Can development aid help a country to escape the poverty trap?

Ngoc Sang PHAM* Thi Kim Cuong PHAM†

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Abstract

This paper fits in the debate on the relationship between development aid and economic growth. It aims to analyze the aid effectiveness for a small recipient country. This country uses capital tax and international aid to finance public investment, which may improve the capital productivity. For the case of a developing country, we analyze the effects of aid, taking account of the corruption in use of aid as well as its characteristics in terms of technology, fixed cost and efficiency of public investment. Given donors' rule, we determine conditions under which the foreign aid can generate good perspectives in the long run for the aid recipient. We also discuss the existence of the poverty trap and the conditions leading to an economic take-off as well as the existence of a middle-income trap and conditions for the economy to convergence to this trap.

Keywords: Aid effectiveness, economic growth, fluctuation, poverty trap, public investment.
JEL Classification: H50, O19, O41

*LEM, University of Lille 3, France. *Email:* pns.pham@gmail.com;

† Corresponding author. BETA, University of Strasbourg, 61 avenue de la Forêt Noire, F-67085 Strasbourg, France. *Email:* kim.pham@unistra.fr; *Tel.*: +33 (0)3 68 85 20 77; *Fax.*: +33 (0)3 68 85 20 70

1 Introduction

At the United Nations Millennium Summit in September 2000, world leaders came to an agreement on eight Millennium Development Goals (MDGs) to be achieved by 2015. Even though several MDG targets have nearly been fulfilled, progress in many areas is far from sufficient.[1] The post-2015 program follows the attainment of these MDGs and proposes new strategies for sustainable development. Developing countries need international aid to achieve their economic development. Many issues are under debate regarding the effectiveness, and the conditionality of foreign aid. Extensive empirical investigations of the effect of aid on growth show conflicting results. Aid may exert a positive and conditional effect on economic growth (Burnside and Dollar, 2000, Collier and Dollar, 2001, 2002) or no significant effects (Hansen and Tarp, 2001, Easterly *et al.* 2004). For example, in Collier and Dollar (2001, 2002), aid may promote economic growth and reduce poverty in recipient countries, but only if the quality of their political economic policy is sufficiently high. Concerning the findings in Guillaumont and Chauvet (2001), Chauvet and Guillaumont (2003, 2009), the incremental effect of aid on growth is contingent on the recipient country's economic vulnerability. They show that while the economic vulnerability is negatively associated with growth, the marginal effect of aid on growth is an increasing function of vulnerability. However, robust evidence of a significant aid effect is not found in Hansen and Tarp (2001) and Easterly *et al.* (2004).

Recently, theoretical frameworks have been developed to examine the relationship between foreign aid and economic development. Chatterjee *et al.* (2003) examine the effects of foreign transfers on economic growth of the recipient country given that foreign transfers are not subject to conditions and positively proportional to the recipient's GDP. It is shown that their effects on growth and welfare are different according the type of transfers, untied or tied to investment in public infrastructures. Chatterjee and Turnovsky (2007) follows this issue by underlying the role of endogeneity of labour supply as a crucial transmission mechanism for foreign aid. Distinguished from the previous studies, Dalgaard (2008) introduces an aid allocation policy rule consistent with empirical literature by considering a flow of aid negatively depending on the recipient's income per capita and on the donor's exogenous degree of inequality aversion. The author considers an OLG growth model applied to the case of a recipient country where government investments are fully financed by aid flows. It is shown that an exogenous increase in foreign aid leads to higher steady-state income and a higher social welfare in the recipient country. Besides, the degree of inequality aversion of the donor determines the characteristics of the transitional dynamics of income per capita.

The goal of this paper is to analyze the effectiveness of development aid for a small recipient country under the poverty trap. Simply put, the main question is to examine how development aid can help a recipient country to escape the poverty trap and then to study the conditions necessary to an economic take-off. To do so, we consider a growth model where public investment, partially financed by aid, may improve the capital productivity. Aid flows depend on the donor rules and the recipient's need which is represented by a low initial endowment. This paper formulates aid flows taking into account the donors' rule. Aid flows are decreasing with the recipient's initial capital and limited by an upper threshold. This implies that a country would no longer receive aid if it was on the growth path in the long run. For the case of a developing country, we consider the possibility of corruption (inefficiency) in use of aid and examine its impact on the aid effectiveness. Other characteristics, such as importance of fixed cost of public investment, its efficiency degree and level of technology, are also taken into account in the analysis.

The main results can be summarized as follows: Firstly, if the initial political and eco-

[1] See the report 2015: http://www.un.org/fr/millenniumgoals/reports/2015

nomic circumstances of the recipient are to a sufficient standard, the country does not need international aid to achieve its development. This result is trivial and corroborates to that of a standard AK model. We analyze then different effects of aid for the case where the recipient economy is under the poverty trap without aid. We conclude that the effects of aid in the long run are complex and conditional to recipient country's characteristics. Aid may help the recipient country to reach economic growth, to surpass its poverty trap or to reduce this threshold. This is conditional to the degree of corruption in use of aid, the technology, the fixed cost and efficiency of public investment, as well as to the donors' rule. Then, our second result shows that if the recipient country has a high quality of political and economic circumstances, international aid may help it to reach economic growth whatever its initial capital. Consequently, there will exist a period where this economy no longer need international aid to stimulate its economic development. Thirdly, by analyzing the case with a low quality of circumstances where the corruption is high and government effort in public investment is low, we show that aid does not affect the threshold for an economic take-off. However, if aid is sufficiently generous, the recipient country may surpass this threshold while it is impossible without international intervention.

The complex results are found in intermediate qualities of circumstances: high corruption and high government effort in public investment (intermediate circumstances 1), low corruption and low government effort (intermediate circumstances 2). It is hard to conclude which situation is better for aid effectiveness. In the first one, aid reduces the threshold for an economic take-off and increases significantly the probability to escape the poverty trap compared to the low circumstances. In the second one, the probability to escape the poverty trap as well as the probability to collapse is lower. In particular, the economy may converge to a middle-income trap or to fluctuate around it.

The remainder of the paper is organized as follows: Section 2 characterizes the case of a small recipient country. Section 3 presents the poverty trap without international aid. In Section 4, we emphasize the role of international aid by analyzing the conditions for the effectiveness of aid. Section 5 concludes. Appendices gather technical proofs.

2 A small economy with foreign aid

This section will consider an economy with infinitively-lived identical individuals. The population size is constant over time and normalized to unity. Labor is exogenous and inelastic. The representative firm produces a single traded commodity, which can be used for either consumption or investment. The Government uses capital tax and international aid to finance public investment which can improve the capital productivity. The waste in spending of aid is considered by the presence of unproductive aid. The latter has no direct effect neither on the household's welfare nor on the production process. The fraction of wasteful aid may reflect the degree of corruption in the recipient government.

2.1 Foreign aid and public investment

Empirical literature on aid effectiveness has a large consensus on the criteria of aid receiving. Countries with high need and high potential marginal effect of aid in terms of economic growth should receive a high amount of aid. In this sense, apart from the initial poverty, Burnside and Dollar (2000), Collier and Dollar, (2001, 2002) focus on the institutional and quality of policy criteria. Following these authors, a country with high institutional quality is more able to use aid in an efficient way. Guillaumont and Chauvet (2001) focus on a fairness argument when they underline the recipient's economic vulnerability, while Guillaumont, McGillivray and Pham (2016), Guillaumont, McGillivray and Wagner (2016) also consider the lack of human capital as a determinant criterion.

We consider a function of aid flows as follows:

$$a_t = (\bar{a} - \phi k_t)^+ \equiv \max\{\bar{a} - \phi k_t, 0\} \tag{1}$$

where $\bar{a} > 0$ is the maximal aid flow that the recipient country can receive. Parameter $\phi > 0$, independent of the per capita capital, may be referred to all exogenous rules imposed by the donor. The couple $(\bar{a}, \phi)$ are taken as given by the recipient country. They represent aid conditionalities. All other characteristics of the recipient country being unchanged, a decrease in ϕ and/or an increase in $\bar{a}$ lead(s) to a higher aid flow. We might interpret ϕ as an indicator of economic vulnerability arisen in Guillaumont and Chauvet (2001) following that more aid should be given to countries with high economic vulnerability (low ϕ) since in these countries aid would be more efficient. This argument also fits in a philosophy of fairness which proposes that aid should compensate the recipient country for its vulnerable initial situation (in macroeconomics conditions or lack of human capital) so that all countries can obtain the same initial opportunities.

Equation (1) also means that the higher the capital and the lower the country is in its need, then the lower the aid flow received. This assumption corroborates with empirical analyzes on the allocation rules. Similar assumption may be found in Carter (2014) and Dalgaard (2008).[2] The form of equation (1) implies that until a certain level of capital, the recipient country no longer receives aid.

