality).

Assumption 2 *Preferences are rationalized by a non-separable utility function* $u(c, P)$*. First and second-order restrictions hold on the sign of derivatives:* $u_c > 0$*,* $u_P < 0$ *and* $u_{cc} < 0$*, jointly with the limit conditions:* $\lim_{c\to 0^+} u_c = \infty$ *and* $\lim_{c\to+\infty} u_c = 0$.

Assumption 2 does not impose any restriction on the sign of the cross-derivative $u_{cP} \lesseqgtr 0$. Following Michel and Rotillon (1995), the household's preferences exhibit a *distaste effect* (*compensation effect*) when pollution decreases (increases) the marginal utility of consumption. If the households enjoy to consume in a pleasant environment, a higher pollution level lowers their consumption demand ($u_{cP} < 0$) giving rise to a *distaste effect* (Michel and Rotillon, 1995). Conversely, household could decide to increase their consumption demand to compensate the utility loss due to a higher pollution level ($u_{cP} > 0$), generating a *compensation effect* (Michel and Rotillon, 1995).

According to Heal (1982), a sufficiently large *compensation effect* promotes a limit cycle near the steady state of a simple environmental Ramsey economy, while saddle-path stability always prevails when preferences exhibit a *distaste effect*. In a recent contribution, Finkelstein *et al.* (2013) provide some empirical evidence of complementarity between health and consumption. Combining that with evidence of negative pollution effects on health (Kampa and Castanas, 2008), we can conclude that a *distaste effect* is more relevant than a *compensation effect*. Following Heal's theoretical approach casts doubts about the plausibility of endogenous cycles. Our contribution aims at challenging this conventional view by showing that Heal's results (1982) fail when the economy experiences an EKC at the steady state.

Let us introduce useful first and second-order elasticities

$$\varepsilon_c \equiv \frac{cu_c}{u} \text{ and } \varepsilon_P \equiv \frac{Pu_P}{u} \tag{4}$$

$$\varepsilon_{cc} \equiv \frac{cu_{cc}}{u_c} \text{ and } \varepsilon_{cP} \equiv \frac{Pu_{cP}}{u_c} \tag{5}$$

$-1/\varepsilon_{cc}$ is the usual consumption elasticity of intertemporal substitution while ε_{cP} captures the effects of pollution on the marginal utility of consumption. According to Assumption 1, $\varepsilon_{cc} < 0$. In terms of elasticity, the *distaste effect* (*compensation effect*) writes $\varepsilon_{cP} < 0$ ($\varepsilon_{cP} > 0$).

In a Ramsey model, the representative household maximizes an intertemporal utility functional $\int_0^\infty e^{-\rho t} u(c, P)\, dt$ under the budget constraint (3) where $\rho > 0$ denotes the rate of time preference. This program is correctly defined under Assumption 2.

Proposition 1 *The first-order conditions of the consumer's program are given by a static relation*

$$\lambda = u_c \tag{6}$$

a dynamic Euler equation $\dot{\lambda} = \lambda(\rho + \delta - r)$ *and the budget constraint (3), now binding,* $\dot{k} = (r - \delta)k + w - c$ *jointly with the transversality condition* $\lim_{t\to\infty} e^{-\rho t}\lambda(t)\, k(t) = 0$. λ *denotes the multiplier associated to the budget constraint.*

Proof. See the Appendix. ■

4 Government

An environment-oriented government spends all the tax revenue to finance depollution through an abatement effort (maintenance m) according to a balanced budget rule:

$$m = \tau K \tag{7}$$

Because of no population growth and inelastic labor supply, $L = 1$ and budget (7) writes in intensive terms $\tau k = m$.

5 Pollution

The aggregate stock of pollution P is a pure externality coming from the aggregate capital stock used to produce (K). In addition, the government takes care of depollution through the abatement expenditures m. To take things as simple as possible, we assume a linear process:

$$\dot{P} = -aP + bK - \gamma m \tag{8}$$

$a \geq 0$, $b \geq 0$ and $\gamma \geqslant 0$ capture respectively the natural rate of pollution absorption, the environmental impact of production and the pollution abatement efficiency. Because $L = 1$, the process of pollution accumulation (8) writes in intensive terms: $\dot{P} = -aP + bk - \gamma m$.

We assume that the abatement efficiency γ also depends on the effort m and that $\gamma \equiv \gamma(m)$ is a derivable non-decreasing function. From a theoretical point of view, two cases matter: constant γ or increasing γ.

Assumption 3.0 $\gamma \geq 0$ is a constant.

Assumption 3.1 $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ *is a* C^1 *increasing function:* $\gamma'(m) > 0$ *for every* $m \geq 0$.

The second case is more justified on the empirical ground: according to Andreoni and Levinson (2001), Managi (2006) and Managi and Kaneko (2009), abatement technology $m\gamma(m)$ exhibits increasing returns with respect to the abatement effort m. Thus, abatement efficiency $\gamma(m)$ increases as depollution expenditures rise. According to Andreoni and Levinson (2001), abatement activities are enhanced by a learning-by-doing process.

In the following, the elasticity of abatement efficiency will play a role:

$$\theta(m) \equiv \frac{m\gamma'(m)}{\gamma(m)}$$

$\theta > 0$ captures the sensitivity of abatement efficiency with respect to maintenance.

6 Equilibrium

At equilibrium, all markets clear. Applying the Implicit Function Theorem on the static relation (6), we obtain the consumption demand $c = c(\lambda, P)$ as a function of multiplier and pollution stock with elasticities:

$$\frac{\lambda}{c}\frac{\partial c}{\partial \lambda} = \frac{1}{\varepsilon_{cc}} < 0 \text{ and } \frac{P}{c}\frac{\partial c}{\partial P} = -\frac{\varepsilon_{cP}}{\varepsilon_{cc}}$$

We observe that the first elasticity is just the negative of the consumption elasticity of intertemporal substitution while the second one captures the *distaste/compensation effects*. More precisely, when preferences exhibits a *distaste effect* (*compensation effect*), then $\partial c/\partial P < 0$ ($\partial c/\partial P > 0$).