The recipient country uses aid and tax on capital to finance public investment, which improves the private capital productivity. As some spending of aid is wasted in most developing countries, there is a significant part of unproductive activity, noted as a_t^U. This is potentially explained by the corruption, administrative fees, etc. Then, the attribution of aid may be written as:

$$a_t = a_t^i + a_t^u \tag{2}$$

If we consider a fixed fraction of aid for each activity, we can rewrite equation (2) as follows:

$$a_t = \alpha_i a_t + \alpha_u a_t \tag{3}$$

with $\alpha_u = 1 - \alpha_i$. Parameter $\alpha_i < 1$ reflects the inefficiency in the use of aid, caused by corruption.

Let us denote B_t as the public investment financed by tax on capital and by aid, B_t may be written as:

$$B_t = T_{t-1} + a_t^i \tag{4}$$

where T_{t-1} is the tax at period $t-1$, $T_{t-1} = \tau K_t$. The positive effect of foreign aid on public investment is an obvious finding in empirical studies (Khan and Hoshino, 1992, Franco-Rodriguez *et al.*, 1998, Ouattara, 2006, Feeny and McGillivray (2010), etc.). For example, using a sample of recipient countries over the period 1980-2000, Ouattara (2006) shows that

[2] Carter (2014) considers that aid flow received by country i is positively correlated with country performance rating as underlined in Collier and Collar (2001,2002) (with index Country Policy and Institutional Assessment) and is negatively associated with income per capita. Dalgaard (2008) assumes that per capita flow of aid at time t is also a reversed function of income of per capita at $t-1$, $a_t = \theta y_{t-1}^{\lambda}, \quad \theta > 0, \lambda < 0$. In this aid function, λ reflects the degree of inequality aversion on the part of the donor. Parameter θ represents exogenous determinants of aid.

aid flows are associated with increases in public investment, but do not reduce tax revenue. Feeny and McGillivray (2010) analyze the interaction between aid flows and different categories of public expenditures and show that for Papua New Guinea, aid flow also increases public investment. However, for this aid recipient, aid flows negatively affect revenue collections.

2.2 Production

The representative firm produces a single commodity with an AK technology. Private capital is referred to a broad concept including for example human capital. Given A, the classical AK form implies that the production function is homogeneous of degree 1 with respect to private capital. In this sense, the marginal product of capital may be interpreted as the total factor productivity.

We assume that technological level is not exogenous, and it depends on public investment B_t, interpreted for example as government effort in financing public investment. Let us consider the following production function:

$$Y_t = F(B_t, K_t) = A\left[1 + (\sigma B_t - b)^+\right] K_t \tag{5}$$

where $A \in (0, \infty)$ is considered as the autonomous and exogenous technology while $A(\sigma B_t - b)^+ = A\max(\sigma B_t - b, 0)$ represents the endogenous technology depending on public investment B_t.[3]

We remark that if $B_t \leq b/\sigma$, this implies that $A(\sigma B_t - b)^+ = 0$, it it features the classical AK model. This means that the positive effect of public investment in technology is observed only from the level b/σ. Parameter $\sigma \in (0, \infty)$ is exogenous and measures the extent to which the public investment translates into technology and the production process. In this sense, σ may reflect the efficiency of public investment and b/σ is considered as a threshold from which the public investment improves the technology. This threshold is decreasing with σ.

At each period t, given government effort B_t, the representative firm maximizes its profit:

$$P_{ft}: \qquad \pi_t \equiv \max_{K_t \geq 0} F(B_t, K_t) - r_t K_t \tag{6}$$

It is straightforward to obtain r_t and π_t for a competitive economy:

$$\begin{aligned} r_t &= A\left[1 + \left(\sigma B_t - b\right)^+\right] \\ \pi_t &= 0. \end{aligned} \tag{7}$$

2.3 Consumption

Let us consider the optimization problem of the representative consumer. She maximizes her intertemporal utility by choosing consumption and capital sequences (c_t, k_t):

$$P_c: \quad \max_{(c_t, k_t)_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \beta^t U(c_t) \tag{8}$$

$$\text{s.t: } c_t + k_{t+1} + T_t \leq (1 - \delta)k_t + r_t k_t + \pi_t \tag{9}$$

[3]Dalgaard (2008) considers a production function in the spirit of Barro (2008) with public spending as a production factor and entirely financed by international aid. The production function is supposed to be homogeneous of degree 1 with respect to private capital and public spending. Given that aid flow is decreasing with income, it converges to a null value in the long run and the economy converges to a steady state where income per capita is constant. Dalgaard (2008) showed that the donors' rule affect only the transitional dynamics of the economy and the steady state income.

where β is the rate of time preference and $U(c_t)$ the consumer's instantaneous utility function depending on consumption c_t. T_t is the tax at t, r_t is the capital return while π_t is the firm's profit at date t. For the sake of simplicity, we assume that the consumer knows that $T_t = \tau k_{t+1}$ and instantaneous utility function is logarithmic, $U(c_t) = \ln c_t$.

According to Lemma 7 in Appendix 6, we establish the relationship between k_{t+1} and k_t

$$k_{t+1} = \beta \frac{1 - \delta + r_t}{1 + \tau} k_t. \tag{10}$$

By the concavity of the utility function, this solution is unique.

2.4 Intertemporal equilibrium

Definition 1. *(Intertemporal equilibrium) Given capital tax rate τ, a list $(r_t, c_t, k_t, K_t, a_t)$ is an intertemporal equilibrium if*

1. *(c_t, k_t) is a solution of the problem P_c, given a_t^i, r_t, π_t.*
2. *(K_t) is a solution of the problem P_{ft}, given B_t and r_t.*
3. *Market clearing conditions are satisfied:*

$$K_t = k_t \tag{11}$$

$$c_t + k_{t+1} + T_t = (1 - \delta)k_t + Y_t. \tag{12}$$

4. *The Government budget is balanced: $T_t = \tau k_{t+1}$.*
5. *$a_t = \max\{\bar{a} - \phi k_t, 0\}$ and $a_t^i = \alpha_i a_t$.*

Combined with (7), the dynamics of capital stock (equation (10)) may be rewritten as follows:

$$k_{t+1} = G(k_t) \equiv f(k_t)k_t \tag{13}$$

$$\text{where} \quad f(k_t) \equiv \beta \frac{1 - \delta + A\Big[1 + \big(\sigma(\tau k_t + \alpha_i(\bar{a} - \phi k_t)^+) - b\big)^+\Big]}{1 + \tau} \tag{14}$$

is the gross growth rate of capital stock. This growth rate depends not only on the level of capital stock but also on other fundamentals.

The following sections analyze the dynamics of k_t over time and the effects of international aid on the long run situation of this economy. Notice that the analysis is far from trivial since the function G is non linear and may not be monotonic. Its properties will be presented in Section 4.1.1.

Before studying the dynamics of capital stock, we introduce the notions of growth and collapse.

Definition 2. .

1. *The economy collapses if $\lim_{t\to\infty} k_t = 0$. It grows without bound if $\lim_{t\to\infty} k_t = \infty$.*
2. *A value k is called a poverty trap if for any $k_0 < k$, we have $\lim_{t\to\infty} k_t = 0$ and for any $k_0 > k$, we have $\lim_{t\to\infty} k_t = \infty$.*

Remark 1. *Let us denote:*

$$r_a \equiv \beta \frac{1-\delta+A}{1+\tau}. \tag{15}$$

We observe that the gross growth rate $f(k_t)$ is higher than $\geq r_a$ for any t. Therefore, it is easy to see that when $r_a > 1$, the economy will grow without bounds.[4]

It should be noticed that when the autonomous technology A is sufficiently high, and higher than $\frac{1+\tau}{\beta} + \delta - 1$ (corresponding to $r_a > 1$), then it may generate growth whatever the levels of other factors such as: initial capital, efficiency of public investment, foreign aid. In this case, the country is not eligible to receive aid. Since our purpose is to look at the impacts of public investment and foreign aid, from now on, we will work under the following assumption.

Assumption 1 (Assumption for the whole paper)**.** $r_a < 1$.[5]

This assumption implies that when autonomous and exogenous technology A is lower than $\frac{1+\tau}{\beta} + \delta - 1$, this economy would never reach economic growth in the long run without government effort B_t in financing public investment. Public investment B_t is then required to raise the technology stock, as necessary for a positive economic growth in the long run.

3 Poverty trap without foreign aid

This section considers an economy which does not receive foreign aid, public investment B_t is entirely financed by tax revenue. We will analyze the dynamics of capital in the long run. From equation (13), we have:

$$k_{t+1} = r_b(k_t)k_t \tag{16}$$

$$\text{where } r_b(k_t) \equiv \beta \frac{1-\delta+A\Big[1+\big(\sigma\tau k_t - b\big)^+\Big]}{1+\tau} \tag{17}$$

Proposition 1. *(Poverty trap) Consider an economy with a low level of autonomous technology, and without foreign aid. The public investment in technology is entirely financed by tax revenue and the dynamics of capital is characterized by (16). There exists a steady-state:*

$$k^{**} = \frac{b+D}{\tau\sigma} \tag{18}$$

$$\textit{where} \quad D = \frac{1}{A}\Big(\frac{1+\tau}{\beta} - (1-\delta)\Big) - 1. \tag{19}$$

We have then three cases:

1. *If $r_b(k_0) > 1$, i.e., $(\sigma\tau k_0 - b)^+ > D$, then (k_t) increases and the economy grows without bounds.*
2. *If $r_b(k_0) < 1$, i.e., $(\sigma\tau k_0 - b)^+ < D$, then (k_t) decreases and the economy collapses.*
3. *If $r_b(k_0) = 1$, i.e., $(\sigma\tau k_0 - b)^+ = D$, then $k_t = k_0$ for any t.*

[4] In this case, $f(k_t) \geq r_a > 1$, then $k_{t+1} > k_t$ for any t.
[5] We ignore the case $r_a = 1$ because this case is not generic.

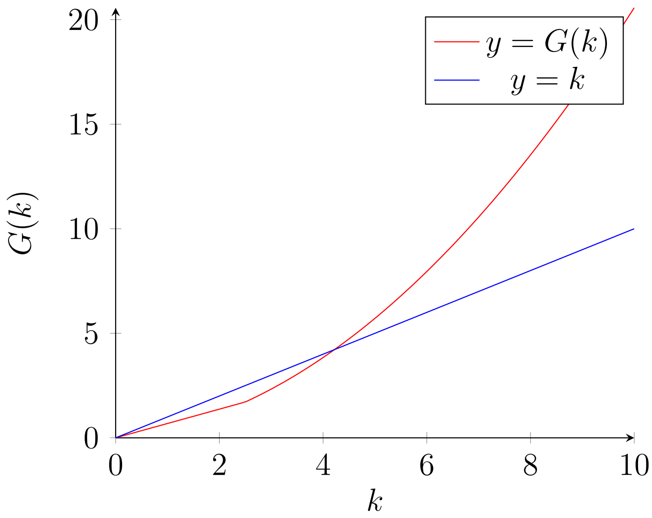

Figure 1: *Poverty trap without foreign aid.* Parameters in function $G(k)$ are $\beta = 0.8; \delta = 0.2; A = 0.5; \tau = 0.4; \sigma = 2$; $\alpha_i = 0.15$; $\bar{a} = 0$; $\phi = 2$; $b = 2$; verifying condition $r_a < 1$.

This result is obtained from the analysis of the dynamics of capital stock given by equation (16) and the fact that $r_b(k_t)$ is an increasing function. We observe that $D > 0$ as $r_a < 1$ (Assumption 1) and $\frac{b+D}{\sigma}$ may be interpreted as the threshold from which public investment τk_0 generates the economic growth. Indeed, as $r_b(k_0) > 1$ is equivalent to $(\sigma\tau k_0 - b)^+ > D$, this is equivalent to $\tau k_0 > \frac{b+D}{\sigma}$. Our result indicates that if the public investment in technology (without aid) is high enough, the economy will grow without bounds.

In another way, we consider b as a fixed cost of public investment. If the return of public investment ($\sigma B_t \equiv \sigma\tau k_0$) is less than b, public investment τk_0 does not make any change on the total factor productivity. Following this interpretation, $b + D$ can be viewed as the threshold so that if the return of public investment in R&D (σB_t) is less than this level, there is no growth of capital stock, i.e. $k_{t+1} < k_t$ for all t.

Figure 1 illustrates Proposition 1. The point of interaction between the convex curve and the first bisector corresponds to the unstable steady-state k^{**} which is considered as a poverty trap for this economy. For all initial capital k_o higher than k^{**} (corresponding to $(\sigma\tau k_0 - b)^+ > D$), the economy will grow without bounds while it collapses if the initial capital is lower than k^{**}. It should be noticed that k^{**} is decreasing in A, σ while it is increasing in b. This means that an economy with a high autonomous technology A, a high efficiency σ and a low fixed cost b in public investment has more chances to surpass its poverty trap as the condition $(\sigma\tau k_0 - b)^+ > D$ is more likely to be satisfied.

4 Role of international aid

Proposition 1 shows that this economy collapses without international aid if the initial capital and the return of public investment in technology ($\sigma\tau k_t$) are low. For that reason, in this section, we will work under the following assumption:

Assumption 2 (Assumption for the whole Section 4)**.**

$$(\sigma\tau k_0 - b)^+ < D \tag{20}$$

D given by equation (17)

We then investigate the role of international aid for this economy which is under its poverty trap. In other words, given the pessimist initial situation of the recipient country, we examine how international aid could generate positive perspectives in the long run.

4.1 Properties of function $G(k)$

Before providing the dynamics of capital stock, it is useful to underline some properties of function $f(k)$ and $G(k)$. Let us recall that $k_{t+1} = G(k_t)$, and

$$G(k) \equiv f(k)k = \beta \frac{1 - \delta + A\Big[1 + \big(\sigma(\tau k + \alpha_i(\bar{a} - \phi k)^+) - b\big)^+\Big]}{1 + \tau} k \tag{21}$$

4.1.1 Monotonicity of function $G(k)$

Lemma 1. .

1. *The function $f_1(k) \equiv (k - a)^+$ is increasing in k.*
2. *The function $f_2(k) \equiv \tau k + \alpha_i(\bar{a} - \phi k)^+$ is increasing on $[0, \infty]$ if $\tau \geq \alpha_i\phi$. When $\tau < \alpha_i\phi$, the function f_2 is decreasing on $[0, \bar{a}/\phi]$ and increasing on $[\bar{a}/\phi, \infty]$.*
3. *$f_2(k) \equiv \tau k + \alpha_i(\bar{a} - \phi k)^+ \geq \bar{a} \min(\alpha_i, \tau/\phi)$.*
4. *$f(k_t) \geq \frac{\beta}{1+\tau}\Big[1 - \delta + A\Big(1 + \big(\sigma\bar{a}\min(\alpha_i, \tau/\phi) - b\big)^+\Big)\Big]$.*

We now study the monotonicity of function G. Let us denote

$$x_1 \equiv \bar{a}/\phi \tag{22}$$

$$x_2 \equiv \frac{\sigma\alpha_i\bar{a} - b}{\sigma(\alpha_i\phi - \tau)} \quad \text{i.e. } x_2 \text{ such that} \quad \sigma f_2(x_2) - b = 0 \tag{23}$$

$$x_3 \equiv \frac{1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b)}{2A\sigma(\alpha_i\phi - \tau)} \quad \text{i.e. } x_3 \text{ such that } f_3'(x_3) = 0. \tag{24}$$

where

$$f_3(x) \equiv \beta \frac{1 - \delta + A\Big[1 + \big(\sigma(\tau x + \alpha_i(\bar{a} - \phi x)) - b\big)\Big]}{1 + \tau} x \tag{17}$$

Lemma 2. .

1. *G is increasing on $[x_1, \infty)$.*
2. *Assume that $x_2 > 0$. We have G increasing on $[x_2, \infty)$.*

Consequently, G is increasing on $[\min(x_1, x_2), \infty)$.

Lemma 3. .

The function G is increasing on $[0, \infty)$ if one of the following conditions is satisfied.

1. *$\tau \geq \alpha_i\phi$.*

2. $\tau < \alpha_i\phi$ *and* $x_2 < 0$.

3. $\tau < \alpha_i\phi$ *and* $x_2 > 0$, *and* $x_3 > \min(x_1, x_2)$.

Lemma 4. .

Assume that $\tau < \alpha_i\phi$ *and* $x_2 > 0$, *and* $x_3 < \min(x_1, x_2)$. *Then G is increasing on* $[0, x_3]$, *decreasing on* $[x_3, \min(x_1, x_2)]$, *and increasing on* $[\min(x_1, x_2), \infty)$.

4.1.2 Steady states

Steady-states are characterized by $G(k) = k$. We will then find all fixed points, i.e. positive solutions of the equation $G(k) = k$. We see that $G(k) = k$ if and only if $f(k) = 1$ which is equivalent to:

$$f_2(k) := \tau k + \alpha_i(\bar{a} - \phi k)^+ = \frac{D+b}{\sigma}. \tag{25}$$

where D is defined in equation (17). The following result is obtained using properties of Lemma 3 and 4.

Lemma 5. .

1. *If* $\sigma\bar{a}\min(\alpha_i, \tau/\phi) > D + b$, *then there is no fixed point.*

2. *If* $\sigma\bar{a}\min(\alpha_i, \tau/\phi) \leq D + b$.