Proposition 2 *Equilibrium dynamics are represented by the following system:*

$$\dot{\lambda} = g_1 (\lambda, k, P) \equiv [\rho + \delta + \tau - R(k)] \lambda \tag{9}$$

$$\dot{k} = g_2 (\lambda, k, P) \equiv [R(k) - (\tau + \delta)] k + w(k) - c(\lambda, P) \tag{10}$$

$$\dot{P} = g_3 (\lambda, k, P) \equiv -aP + bk - \gamma(\tau k) \tau k \tag{11}$$

Proof. Simply consider equations (1) and Proposition 1. ∎

The dynamic system formed by equations (9) to (11) possesses one forward (jump) variable (λ) and two backward (state) variables (k and P).

7 Steady state and EKC

In the following, we highlight a non-monotonic relation between capital and pollution at the steady state. The turning point is a critical capital intensity:

$$\tilde{k} \equiv \frac{1}{\tau} \gamma^{-1} \left(\frac{b}{\tau (1 + \theta)} \right) > 0$$

The existence of an EKC rests on the invertibility of γ ensured by an increasing abatement efficiency.

Proposition 3 *(environmental Kuznets curve) Under Assumption 3.1, at the steady state, $P(k^*)$ is an inverted-U-shaped function of k^* with $P'(k^*) > 0$ if and only if $k^* < \tilde{k}$.*

Proof. See the Appendix. ∎

The capital elasticity of pollution is given by:

$$\pi = \pi(k) \equiv \frac{kP'(k)}{P(k)} \tag{12}$$

with, at the steady state,

$$\pi(k^*) = \frac{b - \tau\gamma^* (1 + \theta)}{b - \tau\gamma^*} \tag{13}$$

Clearly, $k^* < \tilde{k}$ implies $\pi(k^*) > 0$ while $k^* > \tilde{k}$ implies $\pi(k^*) < 0$. In other terms, there exists an inverted-U-shaped relation between the cause (capital) and the effect (pollution), that is an EKC.

The decreasing branch of the EKC means that a wealthier economy becomes also a cleaner one. Nevertheless, an economy could not converge to a steady state exhibiting a negative relation between capital and pollution but, instead, to a more complex attractor (a supercritical limit cycle). Thus, as we will show in the next section, the relation between capital and pollution could become cyclical around a steady state on the decreasing branch of the EKC.

Proposition 4 *(Steady state existence and uniqueness) There exists a unique positive steady state.*

Proof. See the Appendix. ■

Proposition 4 points out that, no matter whether pollution affects negatively or positively the consumption demand and abatement activities increase depollution efficiency, the existence and the uniqueness of the steady state are always ensured.

A simple exercise of comparative statics allows us to connect the effects of taxation on the steady state and the EKC.

Differentiating

$$R(k^*) = \rho + \delta + \tau \tag{14}$$

gives the effect of τ on k^*. More explicitly, using (2), we find

$$\frac{\tau}{k^*}\frac{\partial k^*}{\partial \tau} = -\frac{\sigma}{1-\alpha}\frac{\tau}{\rho+\delta+\tau} < 0 \tag{15}$$

Thus, unsurprisingly, the higher the tax rate (τ) on the physical capital, the lower the capital stock of steady state. The final impact on pollution depends on the EKC.

Proposition 5 *The qualitative impacts of τ on P^* are given by*
(1) $\partial P^*/\partial \tau < 0$ if $k < \tilde{k}$,
(2) $\partial P^*/\partial \tau > 0$ if $k > \tilde{k}$.

Proof. Considering (29) gives:

$$\frac{\tau}{P^*}\frac{\partial P^*}{\partial \tau} = \pi(k^*)\frac{\tau}{k^*}\frac{\partial k^*}{\partial \tau} \tag{16}$$

■

Proposition 5 shows that, when the economy lies on the decreasing branch of the EKC ($k > \tilde{k}$), a higher green tax increases the pollution stock. Indeed, inequality (15) implies that a higher tax rate lowers the capital stock which increases in turn the pollution level (negative EKC effect). Such counter-intuitive effect of pollution tax looks like the one called Green Paradox (Sinn, 2008). Therefore, pollution taxes are not always the right way to clean the environment.

Using (33) and (2), we obtain

$$\frac{\tau}{c^*}\frac{\partial c^*}{\partial \tau} = \frac{\rho + (\rho+\delta+\tau)\frac{1-\alpha}{\sigma}}{\rho + (\rho+\delta+\tau)\frac{1-\alpha}{\alpha}}\frac{\tau}{k^*}\frac{\partial k^*}{\partial \tau} < 0 \tag{17}$$

Even if the effect of taxation on consumption demand is always negative, the effect on welfare is ambiguous. More precisely, we have the following proposition.

Proposition 6 *Under Assumptions 1 and 2, if $u > 0$, the impact of taxation on welfare is positive if and only if*

$$\pi(k^*) > -\frac{\varepsilon_c^*}{\varepsilon_P^*}\frac{\rho + (\rho+\delta+\tau)\frac{1-\alpha}{\sigma}}{\rho + (\rho+\delta+\tau)\frac{1-\alpha}{\alpha}} (> 0) \tag{18}$$

where $\varepsilon_c^*/\varepsilon_P^* < 0$. In particular, on the downward-sloping branch of the EKC, $\pi(k^*) < 0$ and, therefore, the impact of taxation on welfare is negative.

Proof. See the Appendix. ■

Proposition 6 deserves an interpretation. Assume first that the steady state lies on the upward-sloping branch of EKC. A higher pollution tax leads (1) to a lower capital level (see 15) which entails a lower pollution level ($\pi(k^*) > 0$) and a higher utility in turn (Assumption 2), and (2) to lower consumption level (17) and utility at the end (Assumption 2). Hence, the effect of pollution tax on household's utility is ambiguous. According to Proposition 6, the positive effect (1) dominates the negative one (2) if and only if the slope of EKC is very positive (see condition (18)). In this respect, the decrease of consumption demand entailed by a higher tax rate is largely dominated by the drop in pollution. In other terms, when the slope of the EKC is very positive, a higher pollution tax implies a utility gain from pollution drop much larger than the utility loss from consumption decrease: the pollution tax turns out to be welfare-improving. Conversely, when the slope of the EKC is not too positive, condition (18) fails and the negative effect (2) dominates the positive one (1) because a higher pollution tax results in a drop in capital larger than in pollution. We observe that the slope of EKC is very positive if k^* is very close to the origin in the (k, P)-plane: thereby, a pollution tax is more likely welfare-improving in developing countries than in a developed ones.