   (a) *If* $\tau > \alpha_i\phi$, *this implies* $\sigma\bar{a}\alpha_i \leq D + b$, *then*

      i. *the unique fixed point is* $k^* := \frac{\frac{D+b}{\sigma} - \bar{a}\alpha_i}{\tau - \alpha_i\phi} \in (0, \bar{a}/\phi)$ *when* $\sigma\bar{a}\tau/\phi > D + b$.[6]

      ii. *the unique fixed point is* $k^{**} := \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty)$ *when* $\sigma\bar{a}\tau/\phi < D + b$.[7]

   (b) *If* $\tau < \alpha_i\phi$, *this implies* $\sigma\bar{a}\tau/\phi \leq D + b$, *then*

      i. *If* $\sigma\bar{a}\alpha_i < D + b$, *then the unique fixed point is* $k^{**} := \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty)$.

      ii. *If* $\sigma\bar{a}\alpha_i > D + b$, *then there are two fixed points* $k^* := \frac{\bar{a}\alpha_i - \frac{D+b}{\sigma}}{\alpha_i\phi - \tau} \in (0, \bar{a}/\phi)$ *and* $k^{**} := \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty)$.[8]

In the following sections, we will present different effects of aid on recipient perspectives in the long run. The effects of aid may depend on the initial circumstances in the aid recipient (corruption degree, efficiency in management of public investment, etc.) and on the generosity of donors. Hence, the development aid may help the recipient to obtain a positive growth rate for all initial capital, to converge to a steady state with a constant income or to fluctuate around it as well as to surpass it. It is also possible that aid does exert no effect on economic growth and does not conduce the recipient country to more positive perspectives.

[6]This condition guaranties that $k^* < \bar{a}/\phi$
[7]This condition guaranties that $k^* > \bar{a}/\phi$
[8]Condition $\sigma\bar{a}\alpha_i > D + b$ is to ensure that $k^* > 0$.

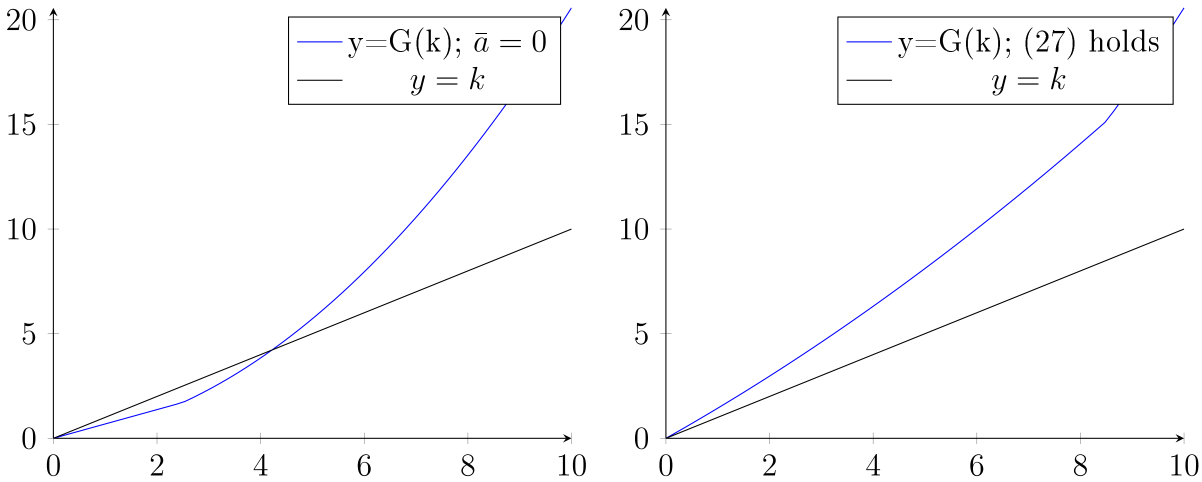

Figure 2: *Growth without bounds.* Parameters in function $G(k)$ are $\beta = 0.8; \tau = 0.4; \delta = 0.2; A = 0.4; \sigma = 2; \alpha_i = 0.15; b = 2, \phi = 2$ verifying conditions $r_a < 1$, $\alpha_i < \tau/\phi$. On the left: $\bar{a} = 0$, and condition (27) does not hold. On the right: $\bar{a} = 17$, and condition (27) holds.

4.2 Growth with generous exogenous aid and low corruption

The aid donors rule is characterized by the pair $(\bar{a}, \phi)$. First, we consider the case with very generous exogenous aid where the maximum level of aid $\bar{a}$ is very high and ϕ is low.

Proposition 2 (Growth)**.** *Considering an aid recipient under poverty trap without aid, characterized by condition (20). The dynamics of capital with foreign aid is characterized by (13). If*

$$r_d \equiv \frac{\beta}{1+\tau}\Big[1 - \delta + A\Big(1 + \big(\sigma\bar{a}\min(\alpha_i, \tau/\phi) - b\big)^+\Big)\Big] > 1 \tag{26}$$

$$\iff \sigma\bar{a}\min(\alpha_i, \tau/\phi) > D + b. \tag{27}$$

1. *the economy will grow without bounds for any level of initial capital k_0.*
2. *$a_t = (\bar{a} - \phi k_t)^+$ decreases in t. Consequently, there exists a time T such that aid amounts $a_t = 0$ for any $t \geq T$.*

This result is obtained from the analysis of $f(k_t)$ in the dynamics of capital defined by equation (13). Following point 4 in Lemma 1, we observe that if condition (27) is satisfied, then $f(k_t)$ and $G(k_t)$ will be increasing in k_t for all k_t. Condition (27) may be written as follows

$$\sigma\bar{a}\frac{\tau}{\phi} > D + b \quad \text{and} \quad \sigma\alpha_i\bar{a} > D + b, \tag{28}$$

where D is given by equation (17). The first condition in (28) means that the foreign aid is generous (high $\bar{a}$ and low ϕ) and/or the efficiency σ is quite high while the second condition may be associated to a low corruption in use of aid. Indeed, this second condition implies that $\alpha_u = 1 - \alpha_i < 1 - \dfrac{D+b}{\sigma\bar{a}}$. In other words, given aid flows, condition (28) is more likely to be satisfied if the recipient country has a high quality of political and economic circumstances, decisive for the effectiveness of aid. Put simply, fixed cost b and degree of corruption in the use of aid should be low and autonomous technology A should be sufficiently high.

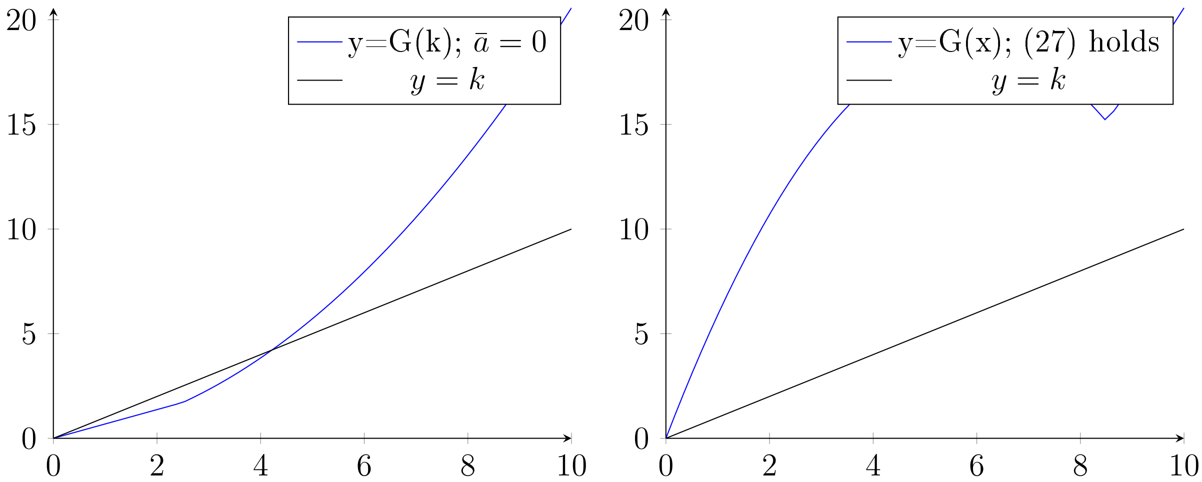

Figure 3: *Growth without bounds.* Parameters in function $G(k)$ are $\beta = 0.8; \tau = 0.4; \delta = 0.2; A = 0.4; \sigma = 2; \alpha_i = 0.8; \bar{a} = 17, b = 2, \phi = 2$ verifying conditions $r_a < 1$, $\alpha_i > \tau/\phi$. On the left: $\bar{a} = 0$, and condition (27) does not hold. On the right: $\bar{a} = 17$, and condition (27) holds.

Proposition 2 gives us the best and ideal scenario for the recipient country but also for donors. Whatever the initial capital, generous aid combined with high quality of initial circumstances could help the recipient country to grow without bounds in the long run. Figures 2 and 3 illustrate this Proposition under condition (27). Figure 2 corresponds to the case $\alpha_i < \tau/\phi$ and Figure 3 to the case $\alpha_i > \tau/\phi$. We observe that, without exogenous aid (corresponding to $\bar{a} = 0$), the dynamics of capital correspond to that in Figure 1 and there is one poverty trap. Thanks to development aid, the dynamics of capital change and are represented by the curve above the first bisector. And we observe that the poverty trap disappears.