Now, assume that the steady state lies on the downward-sloping branch of EKC. Thus, a higher pollution tax results (1) in a lower capital level (see 15) which entails a higher pollution level ($\pi(k^*) < 0$) and a lower utility in turn (Assumption 2) and (2) in lower consumption level (17) and utility at the end (Assumption 2). In this case, the ambiguity is dissipated because both the effects are negative: a higher pollution tax lowers households' utility and social welfare when the economy experiences the negative slope of EKC.

8 Local dynamics

In the following, we are interested in short-run dynamics and fluctuations, and we provide general conditions for local bifurcations and local indeterminacy in the case of a three-dimensional system with two predetermined variables.

The local stability of a the steady state depends on the stability of the eigenvalues of the Jacobian matrix. Computing the derivatives, we obtain a three-dimensional Jacobian matrix:

$$J \equiv \begin{bmatrix} \frac{\partial g_1}{\partial \lambda} & \frac{\partial g_1}{\partial k} & \frac{\partial g_1}{\partial P} \\ \frac{\partial g_2}{\partial \lambda} & \frac{\partial g_2}{\partial k} & \frac{\partial g_2}{\partial P} \\ \frac{\partial g_3}{\partial \lambda} & \frac{\partial g_3}{\partial k} & \frac{\partial g_3}{\partial P} \end{bmatrix} = \begin{bmatrix} 0 & (\mu - \rho) \frac{\alpha}{\sigma} \frac{\lambda}{k} & 0 \\ -\frac{\mu}{\varepsilon_{cc}} \frac{k}{\lambda} & \rho & \mu \frac{\varepsilon_{cP}}{\varepsilon_{cc}} \frac{a}{b - \tau\gamma} \\ 0 & b - \tau\gamma (1 + \theta) & -a \end{bmatrix} \tag{19}$$

where $\alpha = \alpha(k)$, $\sigma = \sigma(k)$, $\gamma = \gamma(m)$, $\theta = \theta(m)$ and

$$\mu \equiv \frac{c}{k} = \rho + (\rho + \delta + \tau) \frac{1 - \alpha}{\alpha}$$

In this and the following sections, all the values are evaluated at the steady state. For notational parsimony, we will omit the asterisk $*$.

The determinant, the sum of minors of order two and the trace are given by:

$$D = -a(\mu - \rho)\frac{\alpha}{\sigma}\frac{\mu}{\varepsilon_{cc}} > 0 \tag{20}$$

$$S = \left[(\mu - \rho)\frac{\alpha}{\sigma} - a\pi\varepsilon_{cP}\right]\frac{\mu}{\varepsilon_{cc}} - a\rho \tag{21}$$

$$T = \rho - a \tag{22}$$

where π is given by (12).

9 Bifurcations

In continuous time, a local bifurcation generically arises when the real part of an eigenvalue $\lambda(p)$ of the Jacobian matrix crosses zero in response to a change in a parameter p. Denoting by p^* the critical parameter value of bifurcation, we get generically two cases: (1) when a real eigenvalue crosses zero: $\lambda(p^*) = 0$, the system undergoes a saddle-node bifurcation (either an elementary saddle-node or a transcritical or a pitchfork bifurcation depending on the number of steady states), (2) when the real part of two complex and conjugate eigenvalues $\lambda(p) = a(p) \pm ib(p)$ crosses zero, the system undergoes a Hopf bifurcation. More precisely, in the second case, we require $a(p^*) = 0$ and $b(p) \neq 0$ in a neighborhood of p^* (see Bosi and Ragot, 2011, p. 76).

The occurrence of a saddle-node bifurcation (elementary saddle-node, transcritical, pitchfork) requires a multiplicity of steady states. In our model, the steady state is unique (Proposition 4). Thus, we leave aside the theory of elementary saddle-node bifurcations to focus exclusively on the general theory of Hopf bifurcations in the case of three-dimensional dynamic systems and on the occurrence of limit cycles.

We eventually observe that system (9-11) is three-dimensional with two predetermined variables (k and P) and one jump variable (λ). Thus, multiple equilibria (local indeterminacy) arise when the three eigenvalues of the Jacobian matrix (19) evaluated at the steady state have negative real parts: either $\lambda_1, \lambda_2, \lambda_3 < 0$ or $\operatorname{Re}\lambda_1, \operatorname{Re}\lambda_2 < 0$ and $\lambda_3 < 0$.

10 Hopf bifurcation

This bifurcation generates limit cycles either attractive (supercritical) or repulsive (subcritical).

Reconsider the Jacobian matrix J and its determinant, sum of minors of order two and trace: $D = \lambda_1\lambda_2\lambda_3$, $S = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3$ and $T = \lambda_1 + \lambda_2 + \lambda_3$. A Hopf bifurcation occurs when the real part of two complex and conjugate eigenvalues $\lambda(p) = a(p) \pm ib(p)$ crosses zero. More precisely, we require $a(p^*) = 0$ and $b(p) \neq 0$ in a neighborhood of p^* (see Bosi and Ragot, 2011, p. 76).

Proposition 7 *(Hopf bifurcation) In the case of a three-dimensional system, a Hopf bifurcation generically arises if and only if $D = ST$ and $S > 0$.*

Proof. See the Appendix. ■

11 Local determinacy

In our economy, there are two predetermined variables (k and P) and a jump variable (λ). As seen above, indeterminacy requires three eigenvalues with negative real parts: either $\lambda_1, \lambda_2, \lambda_3 < 0$ or $\operatorname{Re}\lambda_1, \operatorname{Re}\lambda_2 < 0$ and $\lambda_3 < 0$.

Proposition 8 *If all the eigenvalues are real, the equilibrium is locally indeterminate if and only if $D, T < 0$ and $S > 0$.*

Proof. See the Appendix. ■

Corollary 9 *(local determinacy) The equilibrium is locally unique.*

Proof. Consider inequality (20) and apply Proposition 8. ■

Corollary 9 implies that, when a Hopf bifurcation occurs, from a saddle-point (with a two-dimensional stable manifold) the steady state becomes a source surrounded by a (supercritical) cycle on the central manifold and the new equilibrium remains unique and converges to the cycle. Then, there is no room for stochastic fluctuations due to self-fulfilling expectations.

12 Isoelastic case

In order to provide more economic intuition for local bifurcations, we focus on explicit but standard functional forms:

$$f(k) = Ak^{\alpha} \text{ and } u(c, P) \equiv \frac{(cP^{-\eta})^{1-\varepsilon}}{1-\varepsilon} \tag{23}$$

with $\varepsilon > 0$, $\eta > 0$ and $\alpha \in (0, 1)$.

These functions are interesting because elasticities (4) and (5) write in terms of fundamental parameters: $\varepsilon_c = 1-\varepsilon$, $\varepsilon_P = \eta(\varepsilon - 1)$, $\varepsilon_{cc} = -\varepsilon$, $\varepsilon_{cP} = \eta(\varepsilon - 1)$. This case leads us to analyze either the *distaste effect* (namely $\varepsilon_{cP} < 0$ with $\varepsilon < 1$) or the *compensation effect* (namely $\varepsilon_{cP} > 0$ with $\varepsilon > 1$). Because of the Cobb-Douglas specification for production, we also have $\sigma = 1$. The abatement efficiency is also specified as isoelastic:

$$\gamma(m) = \beta m^{\theta} \tag{24}$$

with $\theta \geq 0$. Depollution efficiency simplifies to $\beta > 0$ when $\theta = 0$.

In this isoelastic context, the unique steady state $(k, P, \lambda)^*$ (Proposition 4) becomes:

$$k^* = \left(\frac{\alpha A}{\rho+\delta+\tau}\right)^{\frac{1}{1-\alpha}} \quad (25)$$
$$P^* = \frac{k^*}{a}\left[b - \beta\tau\left(\tau k^*\right)^{\theta}\right] \quad (26)$$
$$\lambda^* = \left[\rho k^* + (1-\alpha) A k^{*\alpha}\right]^{-\varepsilon} P^{*\eta(\varepsilon-1)} \quad (27)$$

We observe that the positivity of P^* requires the restriction $b > \tau\gamma$. Moreover, if $\theta > 0$, equation (26) explicitly gives the EKC of Proposition (3). The maximization argument of EKC solves

$$P'(k) = \frac{1}{a}\left[b - \tau(1+\theta)\beta(\tau k)^{\theta}\right] = 0 \quad (28)$$

Thus, the isoelastic case allows us to compute also the turning point of EKC:

$$\tilde{k} = \frac{1}{\tau}\left[\frac{b}{\tau(1+\theta)\beta}\right]^{\frac{1}{\theta}}$$

Clearly, if $k < \tilde{k}$, then $b > \tau(1+\theta)\gamma$ and $P'(k) > 0$, while, if $k > \tilde{k}$, then $b < \tau(1+\theta)\gamma$ and $P'(k) < 0$.

According to Proposition 6, welfare analysis holds under the restriction $u > 0$ or, equivalently, $\varepsilon < 1$ (that is the empirically relevant *distaste case*). Inequality (18) applies and the impact of taxation on welfare is positive if and only if

$$\pi(k^*) > \frac{1}{\eta}\frac{\rho + (\rho+\delta+\tau)\frac{1-\alpha}{\sigma}}{\rho + (\rho+\delta+\tau)\frac{1-\alpha}{\alpha}}\ (>0)$$

Proposition 10 *In the isoelastic case (expressions (23) and (24)), the determinant, the sum of minors of order two and the trace write*

$$D = a\alpha(\mu-\rho)\frac{\mu}{\varepsilon} > 0$$
$$S = -a\rho - \left[a\pi\eta(1-\varepsilon) + \alpha(\mu-\rho)\right]\frac{\mu}{\varepsilon}$$
$$T = \rho - a$$

Proof. Consider (19) and replace the elasticities. ■

In the following, we focus the occurrence of Hopf bifurcation and we analyze two main cases: the standard one ($\theta = 0$) corresponding to Assumption 3.0 and the EKC case ($\theta > 0$) corresponding to Assumption 3.1.

A convenient Hopf bifurcation parameter is η. The critical value of this parameter is $\eta_\theta \equiv \eta_0/\pi$ with

$$\eta_0 \equiv \frac{\rho}{\varepsilon-1}\left(\frac{\varepsilon}{\mu} + \frac{\alpha}{a}\frac{\mu-\rho}{\rho-a}\right)$$

and π given by (12).

We observe that neither η_0 nor π depend on η. Indeed, η_0 depends on the fundamental parameters, while $\pi = \pi(k)$ depends on k which, according to expression (25), does not depend on η. Therefore, the critical value η_θ is well-defined. Clearly, when $\theta = 0$, we recover $\eta_\theta = \eta_0$ because $\pi = 1$.

12.1 Standard case

We focus first on the case more considered in literature on endogenous fluctuations: the pollution abatement efficiency does not depends on abatement expenditures (namely $\theta = 0$). In this special case, there is no room for the EKC.

The capital elasticity of pollution (12) simplifies to $\pi(k) = 1$ and D, S and T to:

$$D = a\alpha(\mu - \rho)\frac{\mu}{\varepsilon} > 0$$
$$S = -a\rho - [a\eta(1-\varepsilon) + \alpha(\mu - \rho)]\frac{\mu}{\varepsilon}$$
$$T = \rho - a$$

The two following propositions study the occurrence of deterministic cycles when preferences are characterized by a *distaste effect* or a *compensation effect.*

Proposition 11 *Under a distaste effect ($\varepsilon < 1$) there is no room for Hopf bifurcations.*

Proof. $\varepsilon < 1$ implies $S < 0$. Thus, according to Proposition (7), any Hopf bifurcation is generically ruled out. ∎

Proposition 12 *(Hopf bifurcation) Let $\rho > a$. Under a compensation effect ($\varepsilon > 1$), a limit cycle arises near the steady state through a Hopf bifurcation at $\eta = \eta_0$.*

Proof. According to Proposition (7), a Hopf bifurcation generically occurs if and only if $D = ST$ and $S > 0$. $\eta = \eta_0$ is solution of $D = ST$. In addition, $\eta = \eta_0$ implies $S > 0$ because $\rho > a$. ∎

Propositions 11 and 12 recover the Heal's main result (1982): endogenous deterministic cycles may occur if and only if preferences are characterized by a *compensation effect* ($\varepsilon > 1$).