However, aid is always bounded due to the budget constraint from donors and being subject to conditionalities. In addition, in developing countries, the corruption in use of aid as well as high fixed cost and low technology are often a recurrent problem. For these reasons we are now interested in the case where condition (27) (condition (28)) is not verified. In the following sections, we consider the case:

$$\sigma \bar{a} \min(\alpha_i, \tau/\phi) < D + b. \tag{29}$$

From this condition, we can identify three possibilities:

$$\tau/\phi < \frac{D+b}{\sigma \bar{a}} \text{ and } \alpha_i < \frac{D+b}{\sigma \bar{a}} \tag{30}$$

$$\alpha_i < \frac{D+b}{\sigma \bar{a}} < \tau/\phi \tag{31}$$

$$\tau/\phi < \frac{D+b}{\sigma \bar{a}} < \alpha_i \tag{32}$$

If we focus on the degree of corruption in the use of aid (α_i) and the Government effort in financing public investment (τ) considering constant other parameters (aid and efficiency in public investment, autonomous technology, etc.), equation (30) represents a low quality of circumstances with a high degree of corruption (low α_i) and a low Government effort (low τ). Equation (31) characterizes a high degree of corruption (low α_i) and a high Government effort (high τ) while equation (32) characterizes a low degree of corruption (high α_i) and low Government effort (low τ). These both situations are represented as intermediate circumstances. We note that high circumstances with a low degree of corruption and a high Government effort are already analyzed in Proposition 2 and Figures 2,3 where condition (27) holds.

4.3 Poverty trap: growth or collapse?

Proposition 3 (High corruption and poverty trap). *Considering an aid recipient under poverty trap without aid, characterized by condition (20). The dynamics of capital with foreign aid is characterized by (13) and one of three conditions in Lemma 3 holds. Given aid flows $(\bar{a}, \phi)$, We then have two cases:*

1. *(Low circumstances) If the recipient country has a low quality of circumstances with a high degree of corruption and a low Government effort in financing public investment, so that condition (30) holds then there exists one poverty trap k^{**}*

$$k^{**} = \frac{D+b}{\tau\sigma} \tag{33}$$

2. *(Intermediate circumstances 1) If the recipient country has an intermediate quality of circumstances with a high degree of corruption and a high Government effort in financing public investment, so that condition (31) holds then there exists another poverty trap k^*,*

$$k^* = \frac{\frac{D+b}{\sigma} - \bar{a}\alpha_i}{\tau - \alpha_i \phi}. \tag{34}$$

   and $k^* < k^{**}$.

Then, in both cases

- *If $f(k_0) > 1$, i.e., $\big(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b\big)^+ > D$ then (k_t) increases and the economy grows without bounds. Consequently, there exists a time T such that aid amount $a_t = 0$ for any $t \geq T$.*
- *If $f(k_0) < 1$, i.e., $\big(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b\big)^+ < D$ then (k_t) decreases and the economy collapses. Consequently, there exists a time T_1 such that aid amount $a_t > 0$ for any $t \geq T_1$.*
- *If $f(k_0) = 1$, then $k_t = k_0$ for any t.*

This Proposition gives two initial circumstances with high corruption in the use of aid. Parameters condition (30) reflects a bad situation which is opposite to that given by equation (27) or (28) in Proposition 2. It means that the recipient country should suffer a high corruption (low α_i) and a low government effort in financing public investment.[9]. We remark that the poverty trap is always k^{**}, like in the case without international aid. However, this result does not mean that development aid does not exert any effect on the aid recipient. Indeed, it should be noticed that we are under condition $\sigma(\tau k_0 - b)^+ < D$, i.e. the country is under the poverty trap without development aid. With the same poverty trap, development aid could impede the collapse and help the recipient country to escape poverty if aid flow

[9]Other characteristics such as autonomous technology, efficiency in public investment, or fixed cost b may be identical or worse than in the high circumstances.

is sufficiently high so that condition $\big(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b\big)^+ > D$ for an economic take-off is satisfied. In other words, the development aid might help the aid recipient to surpass its poverty trap while this is impossible without foreign assistance. This result may be considered as a theoretical illustration for the intuition evoked in Kraay and Raddatz (2007) using a Solow model.[10]

We remark that in the intermediate circumstances 1 characterized by a higher government effort, the poverty trap is lower, $k^* < k^{**}$. Simply put, the same aid flows and corruption degree in the use of aid may generate a lower threshold for economic-takeoff. Proposition 3 underlines the fact that given aid flows, the effectiveness of aid is conditional to the initial situations in the recipient country.[11] Besides, analyzing k^*, we can observe that a higher value of $\bar{a}$ and/or lower value of ϕ will reduce the value of k^*.[12] This implies that the more generous donors are, the more likely that the recipient country escapes the poverty trap as the threshold for economic take-off becomes lower. In this case, it is more likely to satisfy the condition $k_0 > k^*$ for getting out of the poverty trap. In other words, the condition $\big(\sigma(\tau k_0 + \alpha_i(\bar{a} - \phi k_0)^+) - b\big)^+ > D$ is more likely to be satisfied for the case with k^* than with k^{**}.

4.4 Middle-income trap: stability or fluctuations?

Let us now consider the last case verifying condition (29). It corresponds to equation (32), which characterizes a low degree of corruption (high α_i) and a high government effort (low τ).

Assumption 3 (Assumptions for the whole Section 4.4)**.** .

1. $\tau/\phi < \frac{D+b}{\sigma\bar{a}} < \alpha_i$.
2. $x_2 > 0$ *and* $x_3 < \min(x_1, x_2)$.[13]

where x_1, x_2, x_3 are given by (22), (23) and (24).

4.4.1 Middle-income trap

Proposition 4 (Low corruption and middle-income trap)**.** *Considering an aid recipient under the poverty trap without aid, characterized by (20) and the conditions in Assumption 3. The dynamics of capital with foreign aid are characterized by (13). Given aid flows* $(\bar{a}, \phi)$*:*

- *(Intermediate circumstances 2) If the recipient country has an intermediate quality of circumstances with a low degree of corruption and a low government effort in financing*

[10]In a Solow model with two exogenous saving rates, there are two steady states which are locally stable. Kraay and Raddatz (2007) indicate that in such a model, if the saving rate is low, foreign aid could help the recipient to accumulate capital. Saving rate might jump to the higher level, and then, the economy would converge to to high steady state.

[11]Other characteristics may be maintained unchanged, or take the values such that condition (31) holds.

[12]As indicated previously, donors' rule is exogenous and represented by the parameter ϕ in function of aid (1). This donors' rule representing aid conditionalities may be determined by macroeconomics conditions of the recipient. For example, referring to Guillaumont and Chauvet (2001, 2003), we may interpret ϕ as the initial situation in recipient country in terms of economic vulnerability. A low value of ϕ may be associated to a high economic vulnerability. Therefore, country with low ϕ will receive more aid given all others variables including initial poverty (low k_o). Guillaumont and Chauvet (2001, 2003) recommend that countries with high economic vulnerability should receive more aid than others as aid is more efficient in these countries.

[13]This condition implies the non-monotonicity of the dynamics of capital.

public investment, so that condition (32) holds, then there exist two steady-states k^*, k^{**} and $k^* < k^{**}$

$$\textit{low steady-state:} \quad k^* = \frac{\bar{a}\alpha_i - \frac{D+b}{\sigma}}{\alpha_i\phi - \tau} \in (0, \bar{a}/\phi) \tag{35}$$

$$\textit{high steady-state:} \quad k^{**} = \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty). \tag{36}$$

Proposition 4 is obtained from Assumption 3, Lemma 4 and Lemma 5. Indeed, according to Lemma 4, the function G is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$, and increasing on $[\min(x_1, x_2), \infty)$, where x_3 is the local maximum of the function G. According to point (2.b) of Lemma 5, there are two steady states $k^* := \frac{\bar{a}\alpha_i - \frac{D+b}{\sigma}}{\alpha_i\phi - \tau} \in (0, \bar{a}/\phi)$ and $k^{**} := \frac{D+b}{\tau\sigma} \in (\bar{a}/\phi, \infty)$.

We notice that the degree of corruption in use of aid and the initial conditions are decisive for the effectiveness of aid. We compare Proposition 3 with Proposition 4. The first one corresponds to high corruption while the second one corresponds to low corruption. On the one hand, given the sam flow of aid, the intermediate circumstances described in Proposition condition 4 give aid effects more satisfying than the bad circumstances. Indeed, in the intermediate circumstances 2, for all initial capital lower than k^{**}, the economy no longer collapses, it may converge to the middle-income trap k^* if this one is stable.[14] Aid may not help to generate growth, but may conduce the economy to a stable steady state where income per capita is constant.