The existence of deterministic cycles is simply interpreted in terms of economic concepts. Assume that the economy is at the steady state at time t and assume an exogenous increase in the pollution stock. This environmental degradation induces consumers to increase their consumption because of the *compensation effect* and, then, to reduce their saving and capital which lowers the next period pollution stock (equation (11)), and so on: deterministic cycles arises.

However, Finkelstein *et al.* (2013) have pointed out complementarity between health and consumption while Kampa and Castanas (2008) the negative pollution effects on health. Thus, empirical evidence advocates for a negative effect of pollution on consumption demand: a higher pollution level worsens health which reduces the consumption demand in turn. This empirically grounded *distaste effect* casts some doubts on the plausibility of the occurrence of endogenous cycles (Proposition 12).

The theoretical interpretation *à la* Heal (1982) provided above shows that the occurrence of endogenous cycles rests on a positive relation between capital and pollution, and rules them out under an (empirically relevant) *distaste effect*. Along the negative slope of EKC capital has a negative effect on pollution: intuition suggests that, in this case, endogenous cycles could become compatible with a *distaste effect*. The next section aims at proving this conjecture.

12.2 EKC case

This section addresses the possibility of endogenous cycles when the economy experiences an EKC at the steady state. The existence of an inverted-U-shaped EKC requires $\theta > 0$. According to (13), D, S and T write

$$\begin{aligned} D &= a\alpha\left(\mu-\rho\right)\frac{\mu}{\varepsilon} > 0 \\ S &= -a\rho - \left[a\eta\left(1-\varepsilon\right)\frac{b-\tau\gamma\left(1+\theta\right)}{b-\gamma\tau} + \alpha\left(\mu-\rho\right)\right]\frac{\mu}{\varepsilon} \\ T &= \rho - a \end{aligned}$$

One of the novelties of the paper is to consider the increasing abatement efficiency ($\theta > 0$) jointly with a capital taxation ($\tau > 0$): this generate a U-shaped EKC. Indeed, if $\tau = 0$, thus $P'(k) > 0$ (Heal's framework, 1982; see (28)). Now, $P'(k) < 0$ iff $k > \tilde{k}$. In order to have $P > 0$ and $P' < 0$, that is $\pi < 0$, we assume

$$\frac{1}{1+\theta}\frac{b}{\gamma} < \tau < \frac{b}{\gamma} \tag{29}$$

Proposition 13 *(Hopf bifurcation) Let $\rho > a$ and $\varepsilon < 1$ (distaste effect). If (29) holds, a limit cycle generically occurs through a Hopf bifurcation at $\eta = \eta_\theta$.*

Proof. According to Proposition (7), a Hopf bifurcation generically occurs if and only if $D = ST$ and $S > 0$. $\eta = \eta_\theta$ is solution of $D = ST$. In addition, $\eta = \eta_\theta$ implies $S > 0$ because $\rho > a$. ■

Notice that, under a Kuznets effect (negative slope of EKC), a *distaste effect* is a necessary condition to observe a Hopf bifurcation. Indeed, a *distaste effect* ($\varepsilon < 1$) implies $\eta_0 < 0$. Thus, a negative capital elasticity of pollution $\pi < 0$ entails a positive bifurcation value $\eta_\theta \equiv \eta_0/\pi$.

As conjectured above, a negative relation between pollution and capital (along the negative-sloped branch of EKC) preserves the scope for endogenous cycles when pollution lowers consumption demand (*distaste effect*). This result is challenging because, to the best of our knowledge, for the first time, the *distaste effect* (empirically grounded contrarily to the *compensation effect*) is pointed out as a potential source of endogenous fluctuations. Conversely, the argument of *compensation effect* no longer works as a source of endogenous cycles along the downward-sloping branch of EKC.

Even now, the existence of limit cycles under a *distaste effect* deserves an interpretation. Assume that the economy is at the steady state at time t. Consider an exogenous increase in the pollution level. Under a *distaste effect*, households reduces their consumption demand while increasing savings. Along the negative-sloped branch of EKC, the rise of capital intensity induced by savings, lowers the next period pollution stock (and so on) giving rise to deterministic (limit) cycles around the steady state.

13 Simulations

In this section, we provide computer simulations for the standard and the EKC case presented above. In both the cases, we consider the following quarterly calibration:

Parameter	A	α	δ	ρ	a	b	τ
Value	1	0.33	0.025	0.01	0.008	0.008	0.1

(30)

This calibration satisfies $\rho > a$ (Proposition 13). To perform the simulations, we use the Matcont package for Matlab.[4]

13.1 Standard case

In the standard case, γ does not depend on pollution abatement ($\theta = 0$). We set:

Parameter	β	ε
Value	0.06	4

The value for ε ensures a *compensation effect*. β is set to keep λ^* low enough.[5] Such a calibration implies $\eta_0 = 18.891$.

Figure 1 represents all the stationary values of λ when $\eta \in (14, 29)$. H denotes the Hopf bifurcation values.

[4] Matcont version 5p4.
[5] Matcont is unable to implement a continuation exercise when λ^* is too high.