On the other hand, comparing point 2 in Proposition 3 with Proposition 4, both describe the intermediate circumstances. For the first one, the corruption is high, but the government effort in financing public investment is also high while for the second one, the corruption is low, but the government effort is also low. Both situations are almost equivalent, these conduce two results which are not comparable. We recall that all other factors being equal, the case with high corruption gives us a low steady-state k^* while the case with low corruption gives us two steady-states k^* and k^{**}. The difficulty of comparing both intermediate situations are justified by two facts. First, if in the first case, k^* is unstable and considered as a poverty trap, in the second case, it may be stable and considered as a middle-income trap. Hence, for all initial capital lower than k^{**}, if the initial situation verifies (31) with high corruption, then the economy collapses while for the initial situation verifying (32) with low corruption, there may not be this risk as there exists a middle-income trap. Second, if the recipient economy begins with an initial capital between k^* and k^{**}, the intermediate situation (31) is better in the sense that an economic-takeoff is possible as k^* is unstable in this case. If the recipient economy begins with an initial capital lower than k^*, then the intermediate situation (32) is better in the sense that the economy does not collapse but may converge to a middle-income trap where the economic growth is null.

4.4.2 Stability of the middle-income trap

We analyze now the conditions for stability of the low steady state k^* in the intermediate circumstances corresponding to Assumption 3.

Proposition 5 (Stability of low steady state)**.** *Under Assumption 3, we have*

1. *If parameters are so that $\sigma\bar{a}\alpha_i < D + b + \frac{1}{A}\left(\frac{1+\tau}{\beta}\right)$,[15] we have: if $k_0 \in (0, k^*)$, then $k_t \in (0, k^*)$ for any t. Moreover, we have $\lim_{t\to\infty} k_t = k^*$.*

[14] Its characteristics will be analyzed in the next section.

[15] This condition corresponds to $x_3 > k^*$, i.e., $A(1 + \sigma\alpha_i\bar{a} - b) > 2\frac{1+\tau}{\beta} - (1 - \delta)$

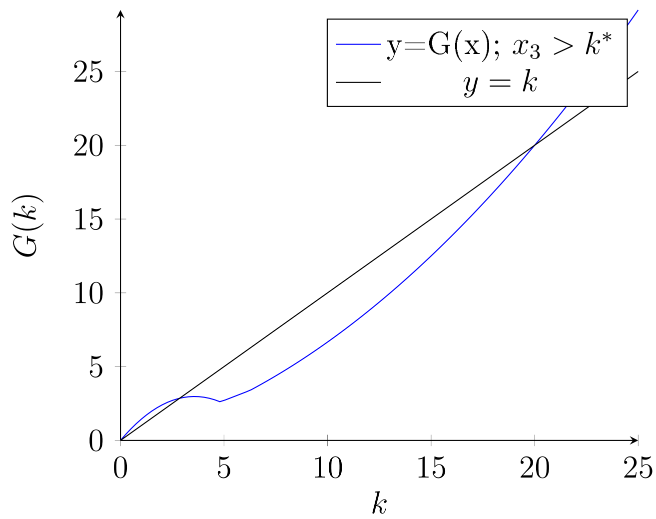

Figure 4: *Stability of low steady-state.* Parameters in function $G(k)$ are $\beta = 0.5, \tau = 0.2; \delta = 0.8, A = 0.5, \sigma = 0.8, \alpha_i = 0.8; \bar{a} = 10, b = 1, \phi = 2$, verifying condition (32) and $x_3 > k^*$. We have $\lim_{t\to\infty} k_t = k^*$ for any $k_0 \in (0, k^*)$.

2. *If parameters are so that* $\sigma\bar{a}\alpha_i > D + b + \frac{1}{A}\left(\frac{1+\tau}{\beta}\right)$,[16] *the steady-state* k^* *is locally stable*[17] *if and only if*

$$\sigma\bar{a}\alpha_i < D + b + \frac{2}{A}\left(\frac{1+\tau}{\beta}\right) \tag{37}$$

We notice that condition (37) is equivalent to

$$3\frac{1+\tau}{\beta} - (1-\delta) > A(1 + \sigma\alpha_i\bar{a} - b). \tag{38}$$

This one is certainly satisfied if the fixed cost b is high enough so that $b \geq 1 + \sigma\alpha_i\bar{a}$. The above figure illustrates the global stability of the low steady state k^* when $\sigma\bar{a}\alpha_i < D + b + \frac{1}{A}\left(\frac{1+\tau}{\beta}\right)$. On the graph, this condition is represented by the fact that the local maximum x_3 of the function $G(k)$ is higher than k^*.

4.4.3 Fluctuations

It should be noticed that when the low steady-state is not locally stable, there will be other possibilities for this economy. In the following section, we consider the case where condition (37) is not satisfied. It is shown that fluctuations around this steady-state may occur. Besides, there is also a probability to obtain a "lucky growth".

Lemma 6. *We assume conditions in Assumption 3 hold. We also assume that* $x_3 < k^*$.

1. *If*

$$\sigma\bar{a}\alpha_i > D + b + \frac{2}{A}\left(\frac{1+\tau}{\beta}\right) \tag{39}$$

[16] This condition is equivalent to $x_3 < k^*$, i.e., $A(1 + \sigma\alpha_i\bar{a} - b) < 2\frac{1+\tau}{\beta} - (1-\delta)$.

[17] It means that there exists $\epsilon > 0$ such that $\lim_{t\to\infty} k_t = k^*$ for any $k_0 \in (k^* - \epsilon, k^* + \epsilon)$.

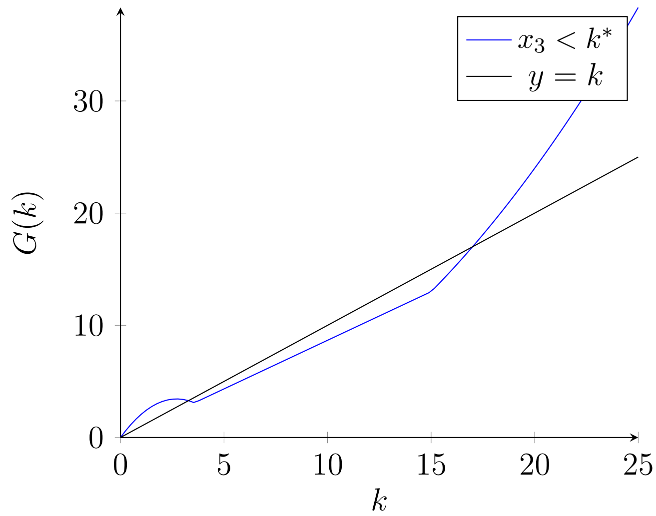

Figure 5: *Local stability of low steady-state.* Parameters in function $G(k)$ are $\beta = 0.8, \tau = 0.2; \delta = 0.8, A = 0.5, \sigma = 1, \alpha_i = 0.8; \bar{a} = 10, b = 3, \phi = 2$, verifying condition (32) and $x_3 < k^*$.

then there exist $y_1 \in (x_3, x^*)$ and $y_2 > 0$ in $(0, x_2)$ such that

$$y_1 \neq y_2, \quad f_3(y_1) = y_2, \quad f_3(y_2) = y_1. \tag{40}$$

2. *If we add assumption that $G(y_1) < x_2$, then the above y_1, y_2 satisfy*

$$y_1 \neq y_2, \quad G(y_1) = y_2, \quad G(y_2) = y_1. \tag{41}$$

Let y_1, y_2 determined in point 2 of Lemma 6. Let us denote

$$\mathcal{F}_0 \equiv \{y_1, y_2\} \tag{42}$$

$$\mathcal{F}_{t+1} \equiv G^{-1}(\mathcal{F}_0), \quad \forall t \geq 0 \tag{43}$$

$$\mathcal{F} \equiv \cup_{t \geq 0} \mathcal{F}_t. \tag{44}$$

The following result is a direct consequence of Lemma 6.

Proposition 6 (Fluctuation around the low steady-state). *We assume that conditions in Assumption 3 hold. Assume also that conditions in point 2 of Lemma 6 hold. We have that: if $k_0 \in \mathcal{F}$, then there exist t_0 such that such that $k_{2t} = y_1$, $k_{2t+1} = y_2$ for any $t \geq t_0$.*

Figure 6 illustrates the fluctuation of the recipient economy around the low steady state following the description in Lemma 6. There exists an interval of capital so that for all initial capital belonging to this interval, there is neither possibility for the recipient country to converge to the middle-income trap, nor the possibility to reach an economic take-off. This result is obtained under condition (39). We observe that this condition is equivalent to $3\frac{1+\tau}{\beta} - (1-\delta) < A(1 + \sigma\alpha_i\bar{a} - b)$. It holds if and only if the two following conditions are satisfied:

$$\text{the fixed cost is not very high: } 1 + \sigma\alpha_i\bar{a} > b \tag{45}$$

$$\text{the productivity is not very low: } A > \frac{3\frac{1+\tau}{\beta} - (1-\delta)}{1 + \sigma\alpha_i\bar{a} - b} \tag{46}$$

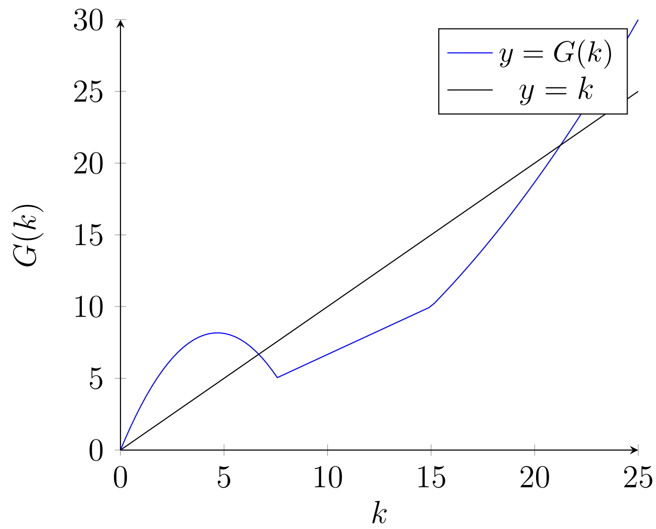

Figure 6: *Fluctuation around the low steady state.* Parameters in function $G(k)$ are $\beta = 0.8, \tau = 0.2; \delta = 0.2, A = 0.2, \sigma = 2, \alpha_i = 0.8; \bar{a} = 17, b = 6, \phi = 2$, verifying condition ((39)).