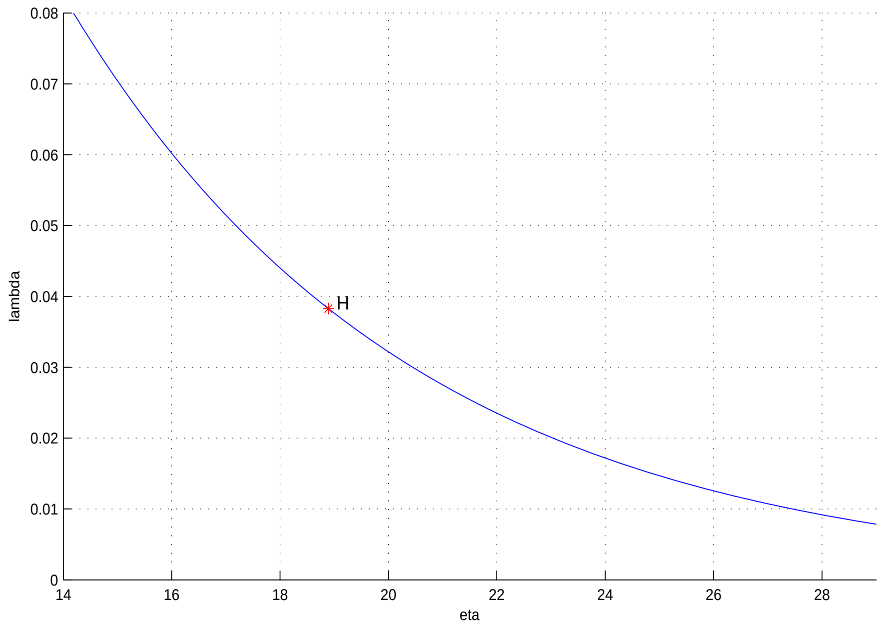

Fig. 1 λ^* in the range $(14, 29) \ni \eta$

Following Matcont, a Hopf bifurcation occurs when $\eta = 18.89068 \approx \eta_0$, the steady state becomes:

$$(k, P, \lambda)^* = (3.7964125, 0.94910312, 0.038280808)$$

The corresponding eigenvalues are: $\lambda_1 = 0.1603i$, $\lambda_2 = -0.1603i$ and $\lambda_3 = 0.002$. The first Lyapunov coefficient $l_1 = -3.493719 * 10^{-3}$ evaluated by Matcont at the Hopf boundary is negative. Therefore, the bifurcation is supercritical and the limit cycle is stable (Figure 2).

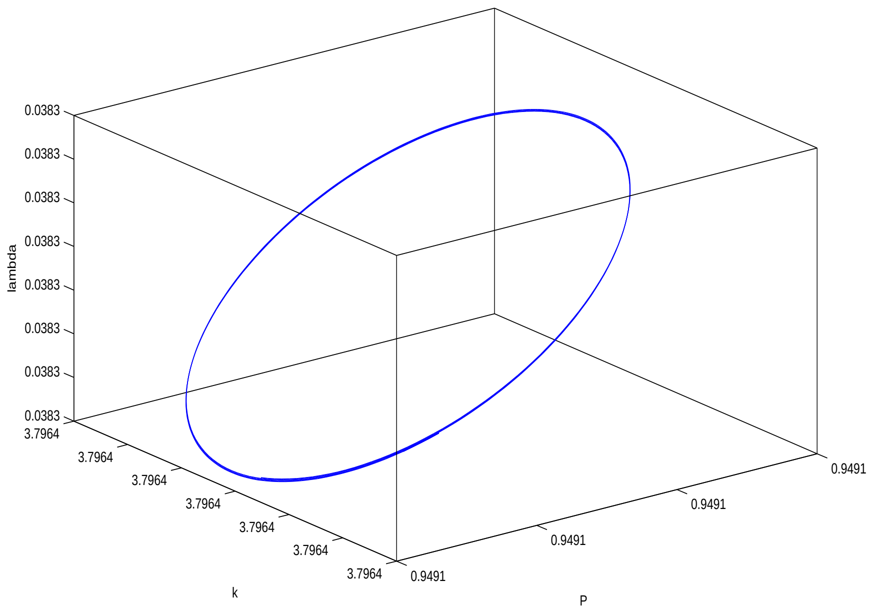

Fig. 2 The stable limit cycle in the (λ, k, P)-space

13.2 EKC case

Focus now on the case with $\theta > 0$ and $\tau > 0$. We complete the calibration table (30) with

Parameter	ε	θ	β
Value	0.5	3	1

ε is now set to ensure a *distaste effect* while θ and β to satisfy inequalities (29). Figure 3 represents the EKC (see (26)).

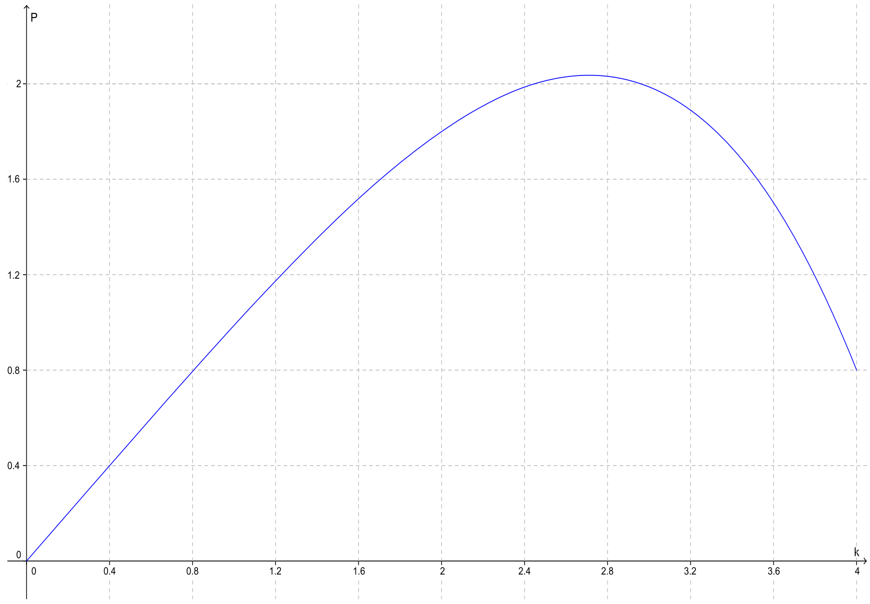

Fig. 3 The EKC in the (k, P)-plane

The new calibration determines $\eta_\theta = 20.591505$ corresponding to the steady state:

$$(k, P, \lambda)^* = (3.796\,4, 1.199\,8, 0.147\,57)$$

We observe that the economy lies on the decreasing branch of EKC when $\eta = \eta_\theta$ (Figure 3). Figure 4 below done with Matcont represents the stationary values of λ when $\eta \in (12, 30)$. As above, H denotes Hopf bifurcation values.