Simply put, comparing this condition (39) to (37), we remark that the economy fluctuates around the middle-income trap, rather than converge to it, if the degree of corruption is lower (higher α_i) and/or the fixed cost b if lower and/or the level of autonomous technology A is higher, for all other parameters being unchanged. It should be noticed that these characteristics always verify condition 32 for the existence of a low and a high steady-state. They are not sufficiently good to verify condition 28 for a growth without bounds as shown in Proposition 2.

4.4.4 Lucky growth vs middle-income trap

In the case $x_3 < k^*$, we can consider two subcases: $G(x_3) \geq k^{**} = G(k^{**})$ corresponding to a strong dynamics of capital and $G(x_3) < k^{**} = G(k^{**})$ a lower dynamics of capital.

Let us denote

$$U_0(x^{**}) := \{x \in [0, x^{**}] : G(x) > x^{**}\} \tag{47}$$

$$U_{t+1}(x^{**}) := G^{-1}(U_t(x^{**})), \quad \forall t \geq 0 \tag{48}$$

$$U(x^{**}) := \cup_{t \geq 0} U_t(x^{**}). \tag{49}$$

Note that $x^* \notin U(x^{**})$, $x_3 \in U(x^{**})$ and $x^* > x_3$.

Proposition 7 (Lucky growth). *We assume that conditions in Assumption 3 hold. Assume that $G(x_3) > x^{**}$. Then we have: $U(x^{**}) \neq \emptyset$ and if there exists $t_0 \geq 0$ such that $k_{t_0} \in U(x^{**})$, then $\lim_{t \to \infty} k_t = \infty$.*

Proof. It is clear. □

Under condition 39, the low steady-state k^* is not stable. Figure 7 illustrates two subcases : $G(x_3) \geq k^{**} = G(k^{**})$ and $G(x_3) < k^{**} = G(k^{**})$. The former corresponding to a strong dynamics of capital (high curve), which may be explained by a higher level of autonomous technology A and/or a higher efficiency σ in management of public investment, compared to the second case. In other words, given the same degree of corruption in the use of aid and the same government effort in financing public investment, the effectiveness of aid (characterized

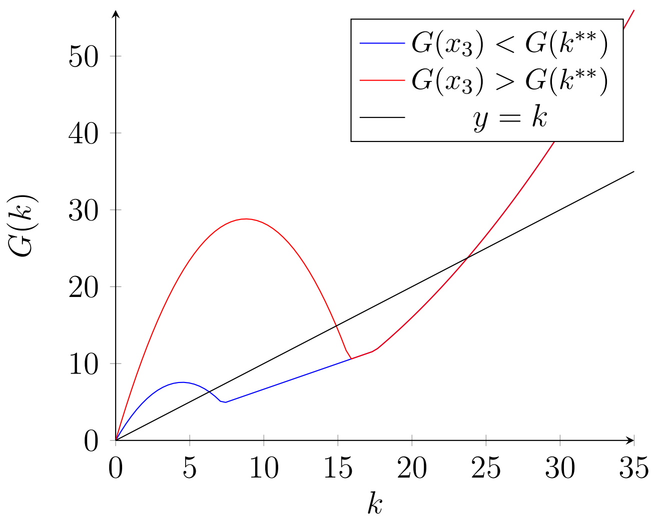

Figure 7: *Lucky growth vs. middle-income trap,.* For $G(x_3) < G(k^{**})$, parameters in function G are $\beta = 0.8, \tau = 0.2; \delta = 0.2, A = 0.2, \sigma = 2, \alpha_i = 0.8; \bar{a} = 17, b = 7, \phi = 2$. For $G(x_3) > G(k^{**})$, $\bar{a} = 32$, other parameters unchanged.

by $\bar{a}$ and ϕ) may be very different following the level of technology and the degree of efficiency of public investment. The same flows of aid may generate a "lucky growth" under condition $G(x_3) \geq k^{**} = G(k^{**})$ or allow the recipient to converge to a middle-income trap (rather than collapse) under condition $G(x_3) < k^{**} = G(k^{**})$. Given a level of initial capital around the value of x_3, foreign aid may help the recipient to surpass the poverty trap k^{**} and reach an economic take-off. Otherwise, the recipient may converge to the middle-income trap with a null growth.

It should be noticed that these characteristics in degree of corruption, government effort, autonomous technology and efficiency of public investment, for a "lucky growth" do not verify condition (28) for a growth without condition on the initial value of capital. This means that there exist only an interval of initial capital around the value of x_3 so that for all k_o belonging to this interval, its value for the following period $G(k_o)$ thanks to development aid, is very high and surpasses the poverty trap k^{**}, then the recipient economy may reach the lucky growth.

5 Conclusions

This paper examines the effectiveness of aid in a recipient country with initial conditions which are not favourable to achieving economic development. Aid flows depend on donors' rule and initial poverty of the recipient. We suppose that the recipient country uses aid to finance its investment in technology which allows to improve the capital productivity.

Considering the case where the recipient is under the poverty trap, , we show that the effectiveness of aid are conditional to the initial situations in terms of autonomous technology, government effort in financing public investment, fixed cost and efficiency of public investment. We discuss the results following different characteristics of the recipient country.

Our first result shows that in case of a very high quality of political and economic circumstances, the development aid may help the recipient country to reach economic growth whatever the initial capital. Consequently, there will exist a period where this economy no longer needs international aid to stimulate its development. Otherwise, in the case of a low quality of political and economic circumstances, the same flows of aid would generate limited results, in particular, the threshold for an economic take-off does not change.

Secondly, in the intermediate circumstances when controlling the degree of corruption and the government effort in public investment, we underline the significant impact of other factors on the effectiveness of aid. We show that from an initial situation under the poverty trap, development aid may generate a middle-income trap to which the recipient economy converge rather collapse.

Finally, our analysis concerning the middle-income trap gives different properties. Following conditions on efficiency in management of public investment and the level of technology, development aid may conduce the economy to a situation with fluctuations around this trap without convergence, or to an economic take-off.

This paper fits in the debate on the effectiveness of aid in terms of economic growth and household's welfare. One of the research perspectives consists to adopt this analysis framework and its results as a starting point for an empirical investigation.

6 Appendix: Formal proofs for Section 2

Euler equation for the program P_c

Lemma 7. *Consider the optimal growth problem*

$$\max_{(c_t,s_t)_t} \sum_{t=0}^{\infty} \beta^t \ln(c_t) \tag{50}$$

$$c_t + s_{t+1} \leq A_t s_t \tag{51}$$

$$c_t, s_t \geq 0. \tag{52}$$

The unique solution of this problem is given by $s_{t+1} = A_t s_t$ *for any* $t \geq 0$.

Proof. Indeed, the Euler condition $c_{t+1} = \beta A_{t+1} c_t$ jointly with the budget constraint becomes $s_{t+2} - \beta A_{t+1} s_{t+1} = A_{t+1}(s_{t+1} - \beta A_t s_t)$. Thus, a solution is given by $s_{t+1} = A_t s_t$. It is easy to check the transversality condition $\lim_{t\to\infty} \beta^t u'(c_t) s_{t+1} = 0$.

By the concavity of the utility function, the solution is unique.

□

7 Formal proofs for Section 4

Proof of Lemma 1. The three first points are obvious. Let us prove the last point. We consider 2 cases. If $k \geq \bar{a}/\phi$, it is easy to see that $f_2(k) \geq \tau k \geq \tau\bar{a}/\phi \geq \bar{a}\min(\alpha_i, \tau/\phi)$.

If $k \leq \bar{a}/\phi$, then $f_2(k) = \alpha_i \bar{a} + (\tau - \alpha_i\phi)k$.

When $\tau - \alpha_i\phi \geq 0$, we have $f_2(k) \geq \alpha_i\bar{a}$.

When $\tau - \alpha_i\phi \leq 0$, we have $f_2(k) \geq \alpha_i\bar{a} + (\tau - \alpha_i\phi)\bar{a}/\phi = \alpha_i\bar{a}/\phi$. □

Proof of Lemma 2. 1. G is increasing on $[x_1, \infty)$ because when $x \geq x_1$, we have

$$G(x) = \beta\frac{1 - \delta + A\Big(1 + (\sigma\tau x - b)^+\Big)}{1 + \tau}x.$$

2. If $x_1 < x_2$, it is trivial that G is increasing on $[x_2, \infty)$ because it is increasing on $[x_1, \infty)$.

   We now consider the case where $x_1 > x_2$. Let x and y such that $x \geq y \geq x_2$. We have to prove that $G(x) \geq G(y)$. It is easy to see that $G(x) \geq G(y)$ when $x, y \in [x_2, x_1]$ or $x, y \in [x_1, \infty)$. We now assume that $x \geq x_1 \geq y$. In this case, we have

$$
\begin{aligned}
G(x) &= \beta \frac{1-\delta+A\Big(1+(\sigma\tau x - b)^+\Big)}{1+\tau} x \geq \beta \frac{1-\delta+A}{1+\tau} x \\
G(y) &= \beta \frac{1-\delta+A\Big(1+(\sigma\alpha_i\bar{a} - b - \sigma(\alpha_i\phi-\tau)y)^+\Big)}{1+\tau} y \\
&= \beta \frac{1-\delta+A}{1+\tau} y
\end{aligned}
$$

   where the last equality is from the fact that $y \geq x_2$. So, it is clear that $G(x) \geq G(y)$.

□

Proof of Lemma 3. 1. When $\tau \geq \alpha_i\phi$, by using point 3 of Lemma 1, we get that G is increasing on $[0, \infty)$.

2. When $\tau < \alpha_i\phi$ and $x_2 < 0$. We consider two cases.

   If $x \leq \bar{a}/\phi$, then $\big(\sigma(\tau x + \alpha_i(\bar{a} - \phi x)^+) - b\big)^+ = (\sigma\alpha_i\bar{a} - b - \sigma(\alpha_i\phi - \tau)x)^+ = 0$ (because $\sigma\alpha_i\bar{a} - b < 0$). So, in this case

$$G(x) = \beta \frac{1-\delta+A}{1+\tau} x.$$

   When $x \geq \bar{a}/\phi$, we have

$$G(x) = \beta \frac{1-\delta+A\Big(1+(\sigma\tau x - b)^+\Big)}{1+\tau} x.$$

   It is easy to see that G is increasing on $[0, \infty)$.

3. We now consider the last case where $\tau < \alpha_i\phi$ and $x_2 > 0$, and $x_3 > \min(x_1, x_2)$.

   First, according to Lemma 2, we observe that G is increasing on $[\min(x_1, x_2), \infty)$.

   Second, we observe that G is increasing on $0, x_3)$. Since $x_3 > \min(x_1, x_2)$, we obtain that G is increasing on $[0, \infty)$.

□

Proof of Lemma 4. According to Lemma 2, we observe that G is increasing on $[\min(x_1, x_2), \infty)$. We now consider G on $[0, \min(x_1, x_2)]$. Let $x \in [0, \min(x_1, x_2)]$. We have

$$G(x) = f_3(x) = \beta \frac{1-\delta+A\big(1+\sigma\alpha_i\bar{a} - b - \sigma(\alpha_i\phi-\tau)x\big)}{1+\tau}. \tag{53}$$

By definition of x_3, we have $f_3'(x_3) \geq 0$ if and only if $x \leq x_3$. Therefore, G is increasing on $[0, x_3]$, decreasing on $[x_3, \min(x_1, x_2)]$. □

7.1 Proofs for Section 4.4

Proof Proposition 5. Part 1. First, we need the following result.

Lemma 8. *Assume that* $\bar{a}\alpha_i > \frac{D+b}{\sigma} > \bar{a}\tau/\phi$ *and* $x_3 < x_2$.

If $x_3 > k^*$, *then* $G(x_3) < x_3$. *And therefore,* $G(x_3) < x_3 < x_2 < k^* = G(k^*)$. *In this case, we have* $G(x) < k^*$ *for any* $x < k^*$.

Proof. It is easy to see that if $x_3 > k^*$, then $G(x_3) < x_3$.

Now, let $x \leq k^*$. If $x \geq k^*$, then $G(x) \leq x \leq k^*$.

If $x \leq k^*$, then we have

$$G(x) \leq \max_{x \leq k^*} G(x) \leq G(x_3) < x_3 \leq k^*. \tag{54}$$

□

We now come back to the proof of Proposition 5.

If $k_0 < k^*$, according to Lemma 8, we have $k_1 = G(k_0) < k^*$. By induction, we have $k_t < k^*$ for any t.

We now prove that $\lim_{t\to\infty} k_t = k^{**}$ for any $k_0 \in (0, k^*)$.

Since G is increasing on $[0, x_3]$, we have $\lim_{t\to\infty} k_t = k^{**}$ for any $k_0 \in (0, x_3]$.

Now consider $k_0 \in (x_3, x_2]$. We see that $k_1 = G(k_0) \leq \max_{x\in[0,x_2]} G(x) = G(x_3) < x_3$. Therefore $k_1 < x_3$, and so $\lim_{t\to\infty} k_t = k^{**}$.

If $k_0 \in [x_2, \bar{a}/\phi]$, we have $k_1 = G(k_0) = \frac{\beta(1-\delta+A)}{1+\tau}k_0$. Since $\frac{\beta(1-\delta+A)}{1+\tau} < 1$, there exists t_0 such that $k_{t_0} < x_2$. Thus $\lim_{t\to\infty} k_t = k^{**}$.

If $k_0 \in [\bar{a}/\phi, k^*]$, we have $G(k_0) < k_0$ which means that $f(k_0) < 1$. Combining with $k_1 = f(k_0)k_0$, there exists t_1 such that $k_1 < \bar{a}/\phi$. This implies that $\lim_{t\to\infty} k_t = k^{**}$.

Part 2. Recall that

$$G(k) = f_3(k) \equiv \frac{\beta}{1+\tau}\Big[1-\delta+A\Big(1+\sigma\alpha_i\bar{a}-\sigma(\alpha_i\phi-\tau)k-b\Big)\Big]k \tag{55}$$

$$= \frac{\beta}{1+\tau}\Big[1-\delta+A\big(1+\sigma\alpha_i\bar{a}-b\big)-A\sigma(\alpha_i\phi-\tau)k\Big]k \tag{56}$$

$$G'(k) = f_3'(k) = \frac{\beta}{1+\tau}\Big[1-\delta+A\big(1+\sigma\alpha_i\bar{a}-b\big)-2A\sigma(\alpha_i\phi-\tau)k\Big]. \tag{57}$$

We have

$$G'(k^*) = \frac{\beta}{1+\tau}\Big[1-\delta+A\big(1+\sigma\alpha_i\bar{a}-b\big)-2A\sigma(\alpha_i\phi-\tau)\frac{\bar{a}\alpha_i-\frac{B+b}{\sigma}}{\alpha_i\phi-\tau}\Big] \tag{58}$$

$$= \frac{\beta}{1+\tau}\Big[1-\delta+A\big(1+\sigma\alpha_i\bar{a}-b\big)-2A\sigma\bar{a}\alpha_i+2A(B+b)\Big] \tag{59}$$

$$= \frac{\beta}{1+\tau}\Big[1-\delta+A(1+b+2B-\sigma\bar{a}\alpha_i)\Big]. \tag{60}$$

It is well-known that k^* is locally stable if and only if $\|G'(k^*)\| < 1$.[18] Since $x_3 < k^*$, have have $G'(k) < 0$. So, k^* is locally stable if and only if $G'(k) > -1$ which is equivalent to

$$3\frac{1+\tau}{\beta} - (1-\delta) + A(b-1-\sigma\alpha_i\bar{a}) > 0. \tag{61}$$

□

[18]See Bosi et Ragot (2011) among others.

Proof of Lemma 6. We will find $y_1, y_2 > 0$ such that (40).

Let us denote $n = 1 - \delta + A(1 + \sigma\alpha_i\bar{a} - b)$ and $m = A\sigma(\alpha_i\phi - \tau)$. y_1, y_2 must satisfy

$$\frac{\beta}{1+\tau}(n - my_1)y_1 = y_2, \quad \frac{\beta}{1+\tau}(n - my_2)y_2 = y_1. \tag{62}$$

Since $y_1 \neq y_2$, we have

$$\frac{\beta}{1+\tau}(n - m(y_1 + y_2)) = -1. \tag{63}$$

So, we obtain

$$H(y_1) \equiv \frac{\beta}{1+\tau}(n - my_1)y_1 + y_1 - \frac{1}{m}\Big(n + \frac{1+\tau}{\beta}\Big) = 0 \tag{64}$$

We have $H(y_1) < 0$. We also see that $H(k^*) > 0$ if condition (39) is satisfied.

Under condition (39), there exists y_1 such that $H(y_1) = 0$. Therefore, y_1 and $y_2 = f_3(y_1)$ satisfy (40). □

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