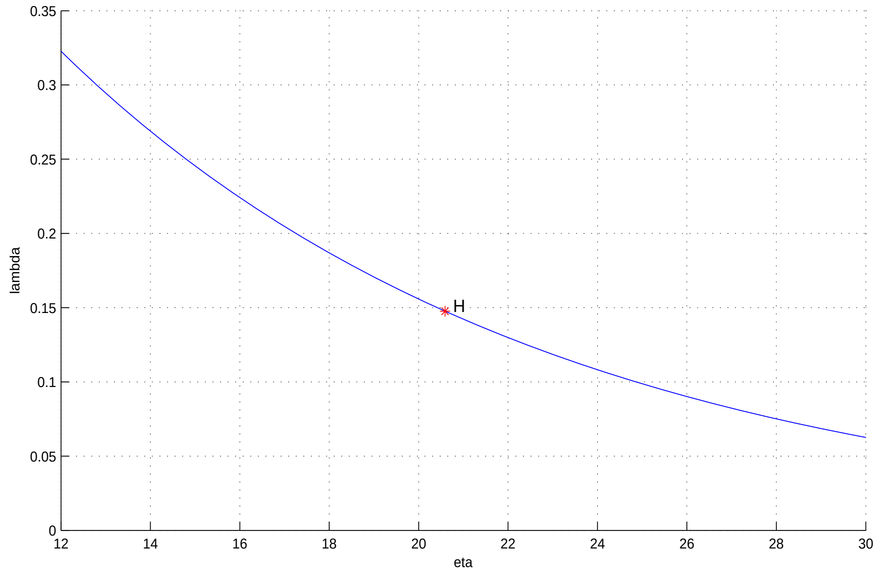

Fig. 4 λ^* in the range $(12, 30) \ni \eta$

The corresponding eigenvalues are $\lambda_1 = 0.453396i$, $\lambda_2 = -0.453396i$ and $\lambda_3 = 0.002$. The first Lyapunov coefficient $l_1 = -6.738557 * 10^{-3}$ evaluated by Matcont at the Hopf boundary is negative. Therefore, the bifurcation is supercritical and the limit cycle is stable (Figure 5).

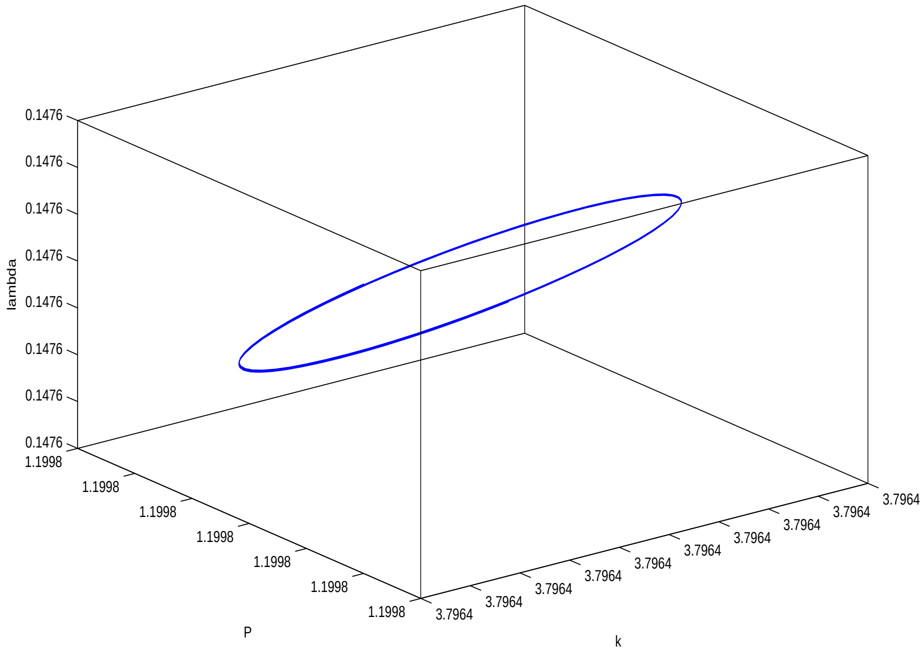

Fig. 5 The stable limit cycle in the (λ, k, P)-plane

14 Conclusion

Departing from a recent empirical evidence on the existence of EKC (Managi (2006) and Managi and Kaneko (2009)) and the negative effects of pollution on consumption demand (Finkelstein *et al.* (2013) and Kampa and Castanas (2008)), we have reconsidered the interplay between pollution, consumption demand and the occurrence of endogenous cycles when the economy experiences an EKC. As in Heal (1982), we show that, without EKC effects, a *compensation effect* leads to persistent cycles through a Hopf bifurcation. Conversely, when the economy lies on the negative-sloped branch of the EKC, limit cycles occurs through a Hopf bifurcation only under *distaste effects*. Our paper reconciles theory and evidence, the theoretical existence of endogenous fluctuations and empirically relevant *distaste effects* under the empirically grounded assumption of EKC.

15 Appendix

Proof of Proposition 1

The consumer's Hamiltonian function writes

$$\tilde{H} \equiv e^{-\rho t} u(c, P) + \tilde{\lambda} [(r - \delta) h + w - c]$$

The first-order conditions are given by $\partial\tilde{H}/\partial\tilde{\lambda} = (r-\delta)h+w-c = \dot{h}$, $\partial\tilde{H}/\partial h = \tilde{\lambda}(r-\delta) = -\tilde{\lambda}'$, $\partial\tilde{H}/\partial c = e^{-\rho t}u_c - \tilde{\lambda} = 0$. Setting $\lambda \equiv e^{\rho t}\tilde{\lambda}$, we find $\dot{\lambda} - \rho\lambda = e^{\rho t}\tilde{\lambda}'$ and, therefore, $\lambda(r-\delta-\rho) = -\dot{\lambda}$. Finally, the budget constraint $\dot{h} = (r-\delta)h + w - c$, now binding, writes at equilibrium $\dot{k} = (r-\delta)k + w - c$. ■

Proof of Proposition 3

Focus on equation (11). $\dot{P} = 0$ implies

$$P = \frac{k^*}{a}\left[b - \gamma(\tau k^*)\tau\right] \equiv P(k^*) \tag{31}$$

with

$$P'(k^*) = \frac{1}{a}\left[b - \tau(1+\theta)\gamma(\tau k^*)\right] \tag{32}$$

Under Assumption 3.1, if $k^* < \tilde{k}$ then $P'(k) > 0$ while if $k^* > \tilde{k}$ then $P'(k^*) < 0$. ■

Proof of Proposition 4

At the steady state, $\dot{\lambda} = \dot{k} = \dot{P} = 0$. Equation (9) gives (14).

Assumption 1 implies that there exists a unique $k^* > 0$ verifying (14). Replacing this value into equation (11) gives $P^* = \left[bk^* - \gamma(\tau k^*)\tau k^*\right]/a$. Since $b > \gamma(m^*)\tau$, there exists a unique $P^* > 0$. Replacing (k^*, P^*) into equation (10), we obtain:

$$c^* = c(\lambda^*, P^*) = \rho k^* + w(k^*) > 0 \tag{33}$$

Equation (6) becomes

$$\lambda^* = u_c(c^*, P^*) = u_c\left(\rho k^* + w(k^*), \left[bk^* - \gamma(\tau k^*)\tau k^*\right]/a\right) > 0$$

■

Proof of Proposition 6

Let

$$W^* \equiv \int_0^\infty e^{-\rho t}u(c^*, P^*)\,dt = u(c^*, P^*)\int_0^\infty e^{-\rho t}\,dt = \frac{1}{\rho}u(c^*, P^*)$$

be the welfare function evaluated at the steady state. We find

$$\frac{\tau}{W^*}\frac{\partial W^*}{\partial \tau} = \frac{c^*}{u^*}\frac{\partial u}{\partial c}\frac{\tau}{c^*}\frac{\partial c^*}{\partial \tau} + \frac{P^*}{u^*}\frac{\partial u}{\partial P}\frac{\tau}{P^*}\frac{\partial P^*}{\partial \tau}$$

Using (16) and (17), we obtain

$$\frac{\tau}{W^*}\frac{\partial W^*}{\partial \tau} = \left[\varepsilon_c^*\frac{\rho + (\rho+\delta+\tau)\frac{1-\alpha}{\sigma}}{\rho + (\rho+\delta+\tau)\frac{1-\alpha}{\alpha}} + \varepsilon_P^*\pi(k^*)\right]\frac{\tau}{k^*}\frac{\partial k^*}{\partial \tau}$$

■

Proof of Proposition 7

Necessity In a three-dimensional dynamic system, we require at the bifurcation value: $\lambda_1 = ib = -\lambda_2$ with no generic restriction on λ_3 (see Bosi and Ragot (2011) or Kuznetsov (1998) among others). The characteristic polynomial of J

is given by: $P(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) = \lambda^3 - T\lambda^2 + S\lambda - D$. Using $\lambda_1 = ib = -\lambda_2$, we find $D = b^2\lambda_3$, $S = b^2$, $T = \lambda_3$. Thus, $D = ST$ and $S > 0$.

Sufficiency In the case of a three-dimensional system, one eigenvalue is always real, the others two are either real or nonreal and conjugated. Let us show that, if $D = ST$ and $S > 0$, these eigenvalues are nonreal with zero real part and, hence, a Hopf bifurcation generically occurs.

We observe that $D = ST$ implies

$$\lambda_1\lambda_2\lambda_3 = (\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)(\lambda_1 + \lambda_2 + \lambda_3)$$

or, equivalently,

$$(\lambda_1 + \lambda_2)\left[\lambda_3^2 + (\lambda_1 + \lambda_2)\lambda_3 + \lambda_1\lambda_2\right] = 0 \tag{34}$$

This equation holds if and only if $\lambda_1 + \lambda_2 = 0$ or $\lambda_3^2 + (\lambda_1 + \lambda_2)\lambda_3 + \lambda_1\lambda_2 = 0$. Solving this second-degree equation for λ_3, we find $\lambda_3 = -\lambda_1$ or $-\lambda_2$. Thus, (34) holds if and only if $\lambda_1 + \lambda_2 = 0$ or $\lambda_1 + \lambda_3 = 0$ or $\lambda_2 + \lambda_3 = 0$. Without loss of generality, let $\lambda_1 + \lambda_2 = 0$ with, generically, $\lambda_3 \neq 0$ a real eigenvalue. Since $S > 0$, we have also $\lambda_1 = -\lambda_2 \neq 0$. We obtain $T = \lambda_3 \neq 0$ and $S = D/T = \lambda_1 \lambda_2 = -\lambda_1^2 > 0$. This is possible only if λ_1 is nonreal. If λ_1 is nonreal, λ_2 is conjugated, and, since $\lambda_1 = -\lambda_2$, they have a zero real part. ■

Proof of Proposition 8

Necessity In the real case, we obtain $D = \lambda_1\lambda_2\lambda_3 < 0$, $S = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 > 0$ and $T = \lambda_1 + \lambda_2 + \lambda_3 < 0$.

Sufficiency We want to prove that, if $D, T < 0$ and $S > 0$, then $\lambda_1, \lambda_2, \lambda_3 < 0$. Notice that $D < 0$ implies $\lambda_1, \lambda_2, \lambda_3 \neq 0$.

$D < 0$ implies that at least one eigenvalue is negative. Let, without loss of generality, $\lambda_3 < 0$. Since $\lambda_3 < 0$ and $D = \lambda_1\lambda_2\lambda_3 < 0$, we have $\lambda_1\lambda_2 > 0$. Thus, there are two subcases: (1) $\lambda_1, \lambda_2 < 0$, (2) $\lambda_1, \lambda_2 > 0$. If $\lambda_1, \lambda_2 > 0$, $T < 0$ implies $\lambda_3 < -(\lambda_1 + \lambda_2)$ and, hence,

$$S = \lambda_1\lambda_2 + (\lambda_1 + \lambda_2)\lambda_3 < \lambda_1\lambda_2 - (\lambda_1 + \lambda_2)^2 = -\lambda_1^2 - \lambda_2^2 - \lambda_1\lambda_2 < 0$$

a contradiction. Then, $\lambda_1, \lambda_2 < 0$. ■

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